DESIGN OF TRAMPOLINE SPRING

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MECHANICAL - 39 – B

OBJECTIVE:

Our objective is to design a trampoline spring for a medium sized 3 m diameter trampoline having **36 springs**. After searching the internet, we found that trampoline springs of our requirement are made of **Galvanized Steel**, have **length of 22 cm** and are **preloaded to 25 cm**. The maximum **vertical deflection** of the trampoline for very heavy weights can be **20 cm**. The springs experience a frequency of **3 Hz** (worst case scenario).

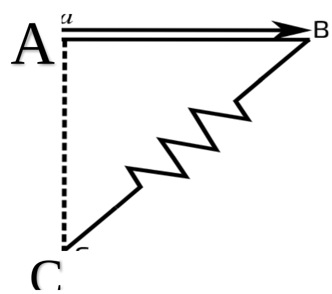
INTRODUCTION

Trampoline is a device consisting of a piece of taut, strong fabric stretched between a steel frames using many coiled springs. People bounce on trampolines for recreational and competitive purposes. The fabric that users bounce on (commonly known as the "bounce mat" or "trampoline bed") is not elastic itself; the elasticity is provided by the springs that connect it to the frame, which store potential energy. So basically a trampoline is a power transmission machines. We'll further do our analysis on trampoline springs.

Design of Trampoline Spring:

Worst Case scenario 3 heavy and fat persons, each having weight 70 kg jump on the trampoline.
So Total Applied Weight = 70 + 70 + 70 = 210 kg
Since there are a total of 36 springs distributed evenly across the mat, so Applied Force F on each spring will be, $F_{max} = \frac{210}{36}$ x 9.81

$$\mathbf{F_{max} = 57.166\ N}$$



Hence Maximum Force that a Single Spring will experience will be
F = 57.225 N
Now,
As we know that,
Fully Stretched Length of Spring = 32 cm
UnStretched Length of Spring =22 cm
So,
Max. Deflection in Spring will be,
x_{max}= Fully Stretched Length of Spring - UnStretched Length of Spring

$$x_{max} = 32-22 = 10 \text{ cm} = 0.1 \text{ m}$$

Finding Spring Constant k, using $F_{max} = kx_{max}$

$k = \frac{57.225}{0.1} = \mathbf{572.25\ N/m}$

We also know that Length of Spring when Fixed in Trampoline = 25 cm
So,
Minimum Deflection,

x_{min} = Length of Spring when Fixed in Trampoline - UnStretched Length of Spring

x_{min} = 25 – 22 = 3 cm (0.03 m)

Now finding minimum Applied Force on each Spring,
Using, $F_{min} = kx_{min}$, As k is constant (572.25), So, F_{min} = 572.25 x 0.03

$$F_{min} = \mathbf{17.1675\ N}$$

THE 1ST ITERATION:

DATA

F_{max} = 57.225N
F_{min} = 17.1675N
k = Spring Constant = 571.666 Nm
d = Spring Wire Diameter = 1.6 mm
C = Spring Index = 7
G = Shear Modulus = 69 GPa
E = Elastic Modulus = 200 GPa
ρ = Density = 8050 kg/m^3
γ = Specific Weight = 78890 N/m^3
D = Mean Diameter of Spring Coil = C × d = (7) × (1.6 mm) = 11.2 mm

$K_B = \frac{4C+2}{4C-3}$

$K_B = \frac{4(7)+2}{4(7)-3}$

$K_B = 1.2$

The preferred range of preload stress can be expressed in terms of the uncorrected torsional stress τ_i as,

$$\tau_i = \frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right) \text{ psi}$$

where C is the spring index.

Hence, finding the minimum preferred preload stress:

$\tau_i = \frac{33500}{\exp(0.105 * 7)} - 1000\left(4 - \frac{7-3}{6.5}\right)$ =13382.6788 psi or 92.270 MPa

This minimum torsional load is compared with the initial preload stress applied. So, calculating the initial preload stress (here the preload is 17.1675 N)

$$\tau_i = \frac{K_B 8 F_i D}{\pi d^3}$$

Putting values of preload (approx. 18 N), D, d and K_B
But,
Hence,

$\tau_B = 1.2 \text{ x } \frac{8 \times 18 \times 0.0112}{\pi \times 0.0016\char`^3}$

τ_B = **150.4012 MPa (PASS)**

FATIGUE STRENGTH

From the table 10-4, Shigley textbook use the values of A and m for the 302 Stainless wire in diameter range of 0.3-2.5. (All tables used are referenced at the end)

$A = 1867$ MPa.mm^m $\qquad\qquad$ $m = 0.146$

The Tensile Strength is given by

$$S_{ut} = \frac{A}{d^m} = \frac{1867}{1.6^{0.146}}$$

$$\mathbf{S_{ut} = 1743.1822\ MPa}$$

For yield strength S_{sy}; we use the table 10-7 from Shigley textbook. The percentage of tensile strength in body torsion for austenitic stainless steel i.e. 35%.

$$S_{sy} = (0.35) \times (S_{ut})$$

$$\mathbf{S_{sy} = 610.113\ MPa}$$

Using Zimmerli Data for an Unpeened Spring; the endurance strength components are S_{sa}= 241 MPa & S_{sm} = 379 MPa

For Fatigue Strength using the ASME Elliptic criterion we have.

$$\left(\frac{S_{sa}}{S_{se}}\right)^2 + \left(\frac{S_{sm}}{S_{sy}}\right)^2 = 1$$

$$\left(\frac{241}{S_{se}}\right)^2 + \left(\frac{379}{610.113}\right)^2 = 1$$

Solving we get, $\mathbf{S_{se} = 307.5333\ MPa}$

NUMBER OF ACTIVE HELICAL TURNS ($15 \leq N_b \leq 30$)

The Spring constant is given as;

$$k = \frac{d^4 G}{8 N_b D^3}$$

$$572.25 = \frac{(0.0016)^4 \times (69 \times 10^9)}{8 \times N_b \times (0.0112)^3}$$

Solving we get;

$$N_b = 70.3071 \text{ turns}$$

Active helical turns for a spring is given as;

$$N_a = N_b + \frac{G}{E}$$

$$N_a = 70.3071 + \frac{69}{200}$$

$$\mathbf{N_a = 70.6921\ turns\ (FAIL)}$$

NATURAL FREQUENCY (f_n>15f (f_n>45))

The weight of the active part of spring is;

$$W = \frac{\pi^2 d^2 D N_a \gamma}{4}$$

$$W = \frac{\pi^2 \times (0.0016)^2 \times (0.0112) \times (70.6921) \times (78890)}{4}$$

$$W = 0.3949 \text{ N}$$

The Natural Frequency is given as; $f = \frac{1}{2}\sqrt{\frac{kg}{W}}$

$$f = \frac{1}{2}\sqrt{\frac{(571.666)\times(9.8)}{(0.3947)}}$$

f = 59.5690 Hz (PASS)

SHEAR STRESSES

The F_a (Amplitude of alternating component of force) and F_m (Midrange steady component of force) are given as;

$$F_a = \frac{F_{max} - F_{min}}{2} \qquad F_m = \frac{F_{max} + F_{min}}{2}$$

$$F_a = \frac{(57.225) - (17.1675)}{2} \qquad F_m = \frac{(57.225) + (17.1675)}{2}$$

F_a = 20.02875 N **F_m = 37.19625 N**

The Bergsträβer factor was calculated as; $K_B = 1.2$

The τ_a (Shear Stress amplitude) and τ_m (Midrange Shear Stress) are given;

$$\tau_a = \frac{K_B 8 F_a D}{\pi d^3} \qquad \tau_m = \frac{K_B 8 F_m D}{\pi d^3}$$

$$\tau_a = \frac{(1.2)\times 8 \times (20.02875) \times (0.0112)}{\pi \times (0.0016)^3} \qquad \tau_m = \frac{(1.2)\times 8 \times (37.19625) \times (0.0112)}{\pi \times (0.0016)^3}$$

τ_a = 167.3529 MPa **τ_m = 310.798 MPa**

FACTOR OF SAFETY (1.2≤n_f≤1.4)

The factor of safety (n) is calculated using the ASME-Elliptic criterion;

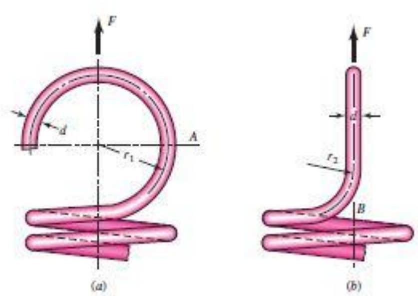
$$\left(\frac{\tau_a}{S_{se}}\right)^2 + \left(\frac{\tau_m}{S_{sy}}\right)^2 = \frac{1}{n^2}$$

$$\left(\frac{167.3529}{307.5333}\right)^2 + \left(\frac{310.798}{610.113}\right)^2 = \frac{1}{n^2}$$

Solving we get; **n = 1.3415 (PASS)**

DESIGN OF HOOK ENDS

Since we are dealing with an extension spring, therefore, hooks are an important part of them. Even if the spring undergoes extension or it is subjected to non-static loading, stresses are also produced in the hooks which might compromise the structural integrity of our design.



STRESSES:

At point A: (Here the hook end could fail due to bending and axial stress)

$r_1 = \frac{D}{2} = \frac{11.2}{2} = 5.6$ mm

$C_1 = \frac{2r_1}{d} = \frac{2\times 5.6}{1.6} = 7$

$K_A = \frac{4C1^2 - C1 - 1}{4C1(C1-1)} = \frac{4\times -7-1}{4\times 7\times(7-1)} = 1.119$

$\sigma_a = F_a\ (K_A(\frac{16D}{\pi d^3}) + \frac{4}{\pi d^2})$

$= 20.02875 \text{ x } (1.119(\frac{16\times 11.2}{\pi(1.6)^3}) + \frac{4}{\pi(1.6)^2})$

= 322.075 MPa

$\sigma_m = F_m\ (K_A(\frac{16D}{\pi d^3}) + \frac{4}{\pi d^2})$

$= 37.19625\ (1.12(\frac{16\times 11.2}{\pi(1.6)^3}) + \frac{4}{\pi(1.6)^2})$

= 598.138 MPa

At point B: (Here the hook end could fail due to torsional stress)

Assuming r_2= 5.5 mm

$C_2 = \frac{2\times r2}{d} = \frac{2\times 5.5}{1.6} = 6.875$

$K_B = \frac{4C2-1}{4C2-4} = \frac{4\times 6.875-1}{4\times 6.875-4} = 1.12765$

$\tau_m = K_B \frac{8FmD}{\pi d^3}$

$= 1.12765(\frac{8\times 37.19625\times 11.2}{\pi(1.6)^3})$

=**292.05972 MPa**

$\tau_a = K_B \frac{8FaD}{\pi d^3}$

$= 1.12765 \frac{8\times 20.02875\times 11.2}{\pi(1.6)^3}$

= **157.2629 MPa**

STRENGTH

S_{ut} = 1743.128 MPa

Then from table 10-7 of Shigley Textbook

S_y = 0.55 x 1743.128 = 958.72 MPa (for normal stresses)
S_{sy} = 0.30 x 1743.128 = 522.954 MPa (for shear stresses)
Then by ASME Elliptic Criteria:

In Torsion: By Zimmerli Data,

$$(\frac{Sa}{Sse})^2 + (\frac{Sm}{Ssy})^2 = 1$$

By simplification:

$$S_{se} = \sqrt{\frac{Sa^2}{1-(\frac{Sm}{Ssy})^2}} = \sqrt{\frac{241^2}{1-(\frac{379}{522.765})^2}} = 349.765 \text{ Mpa}$$

$S_e = \frac{Se}{0.577} = \frac{349.765}{0.577} = 606.178$ Mpa

FATIGUE FAILURE (FOS) (1.2≤n≤1.4)

Due to normal stresses at point A:

$$(\frac{n\times\tau a}{Sse})^2 + (\frac{n\times\tau m}{Ssy})^2 = 1 \quad \text{(ASME Elliptic)}$$

$$\left(\frac{157.26}{349.765}\right)^2 + \left(\frac{292.0597}{522.954}\right)^2 = \frac{1}{n^2}$$

n(shear) = 1.3947 (PASS)

Due to normal stresses at point A:

$$(\frac{n \times \sigma a}{Se})^2 + (\frac{n \times \sigma m}{Sy})^2 = 1$$

$$(\frac{322.075}{606.178})^2 + (\frac{598.138}{985.72})^2 = \frac{1}{n^2}$$

n(normal) = 1.239 (PASS)

Since the number of coils exceeded the optimum limit, we have to design again. Now instead of performing these tedious calculations every time, we programmed it on excel sheets and performed iterations by altering d and C until all criterion were met. (Red represents failure)

d	1.6	1.8	1.8	1.4	1.1	2.5	2.0	2.0
$t_{i(min)(applied)}$	87.4258	77.4474	68.5692	87.42584	47.49295	53.68578	60.68157	53.68578
$t_{(min)(critical)}$	150.401	132.69	146.615	196.4427	505.0868	90.51056	130.0769	141.4228
C	7	8	9	7	12	11	10	11
D	11.2	14.4	16.2	9.8	13.2	27.5	20	22
S_{ut}	1743.18	1713.46	1713.46	1777.5	1841.2	1622.788	1687.306	1687.306
S_{sy}	610.113	599.711	599.711	622.125	644.42	567.9759	590.5572	590.5572
S_{se}	307.533	310.97	310.97	303.9035	297.9835	323.5745	314.2522	314.2522
S_{su}	1167.93	1148.02	1148.02	1190.925	1233.604	1087.268	1130.495	1130.495
k	572.25	572.25	572.25	572.25	572.25	572.25	572.25	572.25
N_b	70.3071	52.9877	37.215	61.51871	9.594498	28.3097	30.14417	22.64776
N_a	70.6921	53.3727	37.6	61.90371	9.979498	28.6947	30.52917	23.03276
W	0.39494	0.48521	0.38455	0.231688	0.031058	0.96099	0.475893	0.394942
f_n	59.6115	53.7813	60.4117	77.82976	212.5745	38.21535	54.30535	59.6116
K_B	1.2	1.17241	1.15151	1.2	1.111111	1.121951	1.135135	1.121951
t_a	167.352	147.645	163.139	218.5829	562.0128	100.7116	144.7373	157.3619
t_m	310.798	274.198	302.974	405.9406	1043.741	187.0363	268.7985	292.2442
n_f	1.34155	1.51711	1.37302	1.029732	0.402243	2.206934	1.544317	1.420422
hooks								
r_1	5.6	7.2	8.1	4.9	6.6	13.75	10	11
r_2	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5
C_1	7	8	9	7	12	11	10	11
K_A	1.11904	1.10267	1.09027	1.119048	1.066288	1.072727	1.080556	1.072727
C_2	6.875	6.11111	6.11111	7.857143	10	4.4	5.5	5.5
K_B	1.12766	1.14673	1.14673	1.109375	1.083333	1.220588	1.166667	1.166667
sigma a	322.087	285.597	316.799	420.6856	1099.758	196.6664	281.9316	307.2913
sigma m	598.163	530.396	588.342	781.2752	2042.413	365.2386	523.5886	570.6853
t_a	157.263	144.411	162.463	202.0753	547.9625	109.5657	148.7578	163.6336
t_m	292.062	268.193	301.718	375.2836	1017.647	203.4797	276.2651	303.8916

S_y(bending)	958.75	942.404	942.404	977.625	1012.66	892.5335	928.0185	928.0185
S_e(bending)	606.177	618.262	618.262	593.7485	574.1564	665.4625	630.0978	630.0978
S_{sy}(torsion)	522.954	514.038	514.038	533.25	552.36	486.8365	506.1919	506.1919
S_{se}(torsion)	349.764	356.737	356.737	342.5929	331.2882	383.9719	363.5664	363.5664
n(bending)	1.22026	1.37342	1.23815	0.936316	0.35952	1.981082	1.38872	1.274114
n(torsion)	1.39472	1.51431	1.34605	1.089017	0.403892	1.975972	1.466029	1.332754

Finally, all the criterion are met for wire diameter d=2.0 mm and spring index C=11

Conclusion:

Finally we have designed a trampoline spring of **wire diameter of 2.0 mm** and **mean coil diameter of 22 mm** with **free length of 22 cm** made of **Galvanized Steel.**

REFERENCES

Table 10-4:

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11

Table 10-7:

	Percent of Tensile Strength		
	In Torsion		In Bending
Materials	Body	End	End
Patented, cold-drawn or hardened and tempered carbon and low-alloy steels	45–50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55

Reference Textbook: *Shigley's Mechanical Engineering Design* (9th edition, Budynas, Nisbett)