# CIRCUIT THEORY I

Lecture 4: The node-voltage and the mesh-current methods for circuit analysis.

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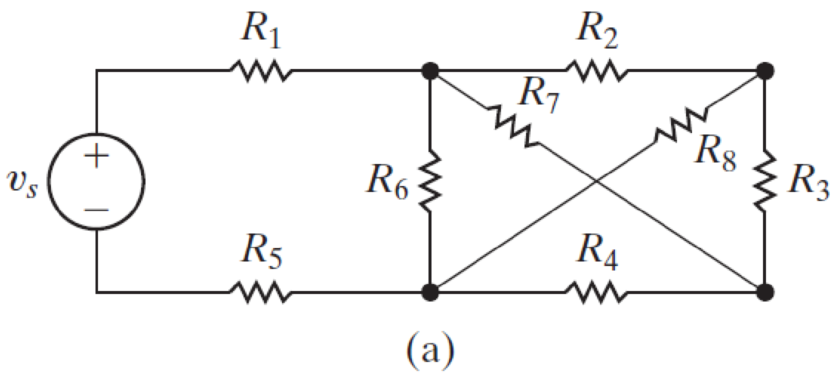
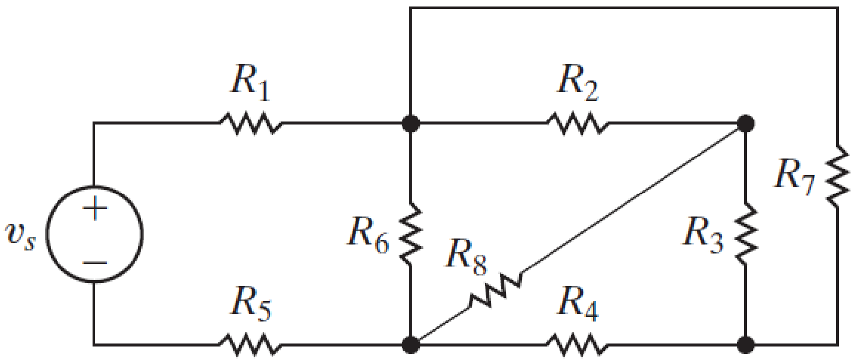
# TERMINOLOGY

Those circuits that can be drawn on a plane with no crossing branches.

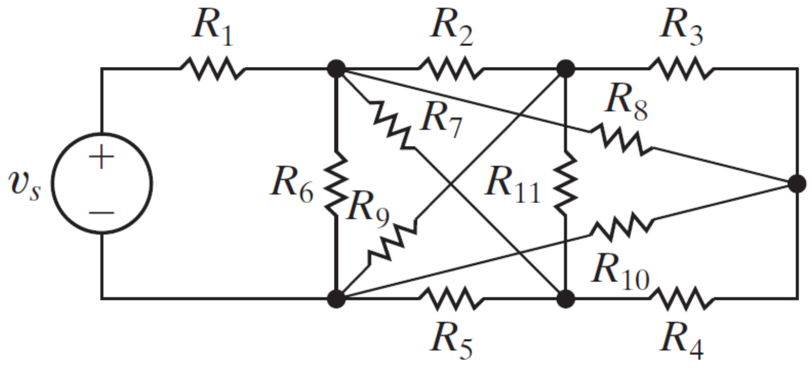



(a) A planar circuit. (b) The same circuit redrawn to verify that it is planar.

# TERMINOLOGY

A nonplanar circuit




a nonplanar circuit cannot be redrawn in such a way that all the node connections are maintained and no branches overlap.

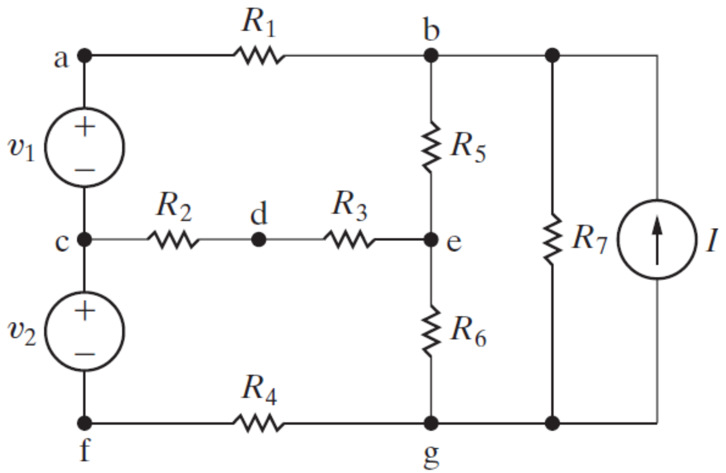
The node-voltage method is applicable to both planar and nonplanar circuits, **whereas the mesh-current method is limited to planar circuits.**

# TERMINOLOGY

| Name | Definition |
|---|---|
| node | A point where two or more circuit elements join |
| essential node | A node where three or more circuit elements join |
| path | A trace of adjoining basic elements with no elements included more than once |
| branch | A path that connects two nodes |
| essential branch | A path which connects two essential nodes without passing through an essential node |
| loop | A path whose last node is the same as the starting node |
| mesh | A loop that does not enclose any other loops |
| planar circuit | A circuit that can be drawn on a plane with no crossing branches |

# TERMINOLOGY

**Example:** Identifying Node, Branch, Mesh and Loop in a Circuit




For the circuit in Figure, identify

a) all nodes. b) all essential nodes.

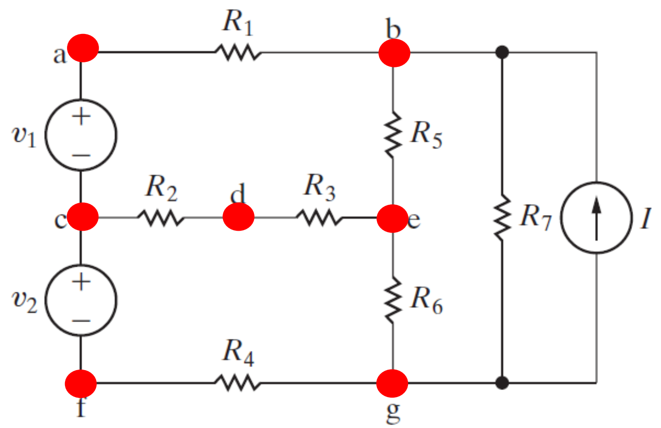
c) all branches. d) all essential branches.

e) all meshes. f) two paths that are not loops or essential branches.

g) two loops that are not meshes.

# TERMINOLOGY

**Solution:**



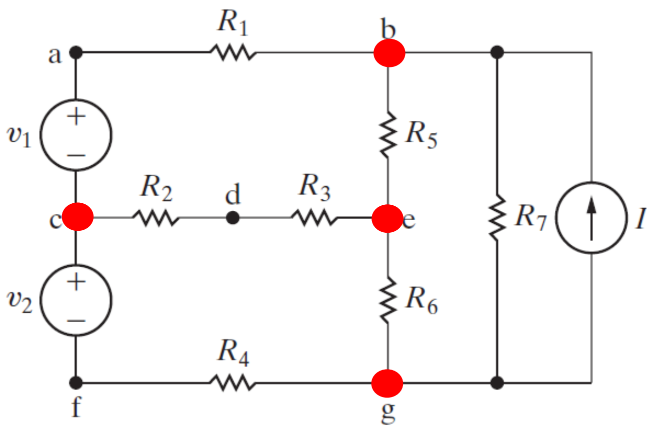

**a)** The nodes are a, b, c, d, e, f, and g.

**b)** The essential nodes are b, c, e, and g.

**c)** The branches are $v_1, v_2, R_1, R_2, R_3, R_7$, and $I$.

# TERMINOLOGY

**Solution:**

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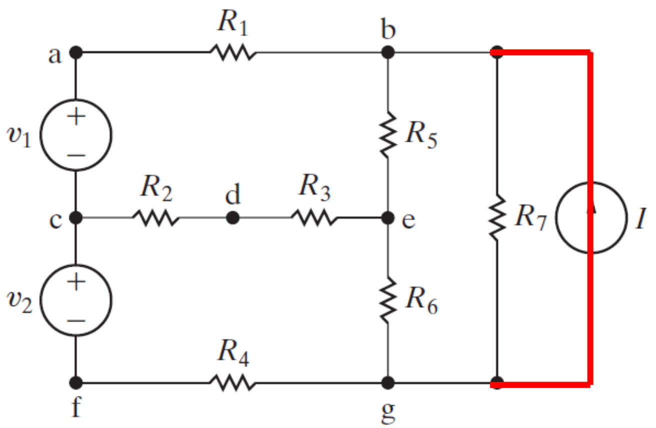
# TERMINOLOGY

**Solution:**




**a)** The nodes are a, b, c, d, e, f, and g.

**b)** The essential nodes are b, c, e, and g.

**c)** The branches are $v_1, v_2, R_1, R_2, R_3, R_7$, and $I$.

# TERMINOLOGY

**d)** The essential branches are $v_1 - R_1$ , $R_2 - R_3$, $v_2 - R_4$, $R_5$, $R_6$, $R_7$ and $I$

**e)** The meshes are $v_1 - R_1 - R_5 - R_3 - R_2$,

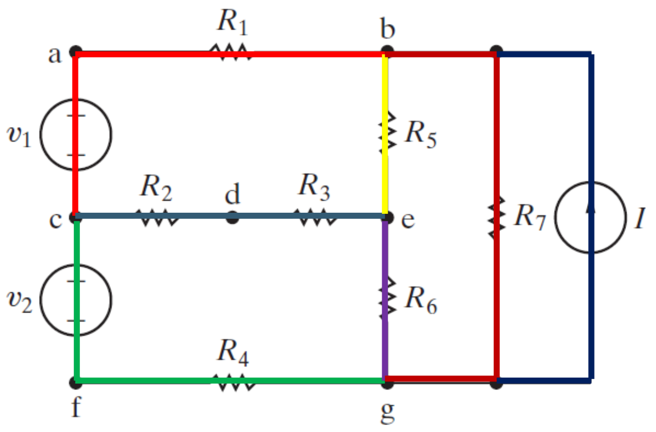
$v_2 - R_2 - R_3 - R_6 - R_4$, $R_5 - R_7 - R_6$,

and $R_7 - I$.

**f)** $R\_1 - R\_5 - R\_6$ is a path, but it is not a loop

(because it does not have the same starting and ending nodes), nor is it an essential branch

(because it does not connect two essential nodes).$v2 - R2$ is also a path but is neither a loop nor an essential branch, for the same reasons.




**g)** $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh, because there are two loops within it.

$I - R_5 - R_6$ is also a loop but not a mesh.

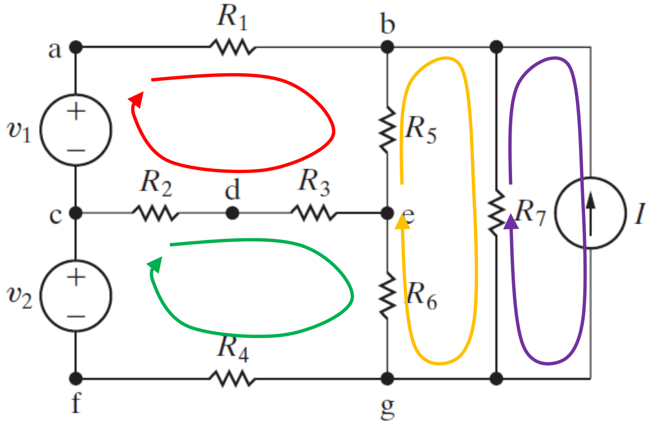
# TERMINOLOGY

**d)** The essential branches are $v_1 - R_1$ , $R_2 - R_3$, $v_2 - R_4$, $R_5$, $R_6$, $R_7$ and $I$

**e)** The meshes are $v_1 - R_1 - R_5 - R_3 - R_2$,

$v_2 - R_2 - R_3 - R_6 - R_4$, $R_5 - R_7 - R_6$,

and $R_7 - I$.




**f)** $R\_1 - R\_5 - R\_6$ is a path, but it is not a loop

(because it does not have the same starting and ending nodes), nor is it an essential branch

(because it does not connect two essential nodes).$v2 - R2$ is also a path but is neither a loop nor an essential branch, for the same reasons.

**g)** $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh, because there are two loops within it.

$I - R_5 - R_6$ is also a loop but not a mesh.

# TERMINOLOGY

**d)** The essential branches are $v_1 - R_1$ , $R_2 - R_3$, $v_2 - R_4$, $R_5$, $R_6$, $R_7$ and $I$

**e)** The meshes are $v_1 - R_1 - R_5 - R_3 - R_2$,

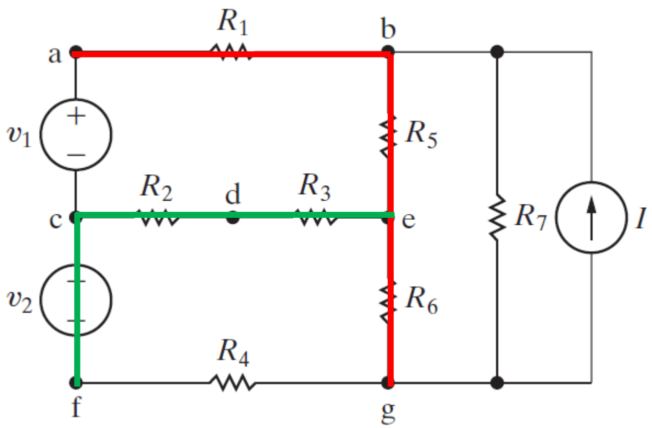
$v_2 - R_2 - R_3 - R_6 - R_4$, $R_5 - R_7 - R_6$,

and $R_7 - I$.

**f)** $R\_1 - R\_5 - R\_6$ is a path, but it is not a loop

(because it does not have the same starting and ending nodes), nor is it an essential branch

(because it does not connect two essential nodes). $v2 - R2$ is also a path but is neither a loop nor an essential branch, for the same reasons.




**g)** $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh, because there are two loops within it.

$I - R_5 - R_6$ is also a loop but not a mesh.

# TERMINOLOGY

**d)** The essential branches are $v_1 - R_1$ , $R_2 - R_3$, $v_2 - R_4$, $R_5$, $R_6$, $R_7$ and $I$

**e)** The meshes are $v_1 - R_1 - R_5 - R_3 - R_2$,

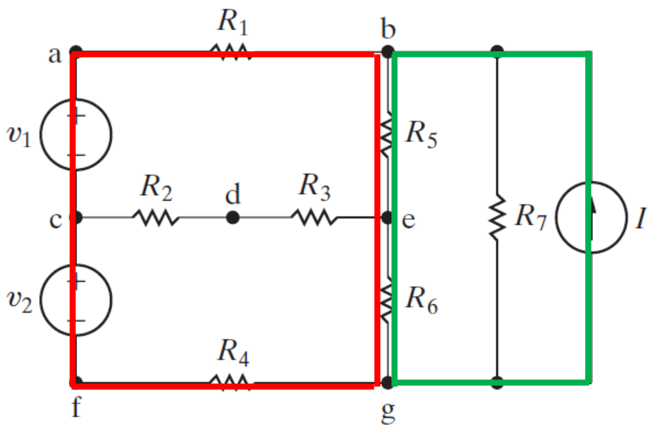
$v_2 - R_2 - R_3 - R_6 - R_4$, $R_5 - R_7 - R_6$,

and $R_7 - I$.

**f)** $R\_1 - R\_5 - R\_6$ is a path, but it is not a loop

(because it does not have the same starting and ending nodes), nor is it an essential branch

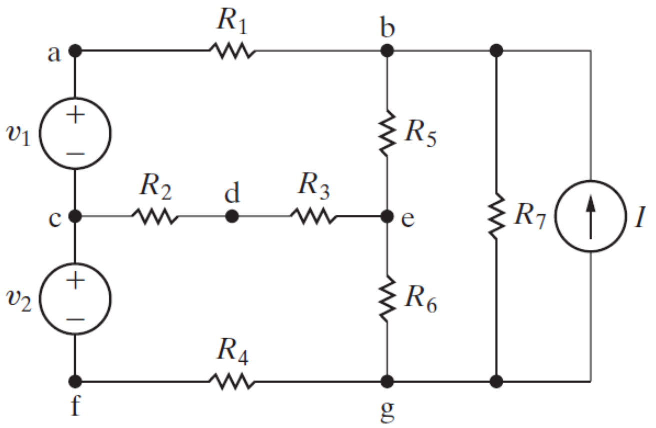
(because it does not connect two essential nodes).$v2 - R2$ is also a path but is neither a loop nor an essential branch, for the same reasons.




**g)** $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh, because there are two loops within it.

$I - R_5 - R_6$ is also a loop but not a mesh.

# TERMINOLOGY



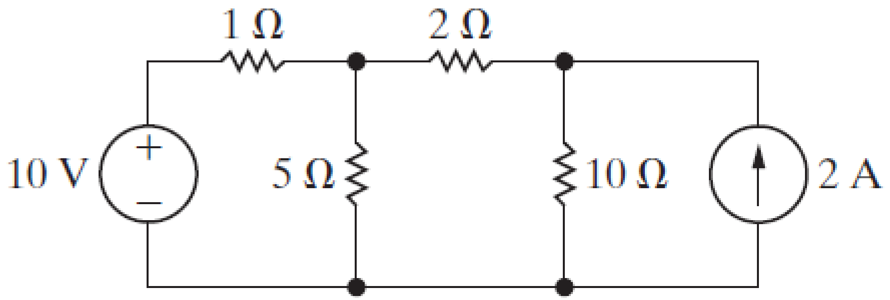

| | | |
|---|---|---|
| node | A point where two or more circuit elements join | a |
| essential node | A node where three or more circuit elements join | b |
| path | A trace of adjoining basic elements with no elements included more than once | $v_1 - R_1 - R_5 - R_6$ |
| branch | A path that connects two nodes | $R_1$ |
| essential branch | A path which connects two essential nodes without passing through an essential node | $v_1 - R_1$ |
| loop | A path whose last node is the same as the starting node | $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ |
| mesh | A loop that does not enclose any other loops | $v_1 - R_1 - R_5 - R_3 - R_2$ |

# NODE-VOLTAGE METHOD

In this method, first we select the reference node, i.e., the ground node, and then determine other nodes.

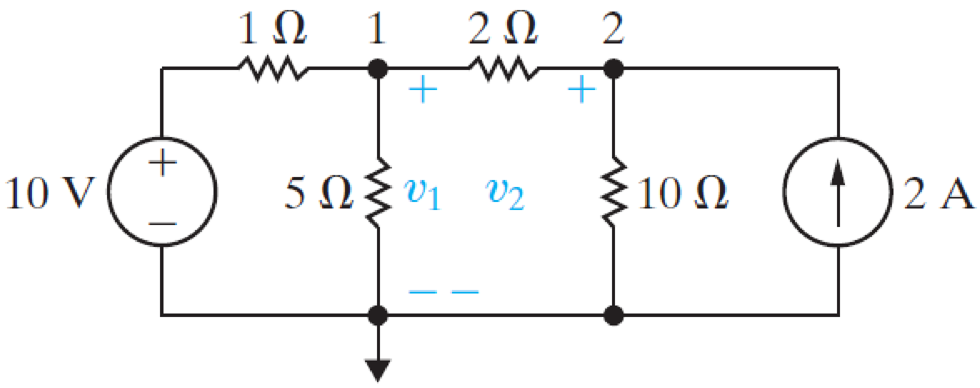
Then, we write the KCL equations for each node in terms of voltages and solve the equation set and determine all the voltages and currents.

**Example:** Analysis the circuit below using node-voltage method.

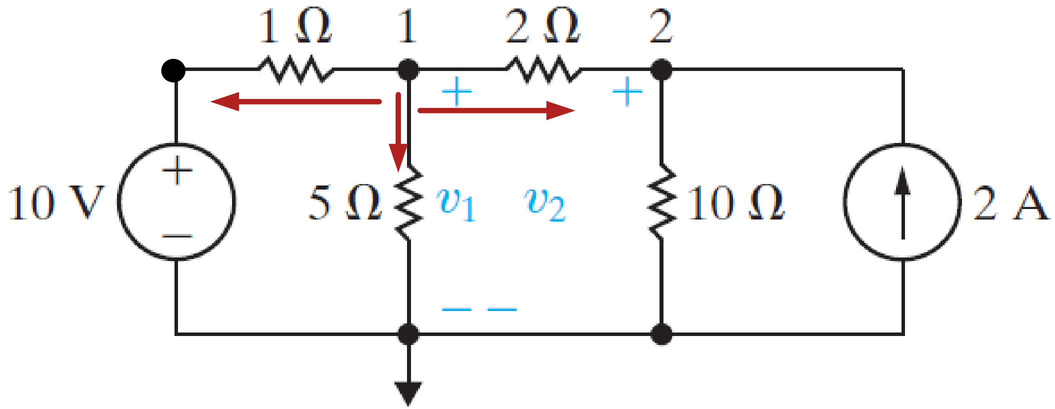


# NODE-VOLTAGE METHOD

**Solution:** We label the nodes and indicate the reference node as in




The currents for node-1 is shown below

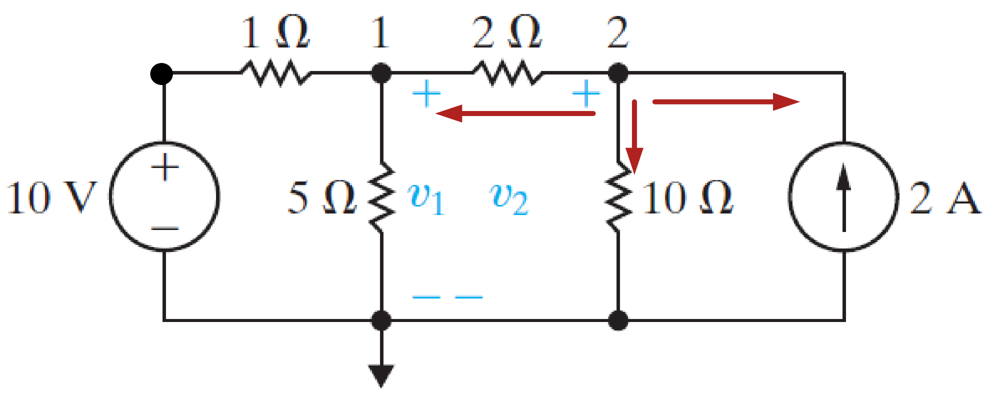



KCL at node-1 gives

$$\frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0 \qquad (Eq1)$$

# NODE-VOLTAGE METHOD

The currents for node-2 is shown below




KCL at node-2 gives

$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0 \qquad (Eq2)$$

Thus we have,

$$\frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0$$

$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0$$

We can solve the above equation set for $v_1$ and $v_2$ and obtain

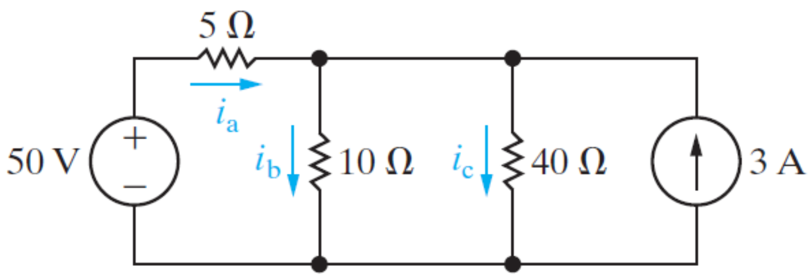
$$v_1 = 9.09\ V \quad v_2 = 10.91\ V$$

# NODE-VOLTAGE METHOD

**Example:**

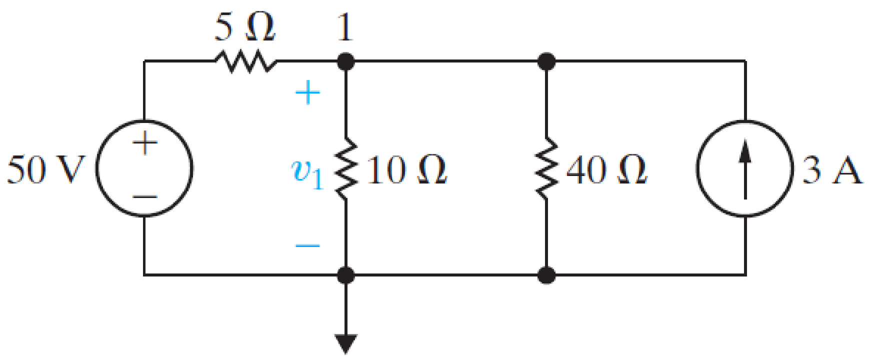
a) Use the node-voltage method of circuit analysis to find the branch currents in the circuit shown below.

b) Find the power associated with each source, and state whether the source is delivering or absorbing power.



# NODE-VOLTAGE METHOD

**Solution:** We label the nodes and indicate the reference node as in




KCL at node-1 gives

$$\frac{v_1 - 50}{5} + \frac{v_1}{10} + \frac{v_1}{40} - 3 = 0$$

from which we get

$$v_1 = 40\ V$$

Then, the currents are founds as

$$i_a = \frac{50 - v_1}{5} = \frac{50 - 40}{5} \to i_a = 2\ A \qquad i_b = \frac{v_1}{10} = \frac{40}{10} \to i_b = 4\ A$$

$$i_c = \frac{v_1}{40} = \frac{40}{40} \to i_b = 1\ A$$

# NODE-VOLTAGE METHOD

The power associated with the 50 $V$ source is

$$p_s = -50i_a = -100W \quad (delivering)$$

The power associated with the 3$A$ $V$ source is

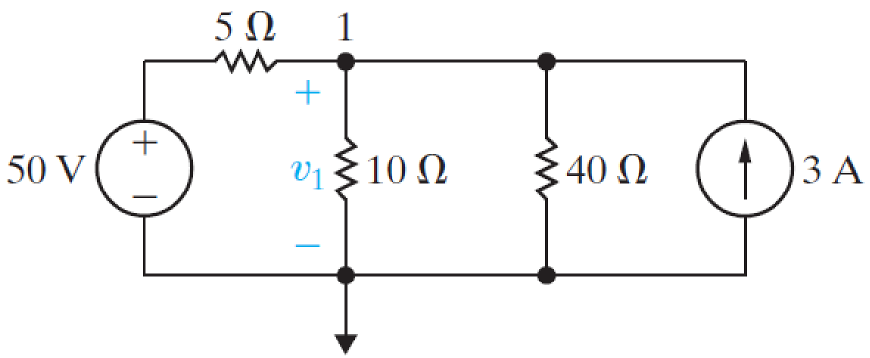
$$p_s = -40 \times 3 = -120W \quad (delivering)$$

The total delivered power is

$$100 + 120 = 220W$$

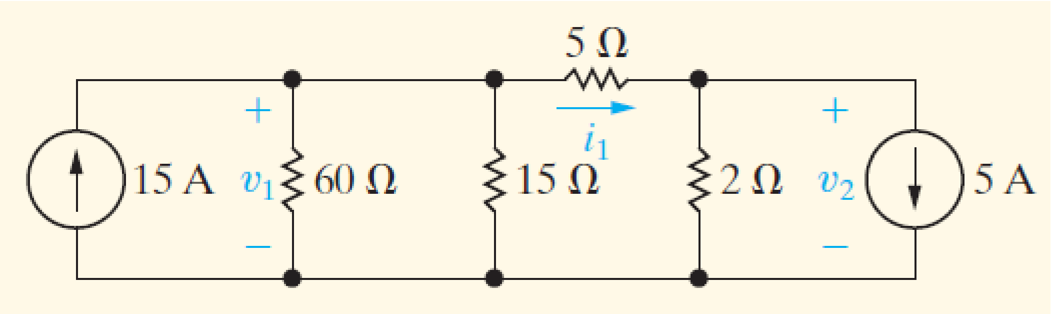
The total power consumed on resistors is

$$i_a^2 \times 5 + i_b^2 \times 10 + i_c^2 \times 40 \ = 220W$$



# NODE-VOLTAGE METHOD

**Example:** For the circuit shown, use the node-voltage method to find $v_1, v_2$ and $i_1$



# NODE-VOLTAGE METHOD

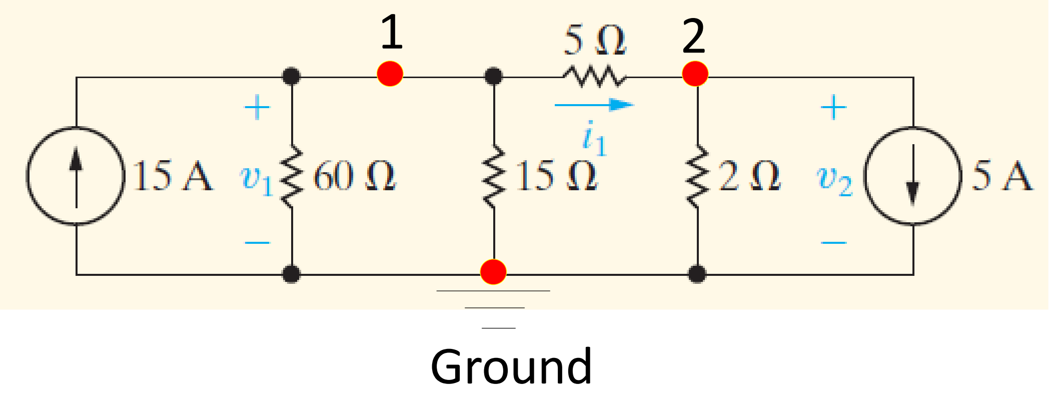
**Solution:** The nodes are labeled as in

KCL at node-1 gives

$$-15+\frac{v_1}{60}+\frac{v_1}{15}+\frac{(v_1-v_2)}{5}=0 \qquad (Eq1)$$

KCL at node-2 gives

$$\frac{(v_2-v_1)}{5}+\frac{v_2}{2}+5=0 \qquad (Eq2)$$




These sets of equations can be solved, and we find

$$v_1=60\ V \qquad\qquad v_2=10V$$

The current $i_1$ can be calculated as

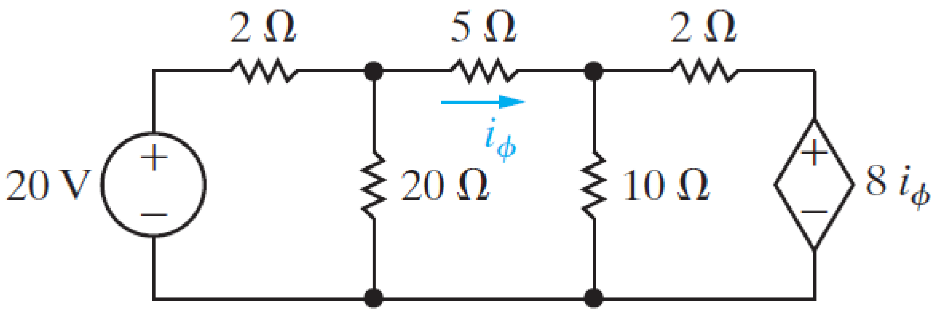
$$i_1=\frac{v_1-v_2}{5}\rightarrow i_1=10\ A$$

# NODE-VOLTAGE METHOD

**The Node-Voltage Method and Dependent Sources**

If the circuit contains dependent sources, the node-voltage equations must be supplemented with the constraint equations imposed by the presence of the dependent sources.

**Example:** Use the node-voltage method to find the power dissipated in the 5Ω resistor in the circuit shown



# NODE-VOLTAGE METHOD

**Solution:** The nodes are labeled  with blue colors

From circuit we can write directly

At node-1

$$v_1 = 20\ V$$

KCL at node-2

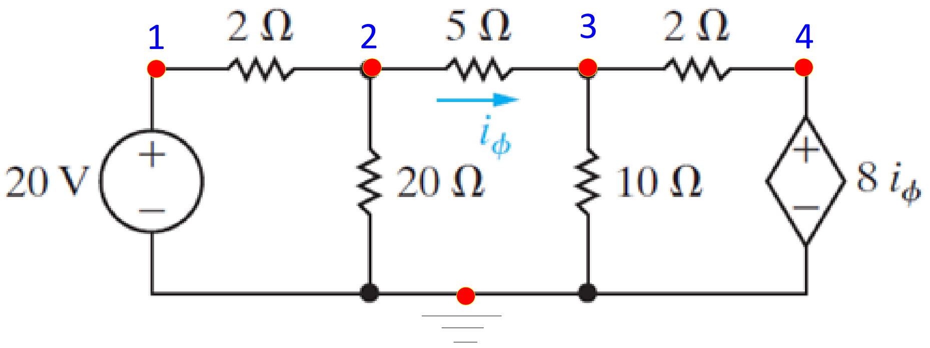
$$\frac{v_2 - 20}{2} + \frac{v_2}{20} + \frac{v_2 - v_3}{5} = 0$$

KCL at node-3

$$\frac{(v_3 - v_2)}{5} + \frac{v_3}{10} + \frac{v_3 - v_4}{2} = 0$$

At node-4

$$v_4 = 8i_\phi \quad i_\phi = \frac{(v_2 - v_3)}{5} \rightarrow v_4 = \frac{8}{5}(v_2 - v_3)$$



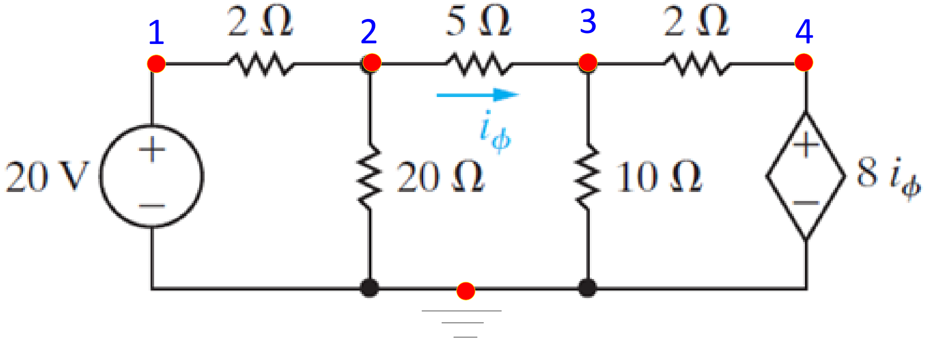
# NODE-VOLTAGE METHOD

**Solution:** The nodes are labeled  with blue colors

$$\frac{v_2 - 20}{2} + \frac{v_2}{20} + \frac{v_2 - v_3}{5} = 0 \qquad (Eq1)$$

$$\frac{(v_3 - v_2)}{5} + \frac{v_3}{10} + \frac{v_3 - v_4}{2} = 0 \qquad (Eq2)$$

$$v_4 = \frac{8}{5}(v_2 - v_3) \qquad (Eq3)$$



# NODE-VOLTAGE METHOD

$$\frac{v_2 - 20}{2} + \frac{v_2}{20} + \frac{v_2 - v_3}{5} = 0 \qquad (Eq1)$$

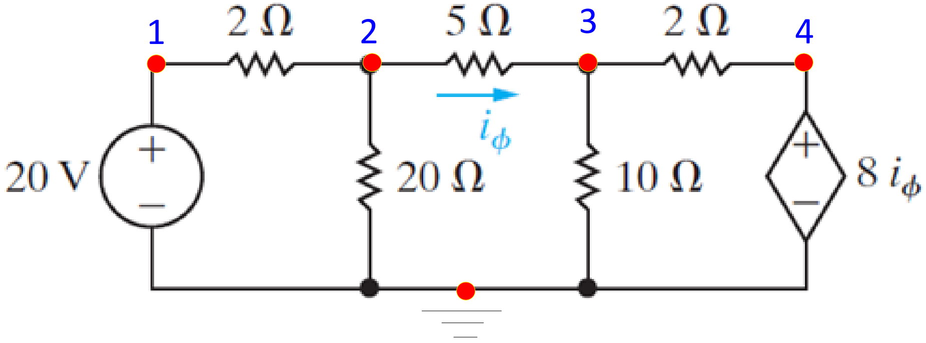
$$\frac{(v_3 - v_2)}{5} + \frac{v_3}{10} + \frac{v_3 - v_4}{2} = 0 \qquad (Eq2)$$

$$v_4 = \frac{8}{5}(v_2 - v_3) \qquad (Eq3)$$

We can substitute $Eq3$ into $Eq2$ and we get

$$\frac{v_2 - 20}{2} + \frac{v_2}{20} + \frac{v_2 - v_3}{5} = 0 \qquad (Eq1)$$

$$\frac{(v_3 - v_2)}{5} + \frac{v_3}{10} + \frac{v_3 - \frac{8}{5}(v_2 - v_3)}{2} = 0 \qquad (Eq2)$$




This equation set can be solved for $v_2$ and $v_3$ and we get

$$v_2 = 16\,V \quad v_3 = 10\,V$$

The current $i_\phi$ is calculated as

$$i_\phi = \frac{(v_2 - v_3)}{5} \rightarrow i_\phi = \frac{16 - 10}{5} = 1.2\,A$$

The power dissipated on 5Ω resistor is

$$p = (v_2 - v_3)i_\phi \rightarrow p = 6 \times 1.2 = 7.2\,W$$

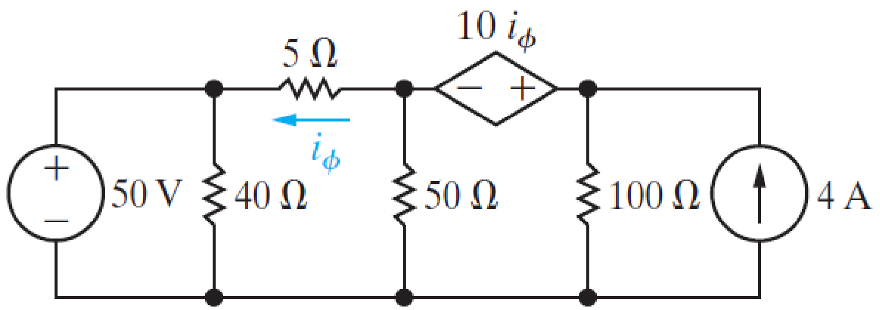
# NODE-VOLTAGE METHOD

**The Node-Voltage Method: Super Node**

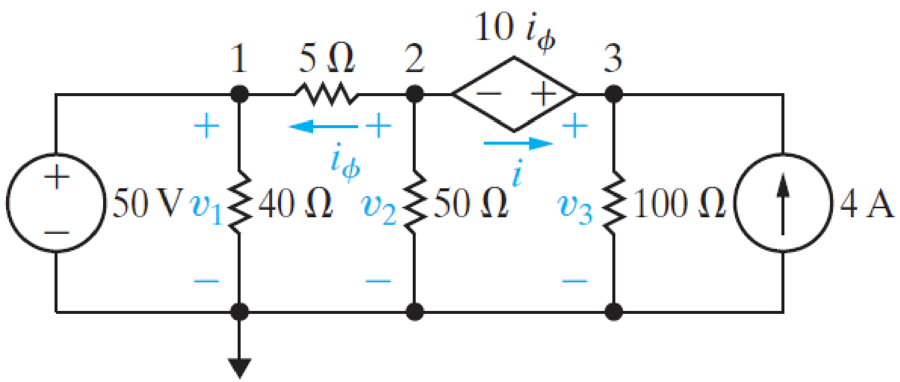
Note that KCL can be used **for any closed surface**, not for nodes only. A super node is formed drawing a closed surface around 2 or more nodes

We accept the closed surface as a single node. That is called super node.

**Example:** Consider the circuit below.

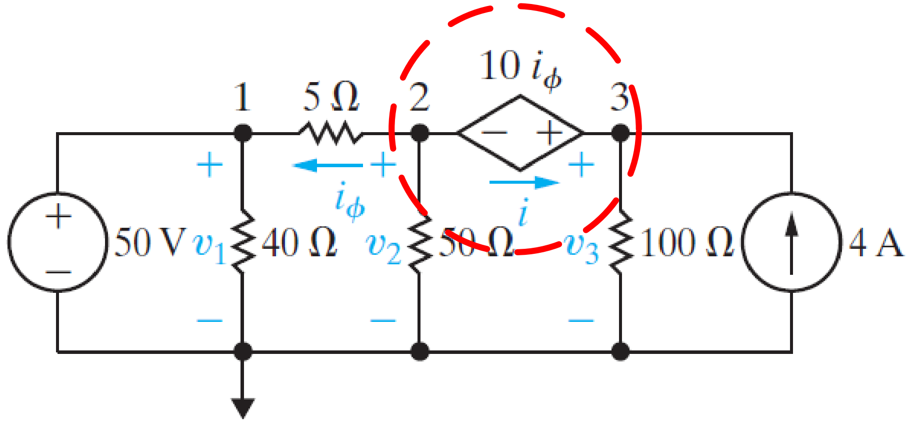



The nodes are labeled as



# NODE-VOLTAGE METHOD

The super-node can be formed as




For the above circuit

$$v_1 = 50\ V$$

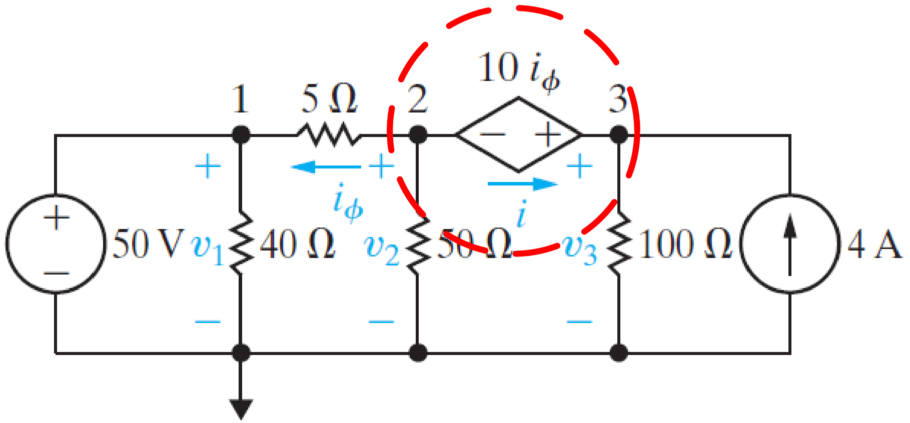
KCL at super-node

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0 \quad (Eq1)$$

and we have

$$v_3 - v_2 = 10 i_\phi \qquad i_\phi = \frac{v_2 - v_1}{5}$$

# NODE-VOLTAGE METHOD




Then

$$v_3 - v_2 = 10 \times \frac{v_2 - v_1}{5} \rightarrow$$

$$v_3 - v_2 = 2v_2 - 2v_1 \rightarrow$$

$$v_3 - 3v_2 + 2v_1 = 0 \qquad (Eq2)$$

Thus we have

$$v_1 = 50\ V$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0 \qquad (Eq1)$$

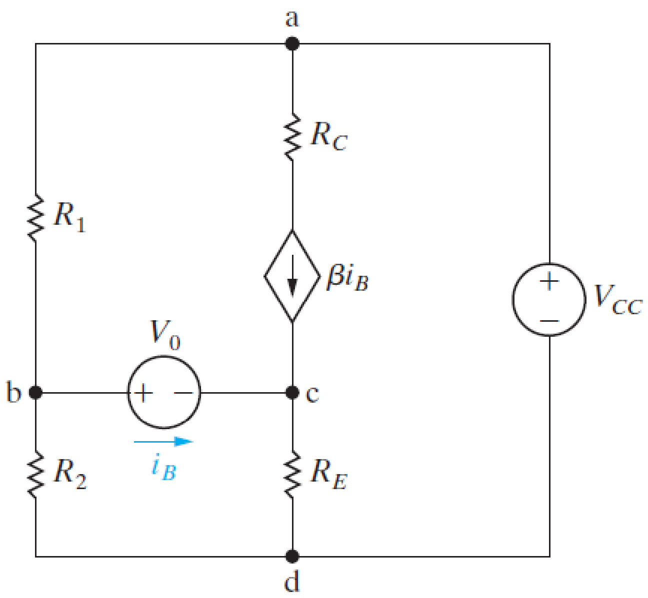
$$v_3 - 3v_2 + 2v_1 = 0 \qquad (Eq2)$$

and this equation set can be solved for $v_2$ and $v_3$ and we find that

$$v_2 = 60\ V \qquad v_3 = 80\ V$$

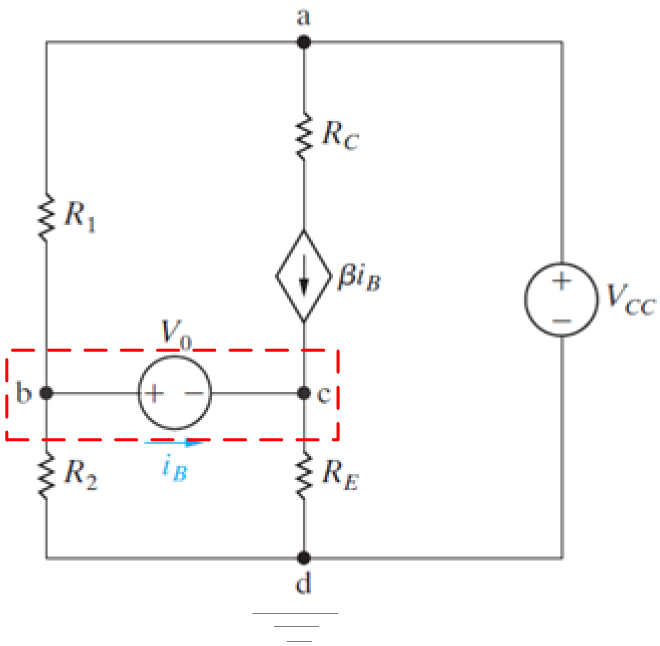
# NODE-VOLTAGE METHOD

**Example:** Find the node voltage $v_b$ for the circuit below



# NODE-VOLTAGE METHOD

**Solution:** The super node is shown in Figure below




$$v_a = V_{CC}$$

KCL at super node

$$\frac{v_b - v_a}{R_1} + \frac{v_b}{R_2} + \frac{v_c}{R_E} - \beta i_\beta = 0 \quad Eq1$$

And we have

$$v_c = (i_\beta + \beta i_\beta) \times R_E \qquad v_b - v_c = V_0$$

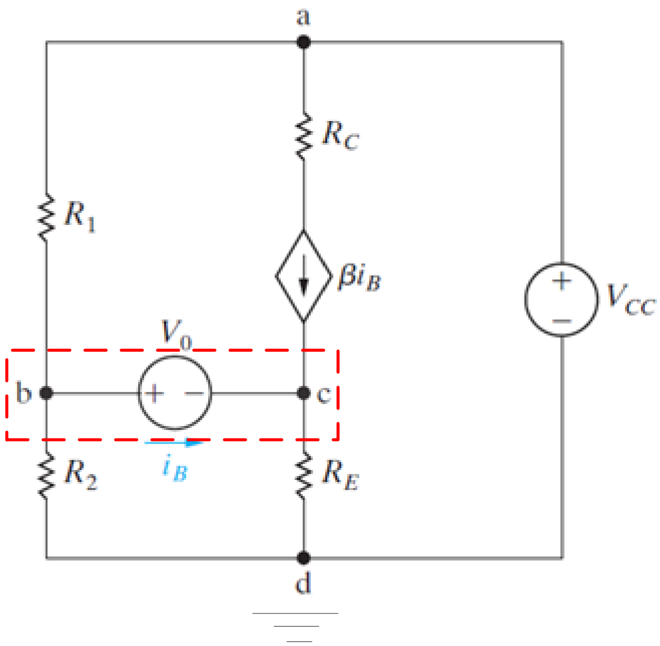
from which we can write

$$v_c = v_b - V_0 \quad i_\beta = \frac{v_c}{(1+\beta)R_E} \rightarrow i_\beta = \frac{v_b - V_0}{(1+\beta)R_E} \quad Eq2$$

When $Eq2$ is used in Eq1, we get

$$\frac{v_b - V_{CC}}{R_1} + \frac{v_b}{R_2} + \frac{v_b - V_0}{R_E} - \beta \frac{v_b - V_0}{(1+\beta)R_E} = 0$$

# NODE-VOLTAGE METHOD




$$\frac{v_b - V_{CC}}{R_1} + \frac{v_b}{R_2} + \frac{v_b - V_0}{R_E} - \beta\frac{v_b - V_0}{(1+\beta)R_E} = 0$$

which can be written as

$$\frac{v_b}{R_1} + \frac{v_b}{R_2} + \frac{v_b}{R_E} - \beta\frac{v_b}{(1+\beta)R_E} = \frac{V_{CC}}{R_1} + \frac{V_0}{R_E} - \beta\frac{V_0}{(1+\beta)R_E}$$

which is simplified as

$$\frac{v_b}{R_1} + \frac{v_b}{R_2} + \frac{v_b}{(1+\beta)R_E} = \frac{V_{CC}}{R_1} + \frac{V_0}{(1+\beta)R_E}$$

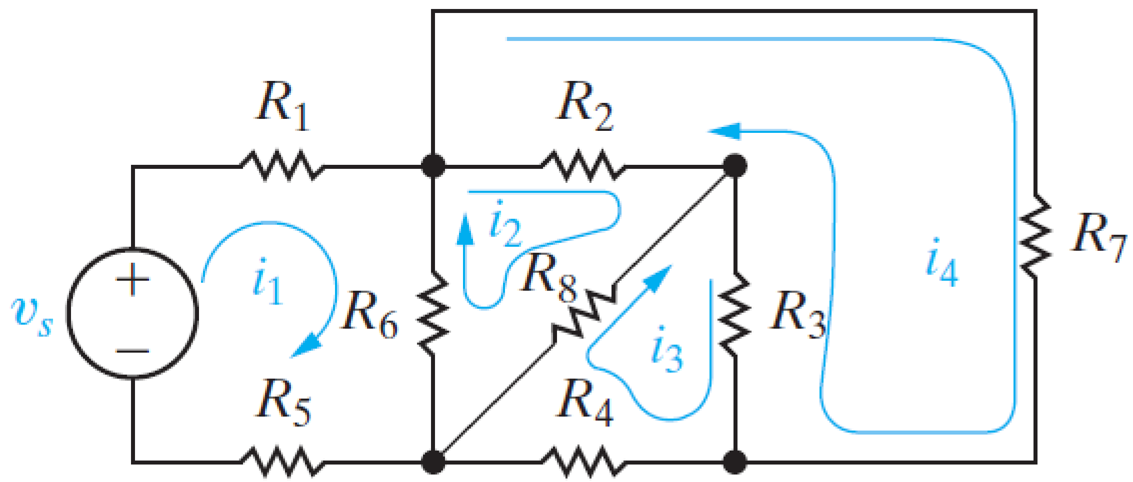
from which we obtain

$$v_b = \frac{V_{CC}R_2(1+\beta)R_E + V_0R_1R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)}$$

# THE MESH-CURRENT METHOD

A mesh current is the current that exists only in the perimeter of a mesh. On a circuit diagram it appears as either a closed solid line or an almost-closed solid line that follows the perimeter of the appropriate mesh.

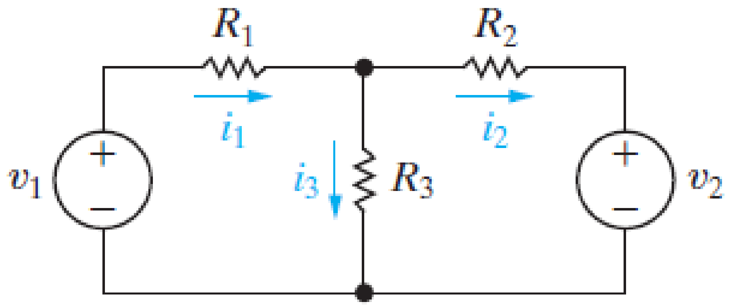
An arrowhead on the solid line indicates the reference direction for the mesh current.



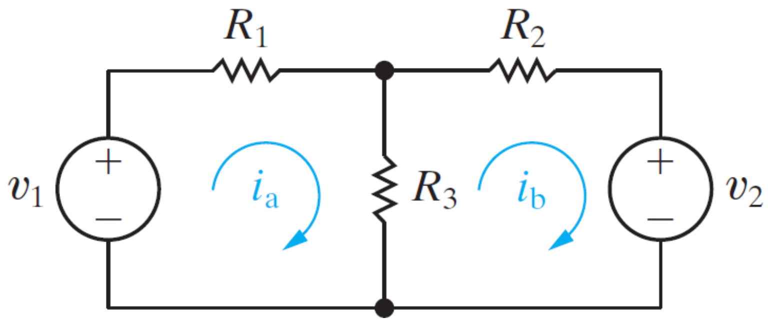

Note that mesh currents are not the branch currents.

# THE MESH-CURRENT METHOD

**Example:** The branch currents are shown in the circuit below




The mesh currents for the above circuit is given as

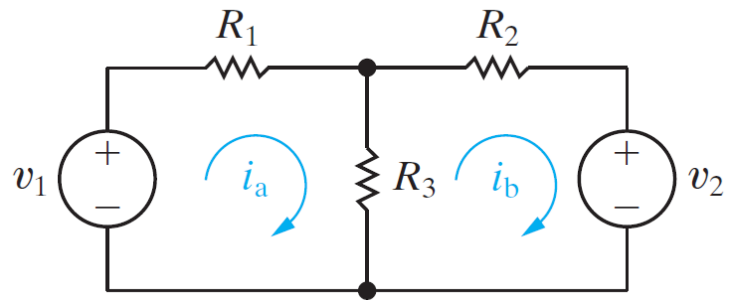



The relationships between branch and mesh currents are given as

$i_1 = i_a$ $\qquad$ $i_2 = i_b$ $\qquad$ $i_3 = i_a - i_b$

# THE MESH-CURRENT METHOD

The mesh equations for the below circuit are written as




KVL for the loop-a

$$-v_1 + i_a R_1 + (i_a - i_b)R_3 = 0$$

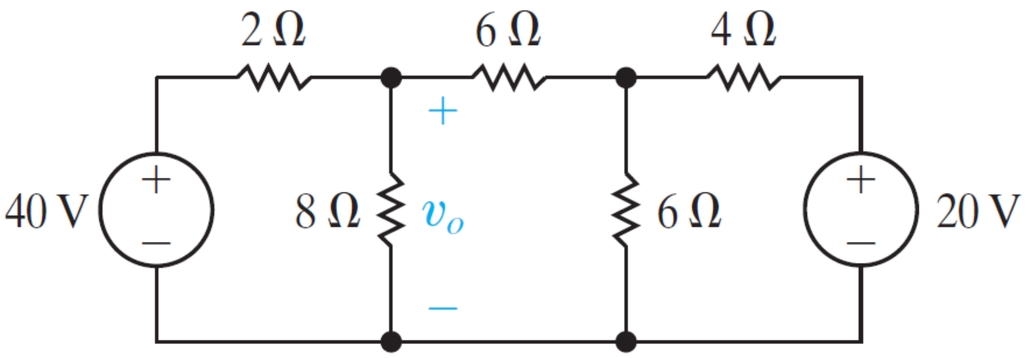
KVL for the loop-b

$$i_b R_2 + v_2 + (i_b - i_a)R_3 = 0$$

These two equations can be used to solve $i_a$ and $i_b$

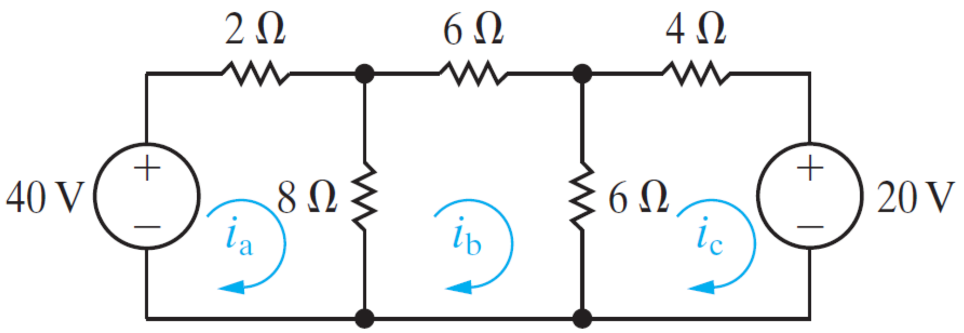
# THE MESH-CURRENT METHOD

**Example:** Use the mesh-current method to determine the power associated with each voltage source in the circuit shown in



# THE MESH-CURRENT METHOD

**Solution:** The mesh currents are shown in the circuit




The mesh equations can b written as

Mesh-a:

$$-40 + 2i_a + 8(i_a - i_b) = 0$$

Mesh-b:

$$8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0$$

Mesh-c:

$$6(i_c - i_b) + 4i_c + 20 = 0$$

The equations can be simplified as

$$10i_a - 8i_b = 40$$

$$-8i_a + 20i_b - 6i_c = 0$$

$$-6i_b + 10i_c = -20$$

when these equations are solved, we get

$$i_a = 5.6\ A \qquad i_b = 2.0\ A \qquad i_c = -0.80\ A$$