Lecture-4

Analog Communications

Abstract:

Pre-envelope

For the real valued signal $g(t)$, the pre-envelope is defined as

$$g_p(t) = g(t) + j\hat{g}(t) \tag{1}$$

We know that

$$FT\{\hat{g}(t)\} = -j\; sign(f)G(f) \tag{2}$$

If we take the Fourier transform of (1), we get

$$G_p(f) = G(f) + jFT\{\hat{g}(t)\}$$

in which using (2), we obtain

$$G_p(f) = G(f) + j\{-j\; sign(f)G(f)\}$$

leading to

$$G_p(f) = G(f) + sign(f)G(f)$$

which can be expressed as

$$G_p(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0. \end{cases} \tag{3}$$

Note that

$$sgn(f) = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0. \end{cases}$$

Summary for the Pre-Envelope Calculation of a Signal

1) The pre-envelope of $g(t)$ can be calculated using

$$g_p(t) = g(t) + j\hat{g}(t).$$

where $\hat{g}(t)$ is the Hilbert transform of $g(t)$.

2) The pre-envelope of $g(t)$ can be calculated first calculating the Fourier transform of $g(t)$, i.e., finding $G(f)$, and next evaluating

$$G_p(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0. \end{cases}$$

and lastly taking the inverse Fourier transform of $G_p(f)$, we get $g_p(t)$, i.e.,

$$g_p(t) = 2\int_0^{\infty} G(f)e^{j2\pi ft}df.$$

Low-Pass Signal

If the magnitude spectrum of a signal is centered around the origin, the signal is called a low pass signal.

The Fourier transform of a typical low-pass signal is as shown in Figure-1.

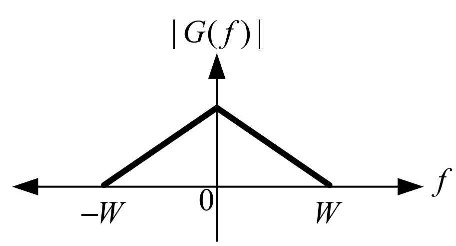

Figure-1

Band-pass Signal

If the magnitude spectrum of a signal is centered around frequencies $\pm f_c$, then the signal is called a band-pass signal.

The magnitude spectrum of a typical band-pass signal is depicted in Figure-2.

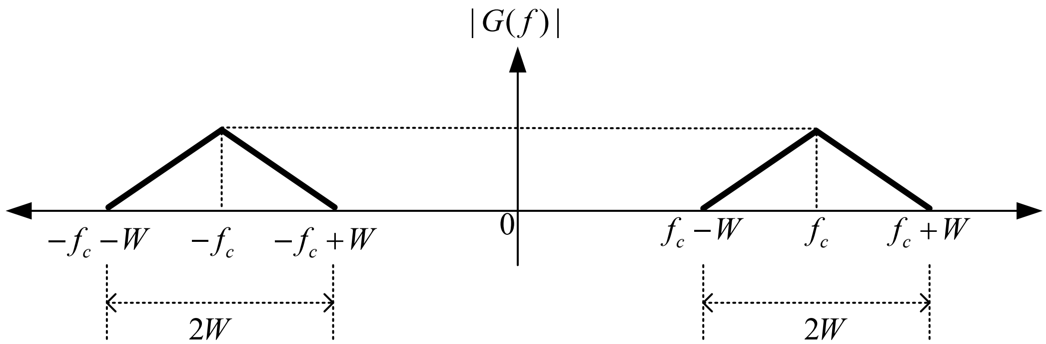

Figure-2

Bandwidth

Signal bandwidth is the difference between the largest and the smallest positive frequencies in the magnitude spectrum of a signal.

Signal bandwidth can be defined in different ways. Two well-known definitions are the null-to-null bandwidth and 3-dB bandwidth.

Null-to-Null Bandwidth

In null-to-null bandwidth definition, the amplitude of the frequencies used for the bandwidth calculation can go up to 0.

The calculation of the null-to-null bandwidth is depicted in Figure-3.

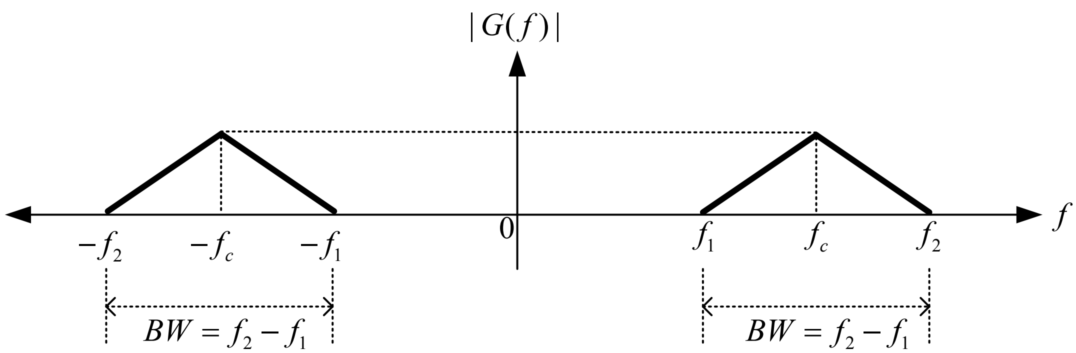

Figure-3

3-dB Bandwidth

In 3-dB bandwidth definition, the amplitude of the frequencies used for the bandwidth calculation should be larger than equal to

$$\frac{A_{max}}{\sqrt{2}}$$

where A_{max} is the maximum value of the magnitude spectrum.

The calculation of the 3-dB bandwidth is depicted in Figure-4.

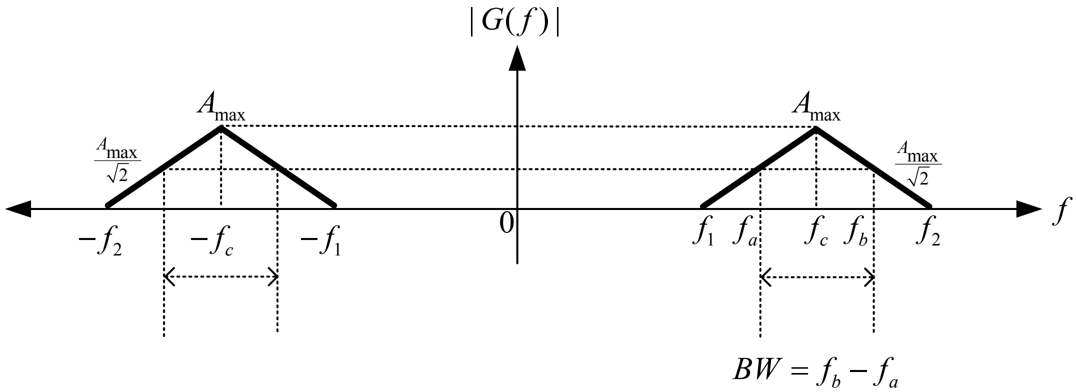

Figure-4

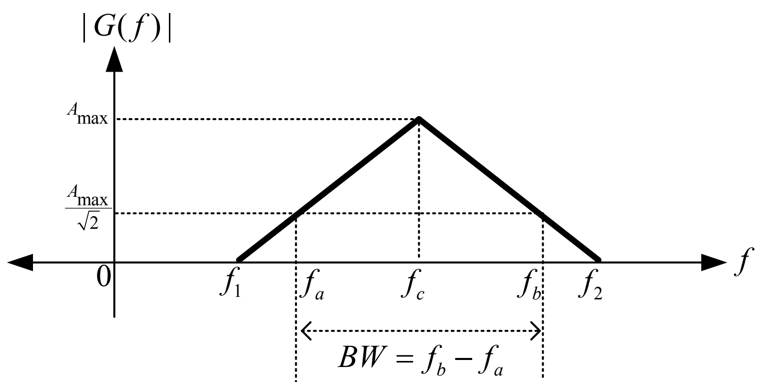

Figure-5

Example: The magnitude spectrum of a low-pass signal is given in Figure-6. What is the bandwidth of this signal?

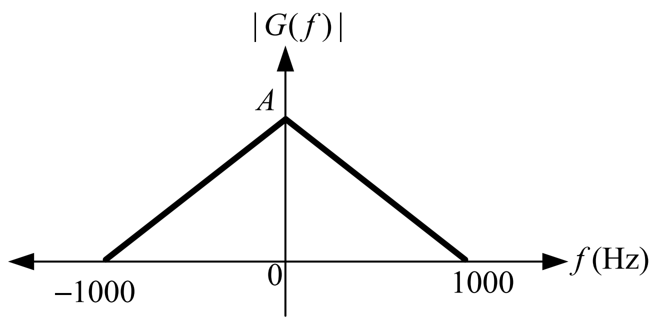

Figure-6

Solution: The bandwidth if the signal is $1000Hz$. This is a low-pass signal.

Complex Envelope

Let $g_p(t)$ be the pre-envelope of $g(t)$. The complex envelope of $g(t)$ is calculated using

$$g_c(t) = g_p(t)e^{-j2\pi f_c t} \tag{4}$$

The Fourier transform of $g_c(t)$ can be expressed as

$$G_c(f) = G_p(f + f_c) \tag{5}$$

Note that $FT\{x(t)e^{j2\pi f_c t}\} = X(f - f_c)$.

From (4), we can write that

$$g_p(t) = g_c(t)e^{j2\pi f_c t}$$

in which substituting $g_p(t) = g(t) + j\hat{g}(t)$ we obtain

$$g(t) + j\hat{g}(t) = g_c(t)e^{j2\pi f_c t} \tag{6}$$

from which we can write that

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\} \tag{7}$$

Property:

If $g(t)$ is a band-pass signal, then its complex envelope $g_c(t)$ is a low-pass signal.

Verification:

Let the magnitude spectrum of the band-pass signal be as in Figure-7.

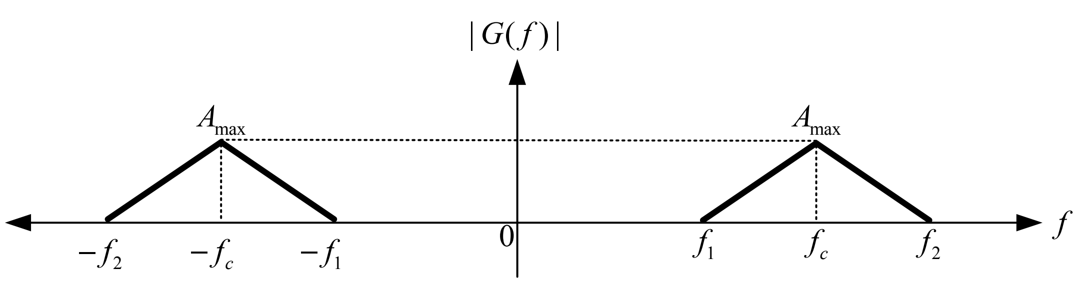

Figure-7

The magnitude spectrum of the pre-envelope signal $g_p(t)$ can be drawn as in Figure-9 using

$$G_p(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0. \end{cases}$$

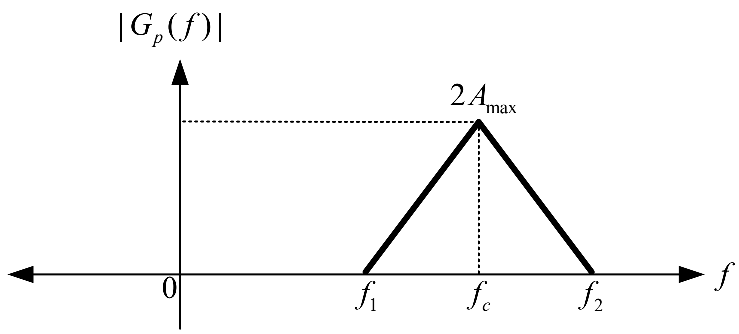

Figure-8

Using (5), i.e.,

$$G_c(f) = G_p(f + f_c)$$

we can draw the graph of $G_c(f)$ as in

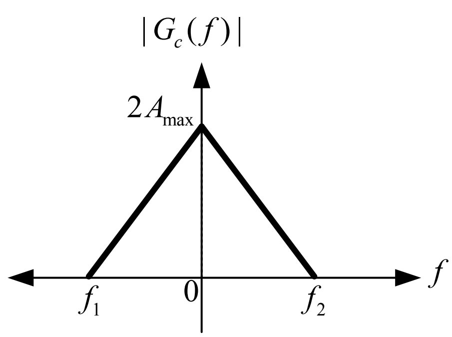

Figure-9

From (6), i.e.,

$$g(t) + j\hat{g}(t) = g_c(t)e^{j2\pi f_c t}$$

it is clear that $g_c(t)$ is a complex number, and we can express $g_c(t)$ as in

$$g_c(t) = g_I(t) + jg_Q(t) \tag{8}$$

Using (8) in

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

we get

$$g(t) = Re\{[g_I(t) + jg_Q(t)][\cos(2\pi f_c t) + j\sin(2\pi f_c t)]\}$$

leading to

$$g(t) = Re\{g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t) + j\ldots\}$$

resulting in

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$$

Result Summary:

If $g_c(t)$ is a low-pass complex signal such that

$$g_c(t) = g_I(t) + jg_Q(t) \tag{9}$$

then

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

is a band-pass signal which can also be written as

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t).$$

The complex envelope signal

$$g_c(t) = g_I(t) + jg_Q(t) \tag{10}$$

can also be written as

$$g_c(t) = |g_c(t)|e^{j\theta(t)} \tag{11}$$

where

$$|g_c(t)| = \sqrt{g_I^2(t) + g_Q^2(t)} \text{ and } \theta(t) = \arctan\frac{g_Q(t)}{g_I(t)}.$$

The equation in (11) can be written as

$$g_c(t) = |g_c(t)|\cos\theta(t) + ja(t)\sin\theta(t) \tag{12}$$

Comparing (10) and (11), we can write that

$$g_I(t) = |g_c(t)|\cos\theta(t) \quad g_Q(t) = |g_c(t)|\sin\theta(t).$$

Besides employing using (11) in

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

we obtain

$$g(t) = |g_c(t)| \cos\big(2\pi f_c t + \theta(t)\big).$$