CIRCUIT THEORY I

Lecture 5: The node-voltage and the mesh-current methods for circuit analysis.

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THE MESH-CURRENT METHOD WITH DEPENDENT SOURCES

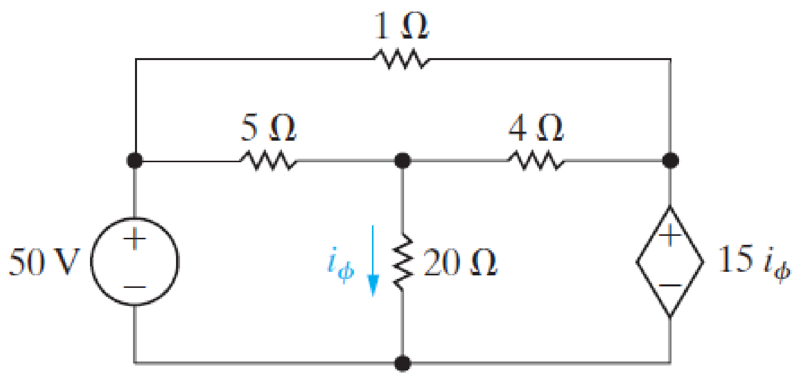
MESH-CURRENT METHOD

1. Identify the meshes with curved directed arrows that follow the perimeter of each mesh.
2. Label the mesh currents for each mesh.
3. Write the KVL equations for each mesh.
4. Solve the KVL equations to find the mesh current values.
5. Solve the circuit using mesh currents from Step 4 to find component currents, voltages, and power values.

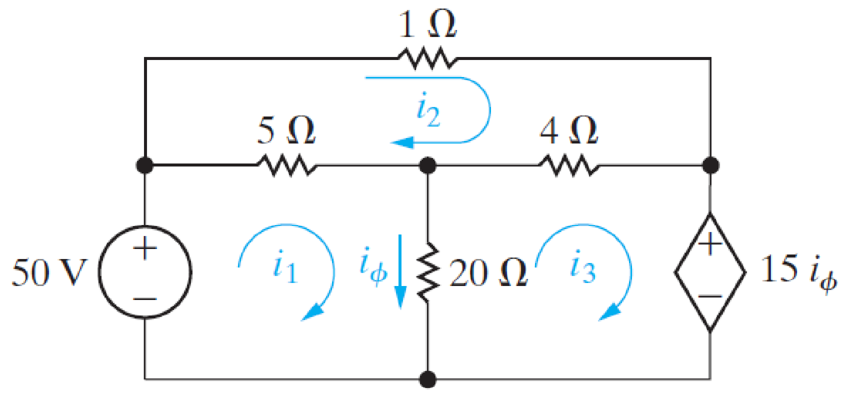
Step 3: Write the KVL equation for each mesh; if the circuit contains a dependent source, write a dependent source constraint equation that defines the controlling variable for the dependent source in terms of the mesh currents.

THE MESH-CURRENT METHOD WITH DEPENDENT SOURCES

Example: Use the mesh-current method of circuit analysis to determine the power dissipated in the 4Ω resistor in the circuit shown in the Figure below



THE MESH-CURRENT METHOD WITH DEPENDENT SOURCES



Solution: The mesh currents are shown in

Let's write the KVL equations for each mesh:

Mesh-1:

$$-50 + 5(i_1 - i_2) + 20(i_1 - i_3) = 0 \qquad i_\phi = i_1 - i_3$$

Mesh-2:

$$i_2 + 4(i_2 - i_3) + 5(i_2 - i_1) = 0$$

Mesh-3:

$$4(i_3 - i_2) + 15i_\phi + 20(i_3 - i_1) = 0$$

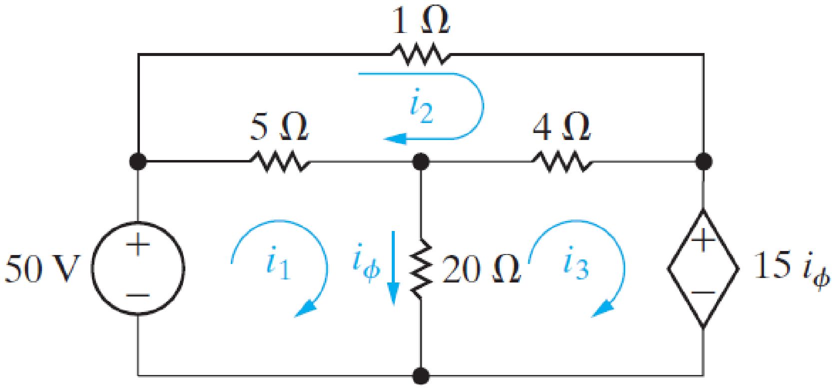
where substituting $i_\phi = i_1 - i_3$

we obtain

$$4(i_3 - i_2) + 15(i_1 - i_3) + 20(i_3 - i_1) = 0$$

THE MESH-CURRENT METHOD WITH DEPENDENT SOURCES

The equations can be rearranged as



$$25i_1 - 5i_2 - 20i_3 = 50$$
$$-5i_1 + 10i_2 - 4i_3 = 0$$
$$-5i_1 - 4i_2 + 9i_3 = 0$$

and when this equation set is solved we get

$$i_2 = 26\ A \quad i_3 = 28\ A \ \ and \quad p = (i_3 - i_2)^2 4 = 16W$$

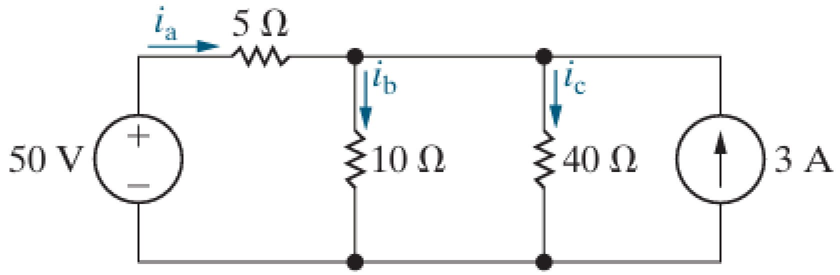
THE MESH-CURRENT METHOD: SOME SPECIAL CASES

When a current source is in a single mesh, the value of the mesh current is known, since it must equal the current of the source. Therefore, we label the mesh current with its value, and we do not need to write a KVL equation for that mesh. This leads to the following modification in Step 2 of the mesh-current method.

Step 2: Label the mesh current for each mesh; if there is a current source in a single mesh, label the mesh current with the value of the current source.

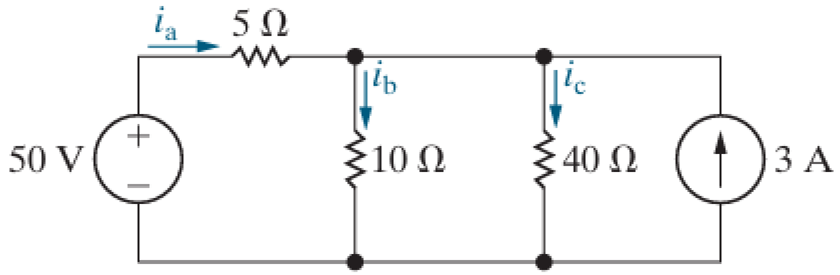
THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Example: Use the mesh-current method to find branch cur rents i_a, i_b, and i_c in the circuit below.

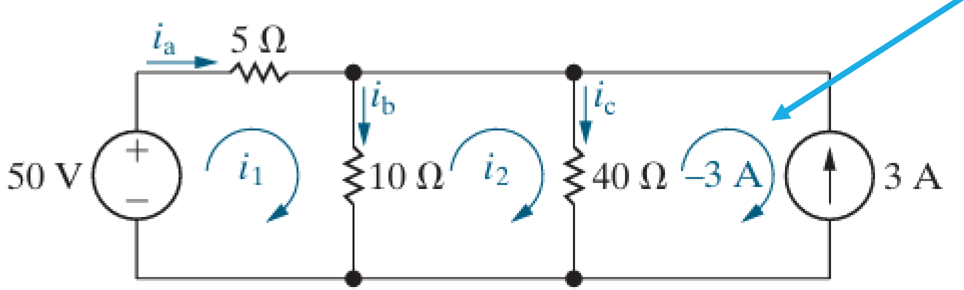


THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Example: Use the mesh-current method to find branch cur rents i_a, i_b, and i_c in the circuit below.



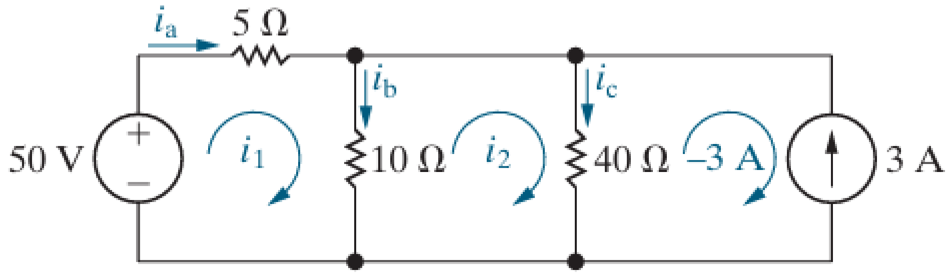
Solution: The mesh currents are shown in



i_3 mesh has a current source that is not shared by any other mesh. Therefore, the i_3 mesh current equals the current supplied by the source.

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

KVL equations for the meshes whose mesh currents are unknown, which in this example are the i_1 and i_2 meshes.



The resulting simultaneous mesh cur rent equations are

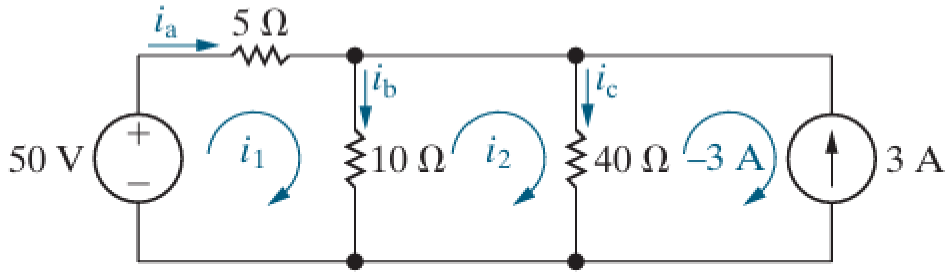
$$-50 + 5i_1 + 10(i_1 - i_2) = 0$$

$$10(i_2 - i_1) + 40(i_2 - (-3)) = 0.$$

Solving the simultaneous mesh current equations gives $i_1 = 2 \text{ A}$ and $i_2 = -2 \text{ A}.$

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

KVL equations for the meshes whose mesh currents are unknown, which in this example are the i_1 and i_2 meshes.



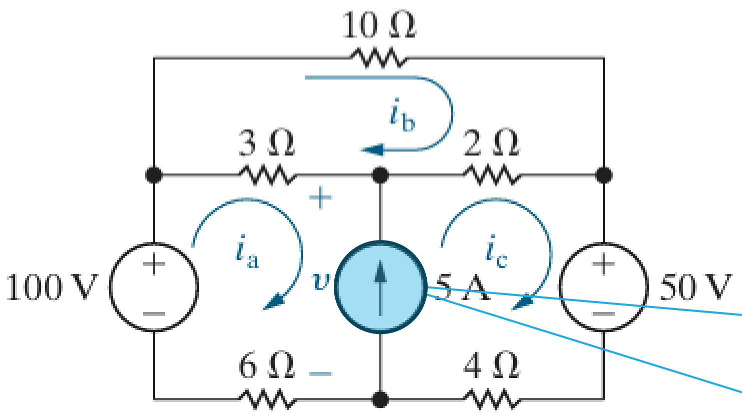
Finally, we use the mesh currents to calculate the branch currents in the circuit, i_a, i_b, and i_c.

$$i_a = i_1 = 2 \text{ A},$$
$$i_b = i_1 - i_2 = 4 \text{ A},$$
$$i_c = i_2 + 3 = 1 \text{ A}.$$

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Current source that is shared between two adjacent meshes.

Exemple:



Applying Steps 1 and 2, we define the mesh currents i_a ,i_b and i_c as well as the voltage across the 5 A cur rent source. In Step 3, we write the KVL equations for each mesh;

Mesh a

$$-100 + 3(i_a - i_b) + v + 6i_a = 0. \qquad \text{Eq. 1}$$

Mesh c

$$50 + 4i_c - v + 2(i_c - i_b) = 0. \qquad \text{Eq. 2}$$

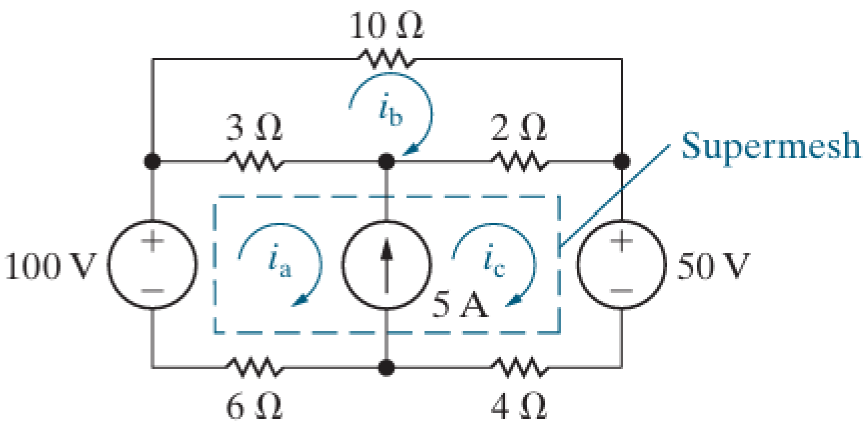
We now add Eqs. 1 and 2 to eliminate v; when simplified, the result is

$$-50 + 9i_a - 5i_b + 6i_c = 0. \qquad \text{Eq. 3}$$

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

The Concept of a Supermesh

When a current source is shared between two meshes, we can combine these meshes to form a supermesh, which traverses the perimeters of the two meshes and avoids the branch containing the shared current source.



KVL equation around the supermesh, using the original mesh currents to give

$$-100 + 3(i_a - i_b) + 2(i_c - i_b) + 50 + 4i_c + 6i_a = 0,$$

which simplifies to

$$-50 + 9i_a - 5i_b + 6i_c = 0.$$

Note that this equation is identical to Eq. 3 (p. 11). Thus, the supermesh has eliminated the need for introducing the unknown voltage across the current source.

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

The KVL equation for the b mesh is

$$10i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0.$$

We have two simultaneous equations, but three unknowns (i_a ,i_b and i_c).

$$\begin{cases} -50 + 9i_a - 5i_b + 6i_c = 0. & \text{Eq. 1} \\ 10i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0. & \text{Eq. 2} \end{cases}$$

The presence of a supermesh in the mesh-current method requires a KVL equation around the supermesh and a supermesh constraint equation that defines the difference between the mesh currents in the supermesh as the value of the shared current source.

The supermesh constraint equation is $i_c - i_a = 5.$ Eq. 3

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

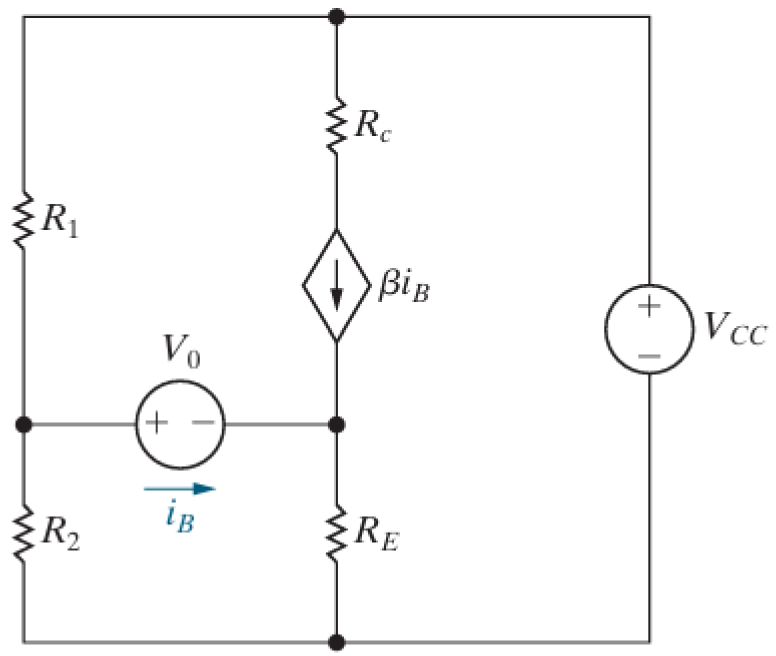
Solve Eqs. 1–3 and confirm that the solutions for the three mesh currents are:

$$i_a = 1.75\ \text{A}, \quad i_b = 1.25\ \text{A}, \quad \text{and} \quad i_c = 6.75\ \text{A}.$$

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

EXAMPLE: Mesh-Current Analysis of the Amplifier Circuit

Use the mesh-current method to find i_B for the amplifier circuit



THE MESH-CURRENT METHOD: SOME SPECIAL CASES

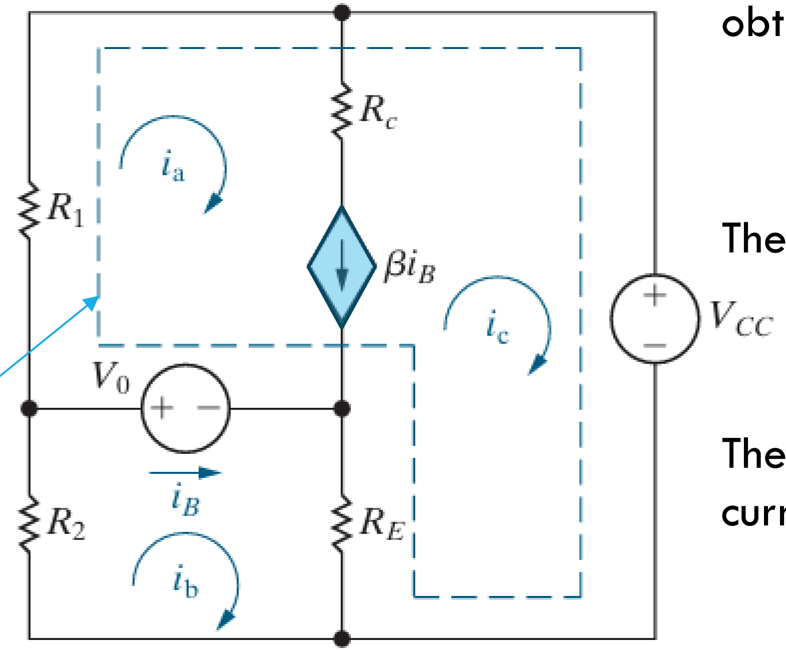
EXAMPLE: Mesh-Current Analysis of the Amplifier Circuit

Use the mesh-current method to find i_B for the amplifier circuit

Use directed arrows that traverse the mesh perimeters to identify the three mesh currents.

Label the mesh currents as i_a , i_b and i_c.
Then recognize the current source that is shared between the i_a and i_c meshes.

Identify the supermesh.



KVL, sum the voltages around the supermesh in terms of the mesh currents i_a , i_b and i_c to obtain:

$$R_1 i_a + v_{CC} + R_E(i_c - i_b) - V_0 = 0. \quad \text{Eq. 1}$$

The KVL equation for mesh b is:

$$R_2 i_b + V_0 + R_E(i_b - i_c) = 0. \quad \text{Eq. 2}$$

The constraint imposed by the dependent current source is

$$i_B = i_b - i_a. \quad \text{Eq. 3}$$

The supermesh constraint equation is $\beta i_B = i_a - i_c.$ Eq. 4

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Use back-substitution to solve Eqs. 1–4.

$$R_1 i_a + v_{CC} + R_E(i_c - i_b) - V_0 = 0. \quad \text{Eq. 1}$$

$$R_2 i_b + V_0 + R_E(i_b - i_c) = 0. \quad \text{Eq. 2}$$

$$i_B = i_b - i_a. \quad \text{Eq. 3}$$

$$\beta i_B = i_a - i_c. \quad \text{Eq. 4}$$

$$i_c = (1 + \beta) i_a - \beta i_b.$$

$$[R_1 + (1 + \beta)R_E] i_a - (1 + \beta) R_E i_b = V_0 - V_{CC},$$

$$-(1 + \beta) R_E i_a + [R_2 + (1 + \beta) R_E] i_b = -V_0.$$

the solution for i_a and i_b gives

$$i_a = \frac{V_0 R_2 - V_{CC} R_2 - V_{CC}(1 + \beta) R_E}{R_1 R_2 + (1 + \beta) R_E (R_1 + R_2)},$$

$$i_b = \frac{-V_0 R_1 - (1 + \beta) R_E V_{CC}}{R_1 R_2 + (1 + \beta) R_E (R_1 + R_2)}.$$

i_B can be found using Eq. 3

$$i_B = i_b - i_a.$$

THE NODE-VOLTAGE METHOD VERSUS THE MESH-CURRENT METHOD

Which one to use?

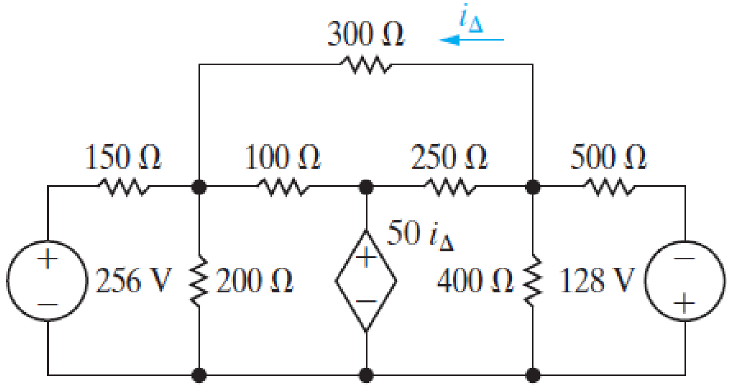
Node-voltage or Mesh-current method?

Choose the method which requires fewer equations.

If your circuit contains more current sources than voltage sources than, mesh current method may be better, otherwise use node-voltage method to analyze your circuit.

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Example: For the circuit shown below, which method is better, node-voltage or mesh-current?

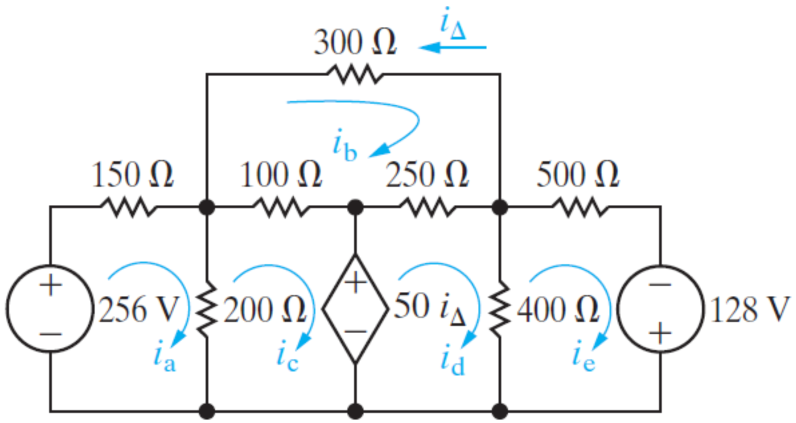


THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Example: For the circuit shown below, which method is better, node-voltage or mesh-current?

Solution:

If we use mesh-current method, the mesh current can be indicated as below



There are 5 meshes and we need to write 5 KVL equations.

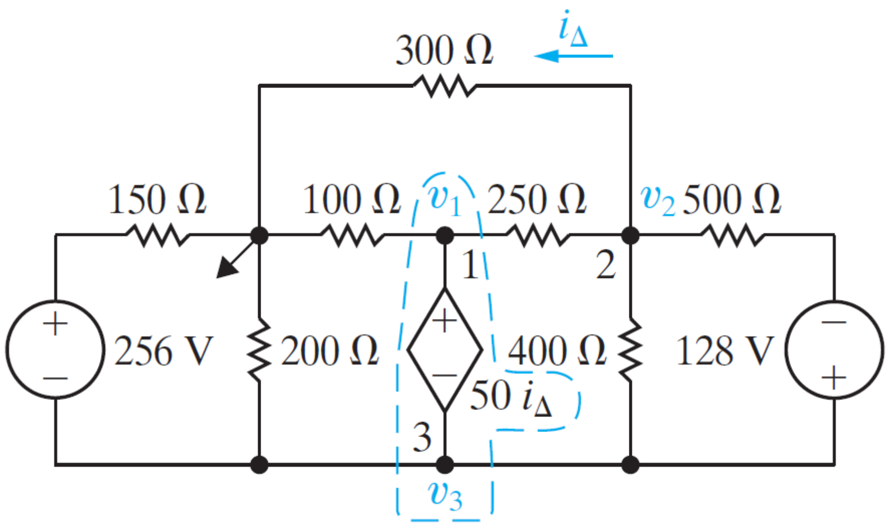
THE MESH-CURRENT METHOD: SOME SPECIAL CASES

Example: For the circuit shown below, which method is better, node-voltage or mesh-current?

Solution:

If we use node-voltage method, the nodes can be indicated as below

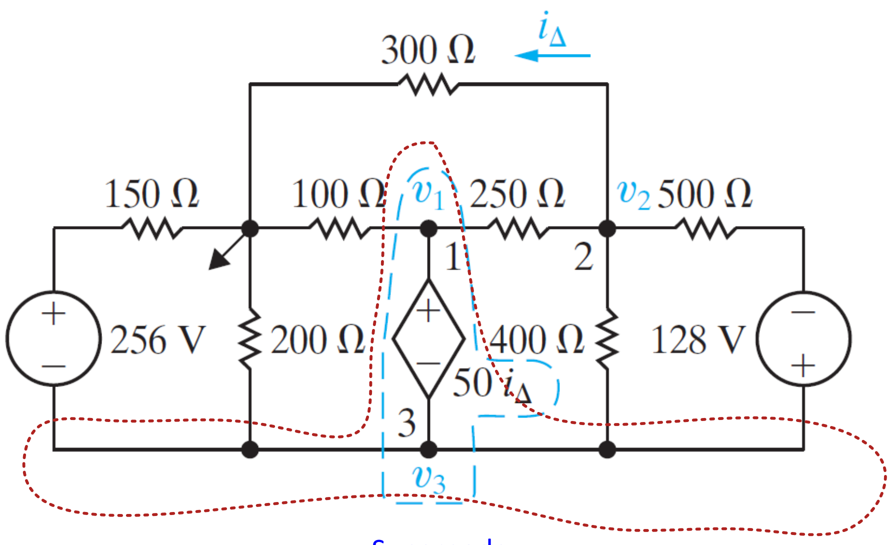
The circuit has four essential nodes, and therefore only three node-voltage equations are required to describe the circuit.



The dependent voltage source between two essential nodes forms a supernode, requiring a KCL equation and a super node constraint equation.

THE MESH-CURRENT METHOD: SOME SPECIAL CASES

The reference node is indicated by an arrow. There are 2 nodes in the circuit. Considering the dependent source, it is sufficient to write 3 equations to analyze the circuit. Hence, for this circuit node-voltage method is preferable.



At the supernode,

$$\frac{v_1}{100} + \frac{v_1 - v_2}{250} + \frac{v_3}{200} + \frac{v_3 - v_2}{400} + \frac{v_3 - (v_2 + 128)}{500} + \frac{v_3 + 256}{150} = 0.$$

At v_2,

$$\frac{v_2}{300} + \frac{v_2 - v_1}{250} + \frac{v_2 - v_3}{400} + \frac{v_2 + 128 - v_3}{500} = 0.$$

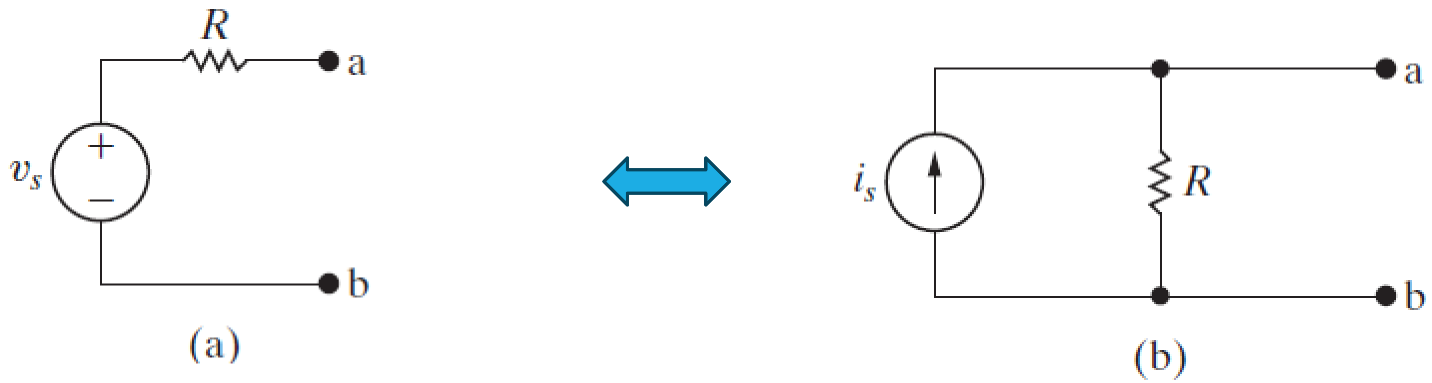
The dependent source constraint equation is $i_\Delta = \frac{v_2}{300}$.

The supernode constraint equation is $v_1 - v_3 = 50 i_\Delta$.

SOURCE TRANSFORMATIONS

A source transformation allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa.

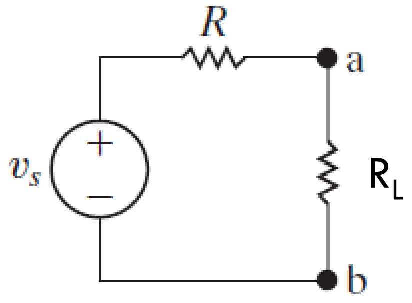
The double-headed arrow emphasizes that a source transformation is bilateral; that is, we can start with either configuration and derive the other.



We need to find the relationship between v_s and i_s that guarantees the two configurations in the figure above are equivalent with respect to nodes a and b. Equivalence is achieved if any resistor R_L has the same current and thus the same voltage drop, whether connected between nodes a and b in Fig. (a) or Fig. (b)

SOURCE TRANSFORMATIONS

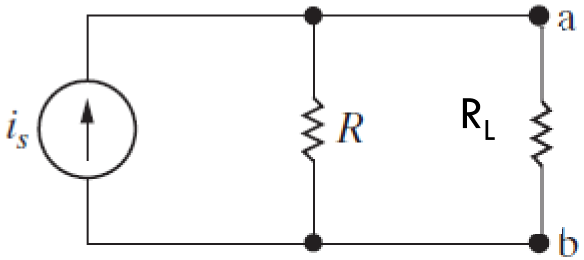
Suppose R_L is connected between nodes a and b in Fig. (a). Using Ohm's law, we find that the current in R_L is



(a)

$$i_L = \frac{v_s}{R + R_L}. \quad \text{Eq. 1}$$

Now suppose the same resistor R_L is connected between nodes a and b in Fig. (b). Using current division, we see that the current in R_L is



(b)

$$i_L = \frac{R}{R + R_L} i_s. \quad \text{Eq. 2}$$

If the two circuits are equivalent, these resistor currents must be the same. Equating the right-hand sides of Eqs. 1 and 2 and sim plifying gives the **condition of equivalence:**

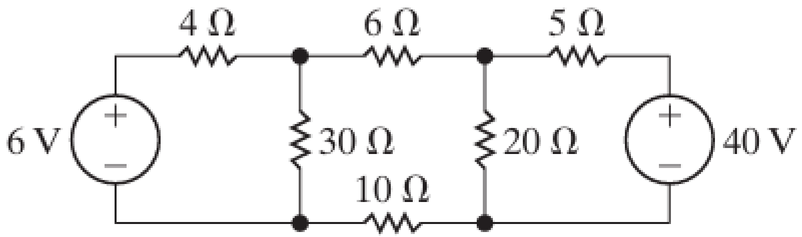
$$i_s = \frac{v_s}{R}.$$

The current in R_L connected between nodes a and b is the same for both circuits for all values of R_L. If the current in R_L is the same for both circuits, then the voltage drop across R_L is the same for both circuits, and the circuits are equivalent at nodes a and b.

SOURCE TRANSFORMATIONS

EXAMPLE: Using Source Transformations to Solve a Circuit

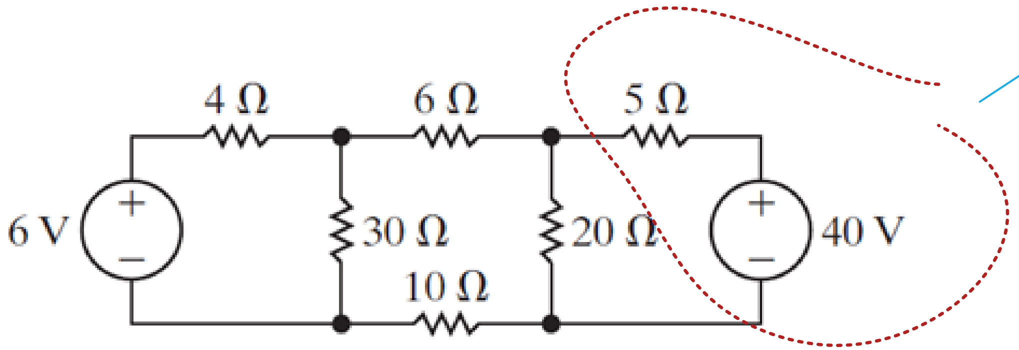
Find the power associated with the 6 V source for the circuit shown in Fig. below and state whether the 6 V source is absorbing or delivering the power.

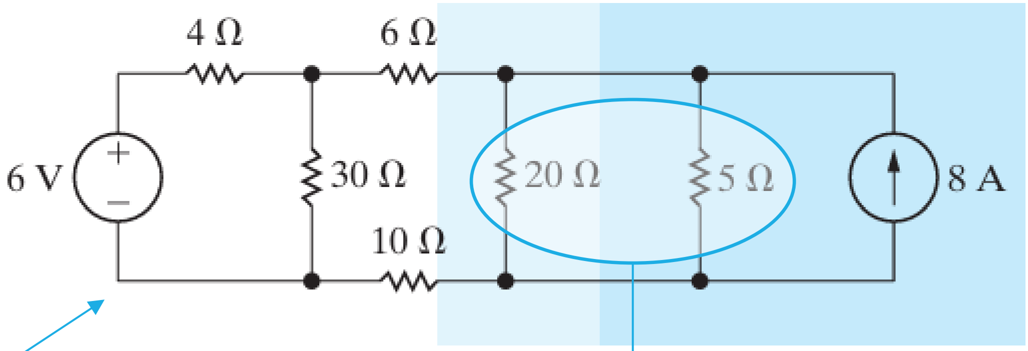


SOURCE TRANSFORMATIONS

Solution:

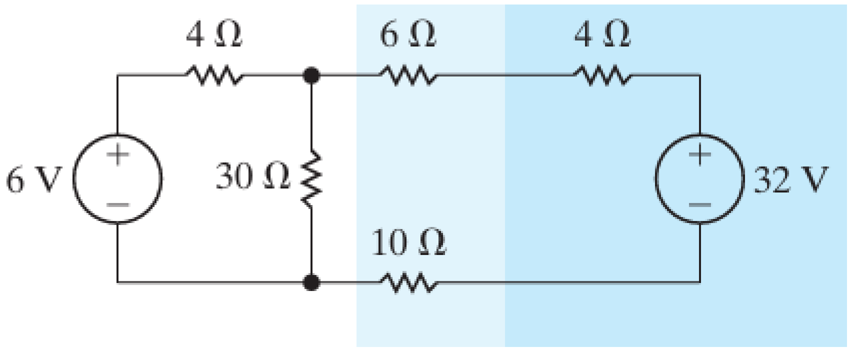
Begin on the right side of the cir cuit with the branch containing the 40 V source. We can transform the 40 V source in series with the 5 Ω resistor into an 8 A current source in parallel with a 5 Ω resistor.





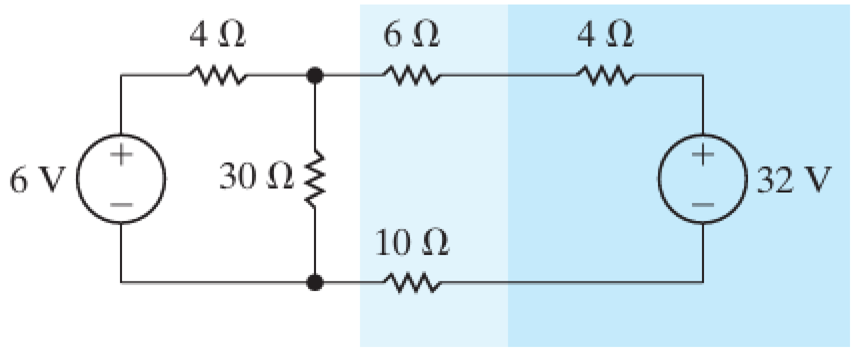
(a) First step

This parallel combination with the 8 A source can be replaced with a 32 V source in series with a 4 Ω resistor.



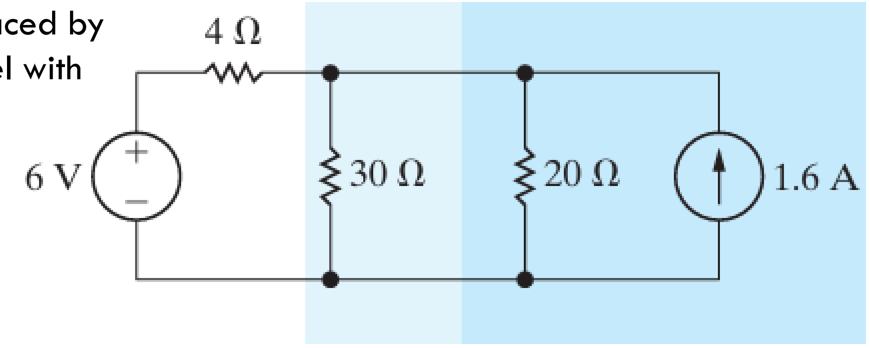
(b) Second step

SOURCE TRANSFORMATIONS



(b) Second step

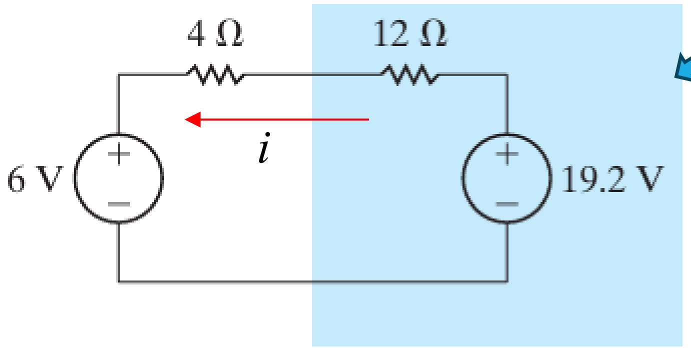
The 32 V source is in series with 20 Ω of resistance and, hence, can be replaced by a current source of 1.6 A in parallel with 20 Ω



(c) Third step

$$i = \frac{19.2 - 6}{16} = 0.825A$$

$$p_{6V} = 6 \times 0.825 = 4.95W$$



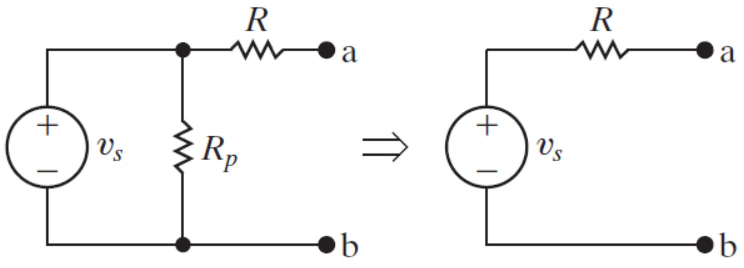
(d) Fourth step

The 20 Ω and 30 Ω parallel resistors can be reduced to a single 12 Ω resistor. The parallel combination of the 1.6 A current source and the 12 Ω resistor transforms into a voltage source of 19.2 V in series with 12 Ω.

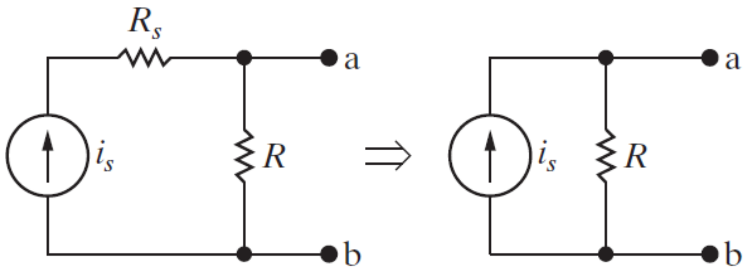
The $6V$ voltage source is absorbing power.

SOURCE TRANSFORMATIONS

What happens if there is a resistance Rp in parallel with the voltage source? • What happens if there is a resistance Rs in series with the current source?



(a)

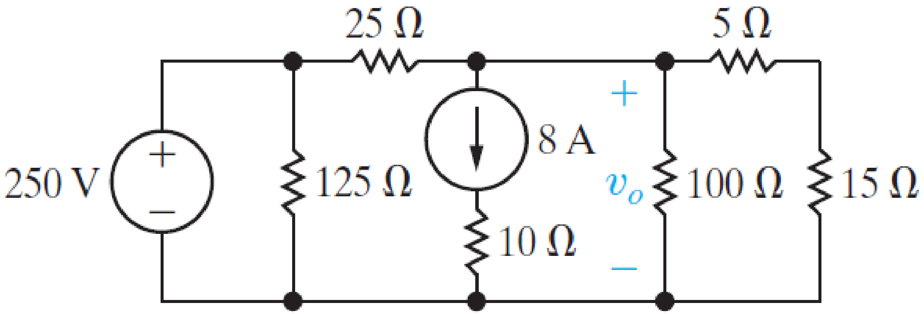


(b)

In both cases, the resistance can be removed to create a simpler equivalent circuit with respect to terminals a and b.
The two circuits depicted in Fig. (a) are equivalent with respect to terminals a and b because they produce the same voltage and current in any resistor R_L inserted between nodes a and b. The same can be said for the circuits in Fig. (b).

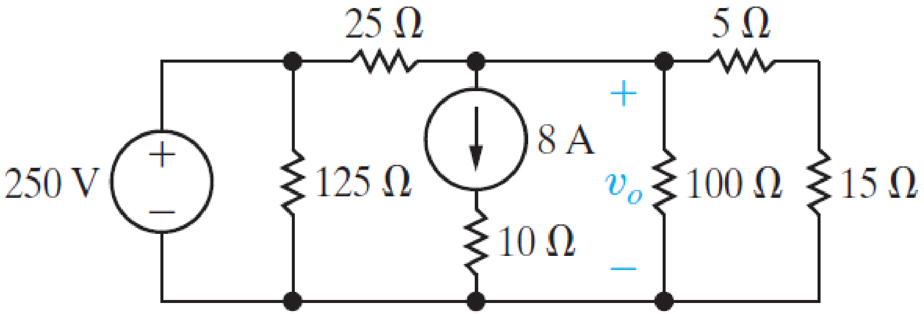
SOURCE TRANSFORMATIONS

Exercise: Use source transformations to find the voltage in the circuit shown

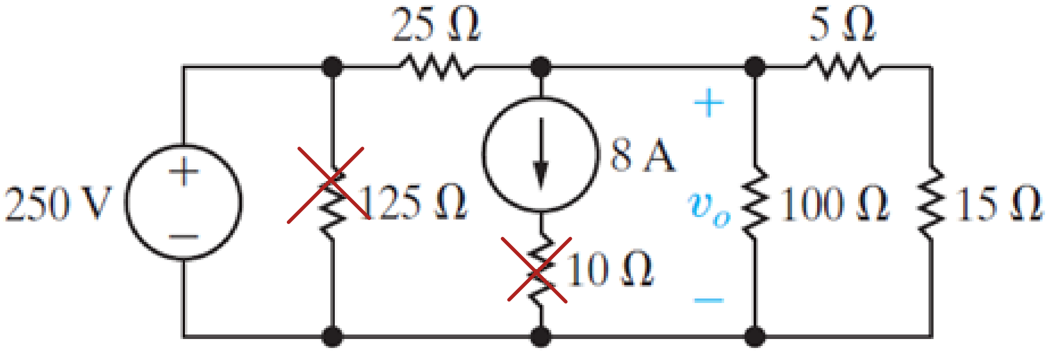


SOURCE TRANSFORMATIONS

Exercise: Use source transformations to find the voltage in the circuit shown

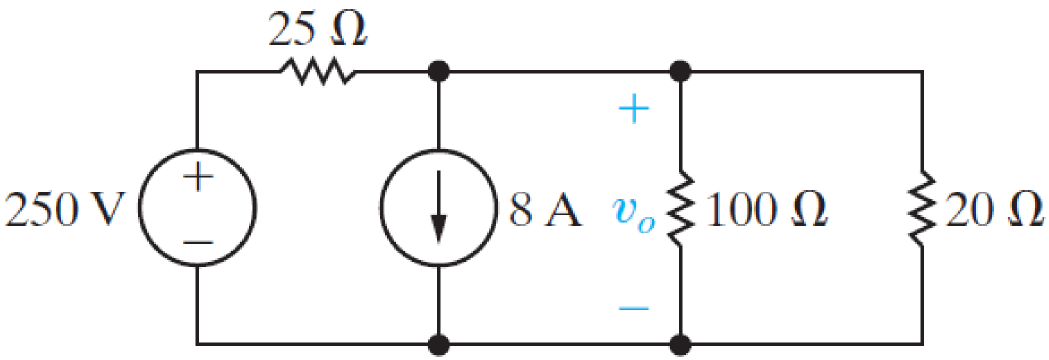


Solution: We begin by removing the 10Ω and resistors 125Ω, because 125Ω the resistor is connected across the 250 V voltage source and the resistor 10Ω is connected in series with the 8 A current source.

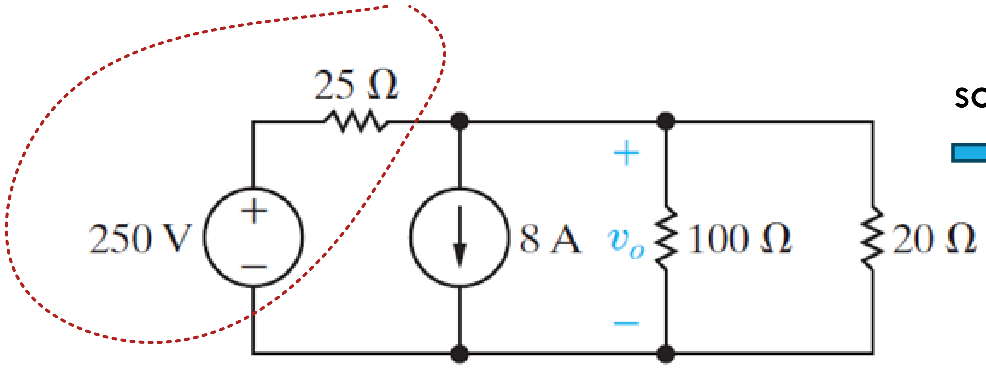


SOURCE TRANSFORMATIONS

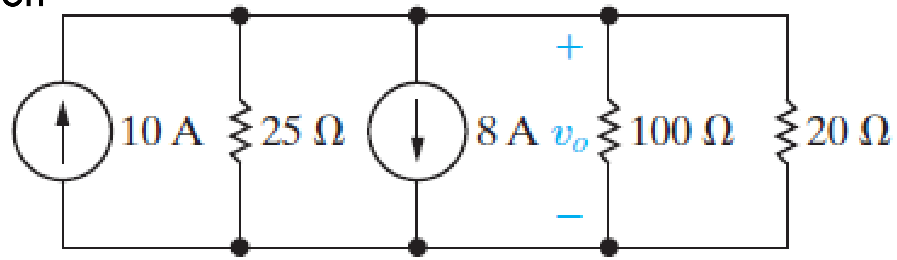
After removing the resistors we get:



We perform source transformation as in below Figure and obtain:

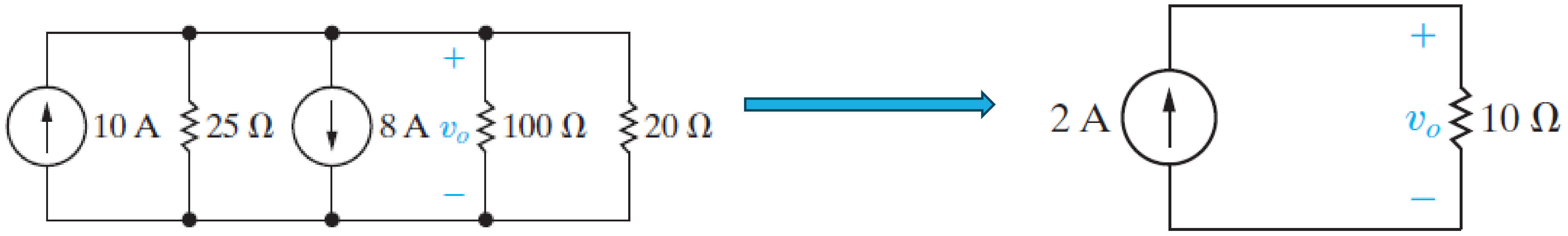


source transformation



SOURCE TRANSFORMATIONS

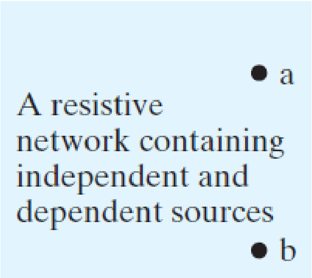
The circuit reduces to:

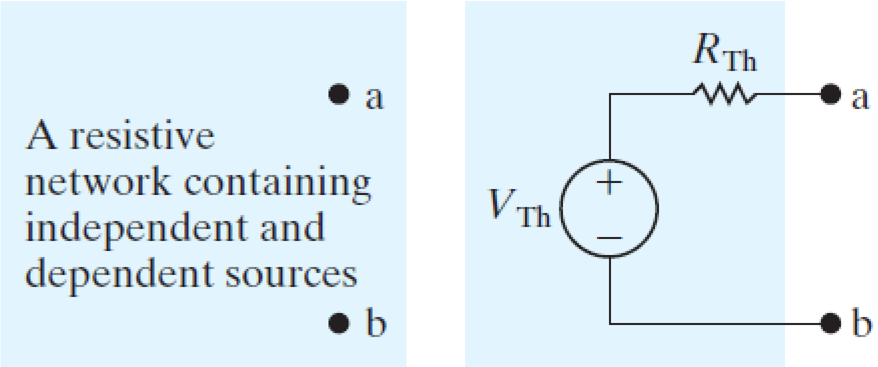


$$v_0 = 2 \times 10 = 20V$$

THÉVENIN AND NORTON EQUIVALENTS

Consider a resistive circuit containing independent and dependent sources as shown below, a and b are two terminals of the circuit.





(a) A general circuit. (b) The Thévenin equivalent circuit.

Assume that we connect a load resistor R_L to the terminals a and b, and a voltage appears on R_L and current flow on R_L.

According to Thévenin theorem, the same voltage and current appears across the load resistor R_L if a single voltage source and resistor is used as in the Figure (b).

THÉVENIN AND NORTON EQUIVALENTS

Then, how to find the Thevenin equivalent circuit of a resistive circuit?

1) Reduce the load resistance to zero, i.e., short circuit the terminals and calculate

$$i_{sc}$$

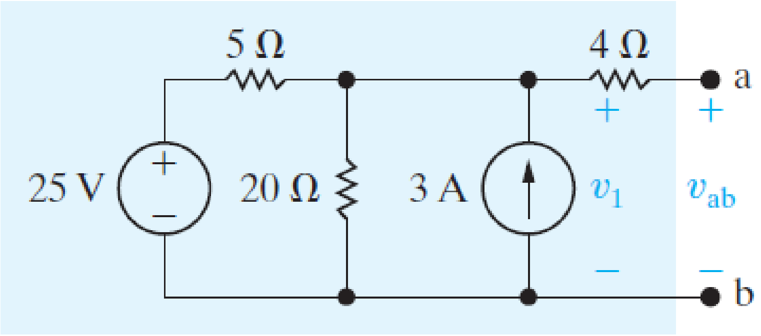
2) Do not connect anything to the terminals and calculate the open-circuit voltage V_{oc} which is equal to V_{th}

3) Calculate R_{Th} from

$$V_{\mathrm{th}} = R_{Th} \times i_{sc}$$

THÉVENIN AND NORTON EQUIVALENTS

Example: Find the Thévenin equivalent of the circuit shown in Figure below for the terminals a and b.



THÉVENIN AND NORTON EQUIVALENTS

Solution:

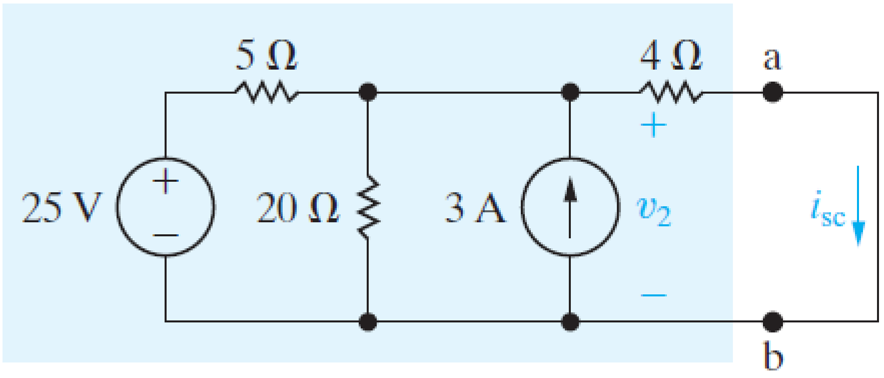
First we open the terminals and calculate V_{oc}. From the above Figure it is clear that

$$v_1 = v_{ab}$$

There is only one node in the circuit, and if we write the node equation, we get

$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0 \rightarrow v_1 = 32V$$

In the next step, we calculate the short circuit current, i_{sc}, as shown



THÉVENIN AND NORTON EQUIVALENTS

Writing node equation

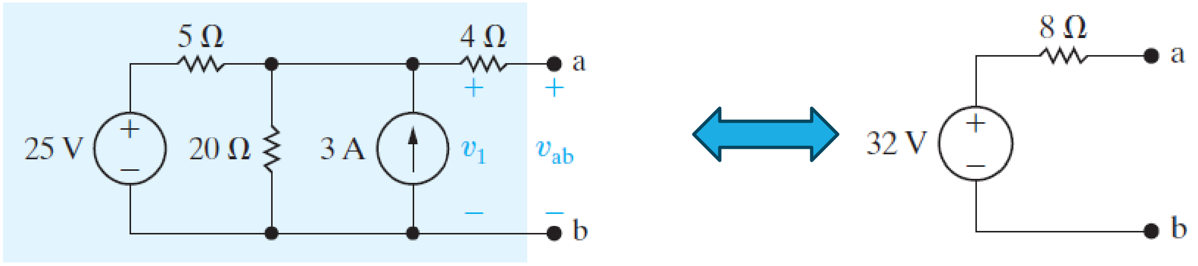
$$\frac{v_2 - 25}{5} + \frac{v_2}{20} - 3 + \frac{v_2}{4} = 0 \rightarrow v_2 = 16V$$

$$i_{sc} = \frac{16}{4} = 4A$$

We can calculate R_{Th} as

$$R_{Th} = \frac{v_{oc}}{i_{sc}} \rightarrow R_{Th} = \frac{32}{4} = 8\Omega$$

Then, the Thévenin equivalent circuit becomes as



SOURCE TRANSFORMATIONS

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