CIRCUIT THEORY I

Lecture 2

Assoc. Prof. Hulusi Açıkgöz

IDEAL SOURCE CONCEPT

Because real sources are complex, circuit theory uses **idealized models** for simplicity.

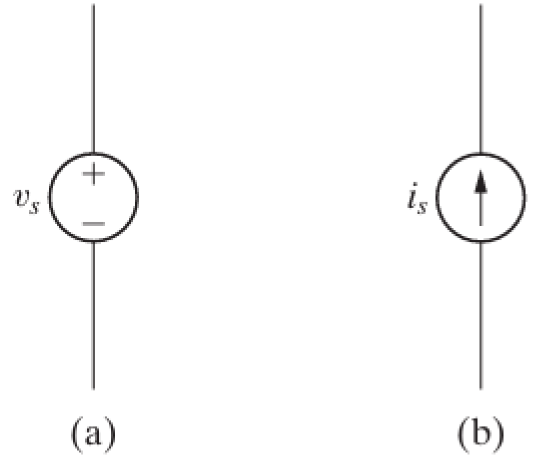
•**Ideal Voltage Source**

- Maintains a *fixed voltage* across its terminals, regardless of the current drawn.
- The current may vary freely, but voltage remains constant.

•**Ideal Current Source**

- Maintains a *fixed current* through its terminals, regardless of the voltage across them.
- The voltage may vary, but current remains constant.

⚠ **Note:** Ideal sources don't exist in reality; they are mathematical abstractions used for analysis.



The circuit symbols for (a) an ideal independent voltage source and (b) an ideal independent current source.

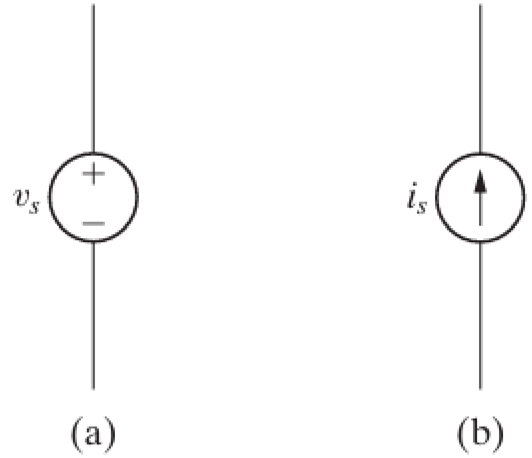
IDEAL SOURCE CONCEPT

Modeling Limitation

•For an **ideal voltage source**, current cannot be determined from voltage alone.

•For an **ideal current source**, voltage cannot be determined from current alone.

•This trade-off simplifies analysis but removes physical realism.



The circuit symbols for (a) an ideal independent voltage source and (b) an ideal independent current source.

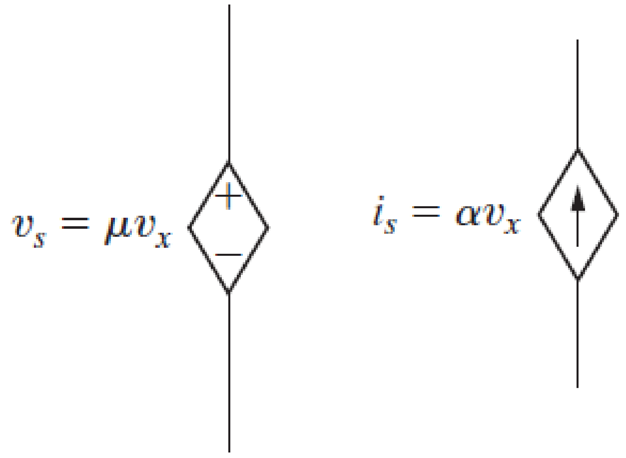
To completely specify an ideal independent voltage source in a circuit, you must include the value of the supplied voltage and the reference polarity.

Similarly, to completely specify an ideal independent current source, you must include the value of the supplied current and its reference direction.

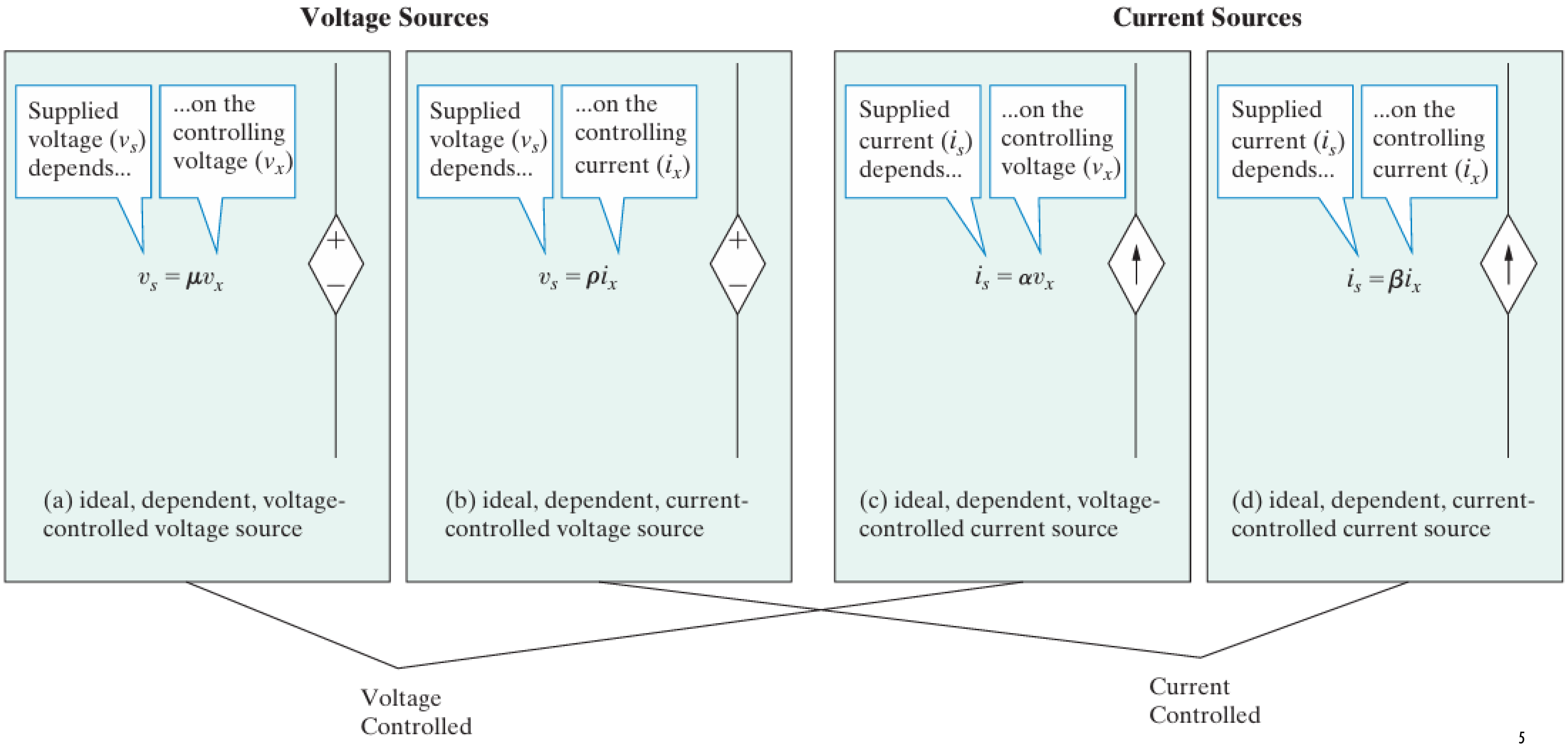
INDEPENDENT VS. DEPENDENT SOURCES

- **Independent Source:**
  - Provides a fixed voltage or current *unaffected by other circuit variables.*
  - Symbol: **Circle** (with polarity or direction and value indicated).

- **Dependent Source (Controlled Source):**
  - Output depends on another voltage or current elsewhere in the circuit.
  - Cannot be specified until the controlling variable is known.
  - Symbol: **Diamond** shape.
  - There are four possible types, since both voltage sources and current sourc controlled by either a voltage or a current elsewhere in the circuit.
  - Dependent sources are also called controlled sources.
    - Voltage-Controlled Voltage Source (VCVS)
    - Voltage-Controlled Current Source (VCCS)
    - Current-Controlled Voltage Source (CCVS)
    - Current-Controlled Current Source (CCCS)



INDEPENDENT VS. DEPENDENT SOURCES



INDEPENDENT VS. DEPENDENT SOURCES

Type	Output	Controlled by	Equation	Constant	Units
VCVS	Voltage source	Voltage (v_x)	($v_s = \mu\ v_x$)	(μ)	Dimensionless
CCVS	Voltage source	Current (i_x)	($v_s = i_x$)	()	Volts per ampere (Ω)
VCCS	Current source	Voltage (v_x)	($i_s = \ v_x$)	()	Amperes per volt (S)
CCCS	Current source	Current (i_x)	($i_s = \ i_x$)	()	Dimensionless

Each constant (μ, ρ, α, β) defines the *gain or transconductance* between the controlling and controlled quantities.

DC (DIRECT CURRENT) SOURCES

- Ideal sources (independent or dependent) are **time-invariant —** they produce **constant voltages or currents**.

- These are referred to as **DC sources**:

  - "DC voltage" → constant voltage
  - "DC current" → constant current

- Historically, *direct current* meant current produced by a steady voltage (as opposed to alternating). The "DC" label now simply means **constant**, not necessarily "non-reversing."

Active vs. Passive Elements

- **Active elements**: capable of **generating electrical energy** (e.g., voltage and current sources).
- **Passive elements: cannot generate** energy; they **store or dissipate** it (e.g., resistors, capacitors, inductors).
- Thus:
  - *Sources → Active*
  - *R, L, C → Passive*

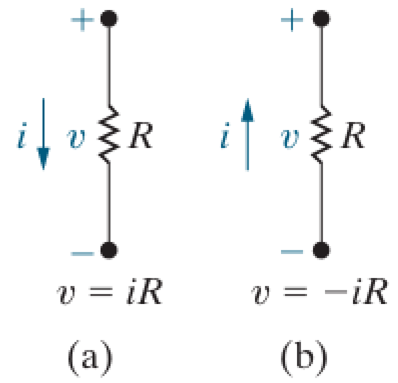
OHM'S LAW AND ELECTRICAL RESISTANCE

Mathematical Model — Ohm's Law

- The **ideal resistor** is defined by a simple linear relationship between **voltage (v)** and **current (i)**:

$$v = iR$$

- where:
  - v= voltage across the resistor (V)
  - i= current through the resistor (A)
  - R= resistance (Ω)
- This equation is known as **Ohm's Law,** discovered by **Georg Simon Ohm** in the early 19th century.



Passive Sign Convention

- The direction of current and voltage is taken such that **current enters the positive terminal** of the resistor.
  - In this orientation, the resistor **absorbs energy** and **dissipates power as heat.**

OHM'S LAW AND ELECTRICAL RESISTANCE

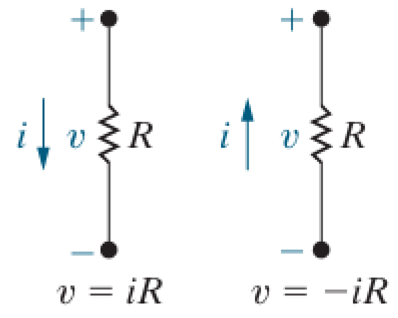
Polarity and Sign Convention

- Depending on the **direction of current** and **voltage reference,** Ohm's law can take two algebraic forms:

  $v = iR$ (current enters the positive terminal — passive sign convention)
  $v = -iR$ (current enters the negative terminal — reversed polarity)

- Both equations describe the same physical resistor but with **opposite reference directions.**

- The **passive sign convention** (PSC) is used throughout analysis:
  → Current enters the positive voltage terminal → the element **absorbs power.**



(a) (b)

OHM'S LAW AND ELECTRICAL RESISTANCE

Units and Notation

- Resistance is measured in **ohms (Ω).**
- **1 ohm (1 Ω)** = the resistance that allows **1 A** of current when **1 V** is applied.
- Example: An **8 Ω resistor** symbol would be labeled "8 Ω" on the circuit diagram.

OHM'S LAW AND ELECTRICAL RESISTANCE

Inverse Relationship — Conductance

- Ohm's Law can also express **current** as a function of **voltage**:

$$i = \frac{v}{R} \; or \, i = -\frac{v}{R}$$

- The **reciprocal** of resistance is called **conductance (G):**

$$G = \frac{1}{R}$$

- Units: **siemens (S)** (formerly mho, symbol ℧).
- Example: An **8 Ω resistor** has a **conductance of 0.125 S.**

OHM'S LAW AND ELECTRICAL RESISTANCE

The Ideal Resistor Model

- **Ideal resistors** are **simplified representations** of real resistive devices.
- Key assumption: **Resistance (R) is constant** over time and does not change with:
    - Temperature
    - Voltage
    - Current
- In reality, most resistors' values vary slightly due to **temperature changes,** but in circuit analysis, this variation is negligible.
- Thus, the **ideal resistor** assumes a **linear, time-invariant relationship** between voltage and current.

POWER IN A RESISTOR

Power Definition

- The **instantaneous power** at the terminals of any element is:

$$p = vi$$

- For a resistor using the **passive sign convention:**
  - If current enters the **positive voltage terminal,**

$$p = vi$$

  - If current enters the **negative voltage terminal,**

$$p = -vi$$

POWER IN A RESISTOR

Power in Terms of Current

- Substitute **Ohm's Law** ($v = iR$) into $p = vi$:

$$p = (iR)i = i^2R$$

Thus,

$$\boxed{p = i^2R}$$

- This shows that **power dissipated** in a resistor is **always positive,** regardless of current direction or voltage polarity — resistors always **absorb power** (convert electrical energy into heat).

POWER IN A RESISTOR

Power in Terms of Voltage

- Substitute $i = \frac{v}{R}$ into $p = vi$:

$$p = v\left(\frac{v}{R}\right) = \frac{v^2}{R}$$

Hence,

$$\boxed{p = \frac{v^2}{R}}$$

- This form is useful when **voltage** is known but **current** is not.

POWER IN A RESISTOR

Power in Terms of Conductance

- Since $G = \frac{1}{R}$,power expressions can also be written as:

$$p = i^2/G \; and p = v^2G$$

- These are alternative formulations when a resistor's **conductance** is specified instead of its resistance.

Interpretation

- These equations provide **multiple equivalent ways** to calculate the **same power dissipation**:

$$p = vi = i^2R = \frac{v^2}{R} = \frac{i^2}{G} = v^2G$$

- Regardless of sign conventions, **power in a resistor is always positive,** meaning resistors **absorb** energy rather than generate it.
- Physically, this energy is converted into **thermal energy (heat),** which is dissipated into the surroundings.

KIRCHHOFF’S LAWS

Purpose of Kirchhoff’s Laws

- **Ohm’s law** describes the relationship between voltage, current, and resistance for a single element, but it’s **not sufficient** to analyze entire circuits.

- To find all **voltages and currents** in a network, two additional algebraic relationships are needed:

  - **Kirchhoff’s Current Law (KCL)**
  - **Kirchhoff’s Voltage Law (KVL)**

Together, these laws enable the complete **solution of circuits**.

KIRCHHOFF'S LAWS

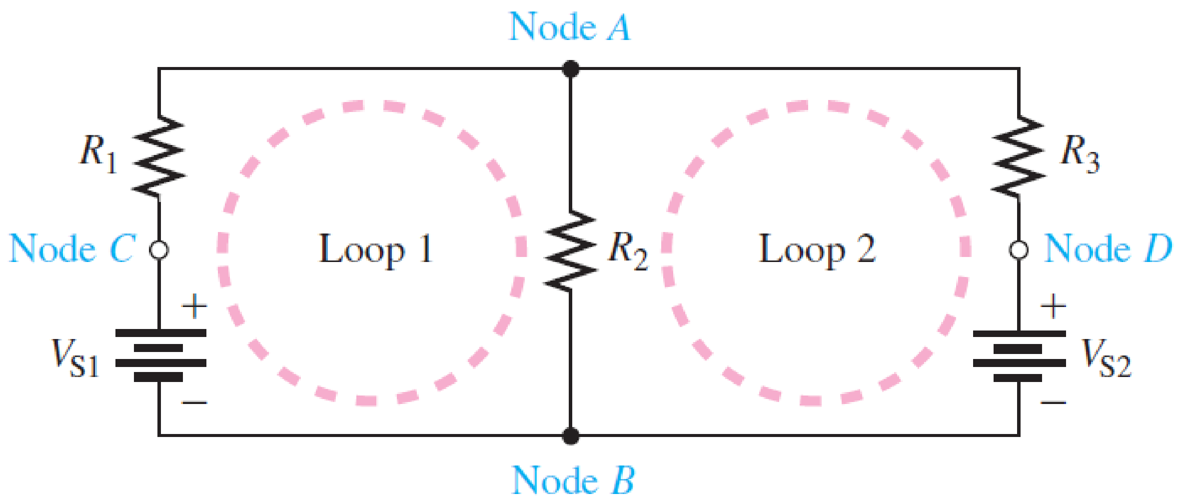
Introducing Kirchhoff's Laws

- To obtain the missing equations, we apply **Kirchhoff's laws**, which are based on fundamental **conservation principles**:
    - **Kirchhoff's Current Law (KCL)** — based on **conservation of charge.**
    - **Kirchhoff's Voltage Law (KVL)** — based on **conservation of energy.**
- These laws were first proposed by **Gustav Robert Kirchhoff** in **1848.**

KIRCHHOFF'S LAWS

Node
A node is a point where two or more circuit elements meet.

Loop
A loop is a complete current path within a circuit



KIRCHHOFF’S LAWS

Kirchhoff’s current law (KCL)

The algebraic sum of all the currents entering or leaving a closed surface equals zero.

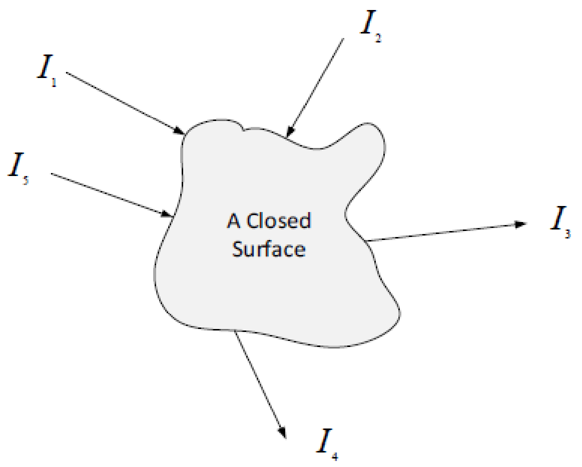
Node can be considered as the smallest closed surface.

Kirchhoff's current law can also be stated as: The algebraic sum of all the currents at any node with a reference direction (all entering or all leaving) in a circuit equals zero.

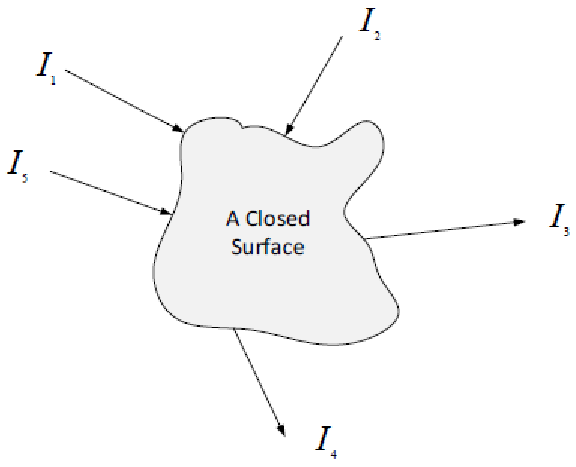
That is,

Energy flow in = Energy flow out

Energy flow in - Energy flow out = 0



KIRCHHOFF'S LAWS



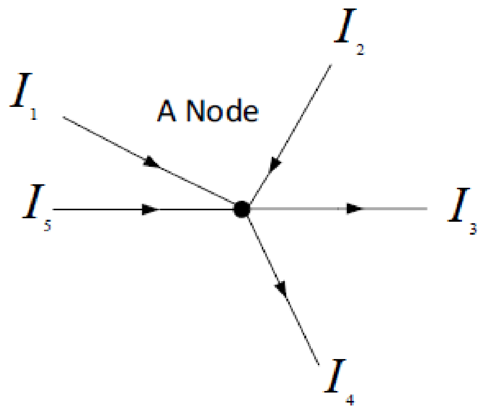
The sum of the current entering the closed surface equals zero, i.e.,

$$I1 + I2 - I3 - I4 + I5 = 0 \qquad E1$$

The sum of the current leaving the closed surface equals zero, i.e.,

$$-I1 - I2 + I3 + I4 - I5 = 0 \qquad E2$$

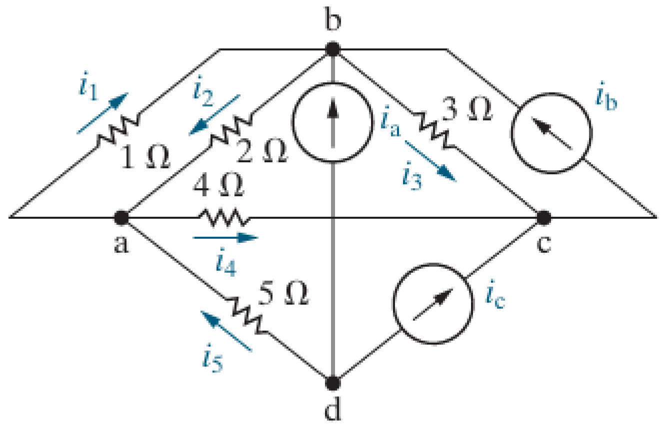
In fact, $E1 = -E2$



KIRCHHOFF'S LAWS

EXAMPLE: Using Kirchhoff's Current Law

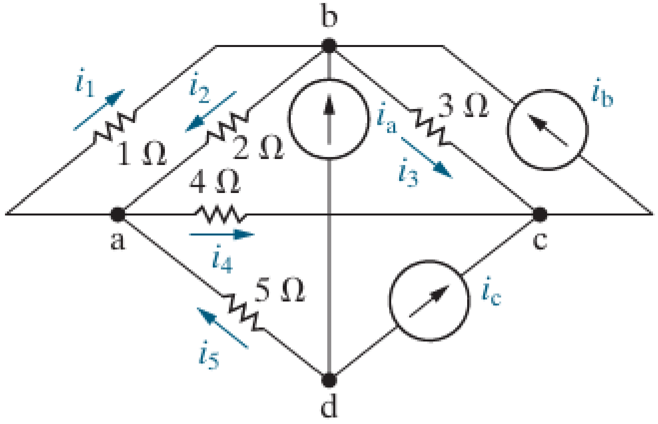
Sum the currents at each node in the circuit shown. Note that there is no connection dot (•) in the center of the diagram, where the 4 Ω branch crosses the branch containing the ideal current source i_a.



KIRCHHOFF'S LAWS

EXAMPLE: Using Kirchhoff's Current Law

Sum the currents at each node in the circuit shown. Note that there is no connection dot (•) in the center of the diagram, where the 4 Ω branch crosses the branch containing the ideal current source ia.



Solution

In writing the equations, we use a positive sign for a current leaving a node. The four equations are

$$\text{node a} \quad i_1 + i_4 - i_2 - i_5 = 0,$$

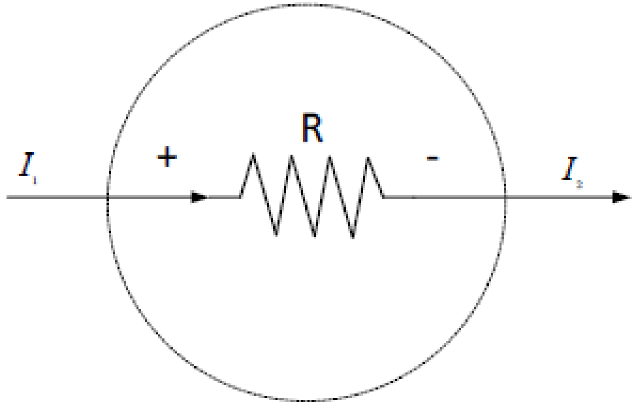
$$\text{node b} \quad i_2 + i_3 - i_1 - i_b - i_a = 0,$$

$$\text{node c} \quad i_b - i_3 - i_4 - i_c = 0,$$

$$\text{node d} \quad i_5 + i_a + i_c = 0.$$

KIRCHHOFF'S LAWS

Example:



For the above circuit if we use KCL

$$i_1 - i_2 = 0$$

KIRCHHOFF'S LAWS

Kirchhoff's Voltage Law (KVL)

The algebraic sum of all the voltages around any closed path in a circuit equals zero.

- Kirchhoff's Voltage Law (KVL) deals with the voltages around a closed path (loop) in a circuit.
- It is based on the principle of energy conservation:

The algebraic sum of all voltages around any closed loop is zero.

$$\sum v = 0$$

KIRCHHOFF'S LAWS

Assigning Voltage Signs

To apply KVL correctly, each voltage must have a **reference direction** (sign convention).

As you trace a **closed loop** in the circuit:

- When the **tracing direction moves from the negative terminal to the positive terminal** of an element → it is a **voltage rise (+v).**
- When the **tracing direction moves from the positive terminal to the negative terminal** → it is a **voltage drop (−v).**

KIRCHHOFF'S LAWS

Sign Convention Options

- You can choose either convention — just be **consistent** throughout the loop:
  - If you define a **voltage rise as positive**, then a **voltage drop is negative**.
  - If you define a **voltage rise as negative**, then a **voltage drop is positive**.

KVL ensures that the **sum of all rises and drops** in a loop equals zero because any energy gained (from sources) is exactly balanced by energy lost (across resistors and loads).

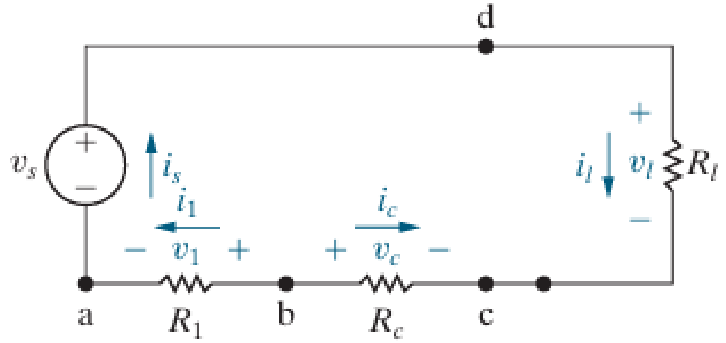
$$\Sigma\ (Voltage\ Rises) = \Sigma\ (Voltage\ Drops)$$

KIRCHHOFF'S LAWS

Step-by-step KVL example

1. Choose current direction & reference polarities

Assume a clockwise loop current i. Use the passive sign convention on each resistor (current enters the $+$terminal), so:



$$v_1 = iR_1, v_c = iR_c, v_l = iR_l.$$

Let the source be oriented as a **rise** of $+v_s$ when we go clockwise through it.

2. Write KVL around the closed loop

Traverse clockwise starting at the source:

- Source: **voltage rise** $+v_s$
- Resistor drops: $-v_1$,$-v_c$,$-v_l$

So,

$$+v_s - v_1 - v_c - v_l = 0.$$

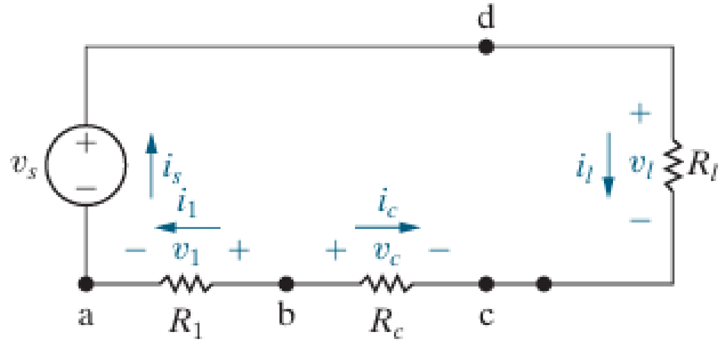
KIRCHHOFF'S LAWS

3. Substitute Ohm's law

$$v_s - (iR_1) - (iR_c) - (iR_l) = 0 \;\Rightarrow\; v_s - i(R_1 + R_c + R_l) = 0.$$

4. Solve for the loop current

$$\boxed{i = \frac{v_s}{R_1 + R_c + R_l}}$$



5. Back-solve element voltages

$v_1 = iR_1 = \frac{R_1}{R_1+R_c+R_l}\, v_s, v_c = \frac{R_c}{R_1+R_c+R_l}\, v_s, v_l = \frac{R_l}{R_1+R_c+R_l}\, v_s$. (Each drop is a fraction of $v_s$proportional to its resistance.)

6. Sign consistency note

If you traverse the loop the **opposite** way, treat the source as a **drop** $(-v_s)$and each resistor as a **rise** $(+v_k)$; you'll get $-v_s + v_1 + v_c + v_l = 0$,which is algebraically identical.

KIRCHHOFF'S LAWS

Quick numeric check

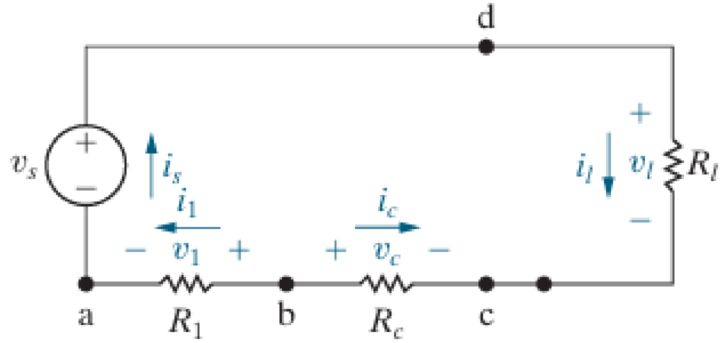
Let $v_s = 3\ V, R_1 = 1\ \Omega, R_c = 2\ \Omega, R_l = 3\ \Omega$.

$$i = \frac{3}{1+2+3} = \frac{3}{6} = 0.5\ A$$

$$v_1 = 0.5\ V, v_c = 1.0\ V, v_l = 1.5\ V$$

KVL check:

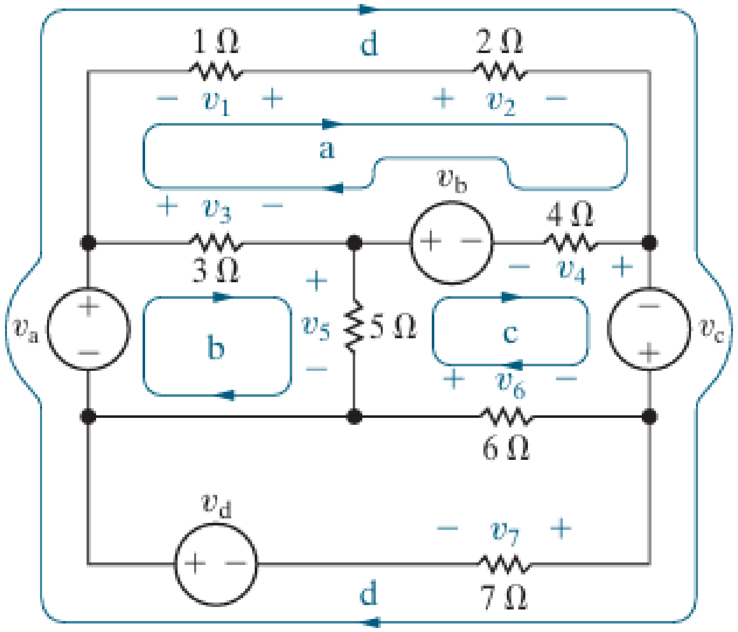
$$0.5 + 1.0 + 1.5 = 3.0\ V = v_s$$



KIRCHHOFF'S LAWS

EXAMPLE: Using Kirchhoff's Voltage Law

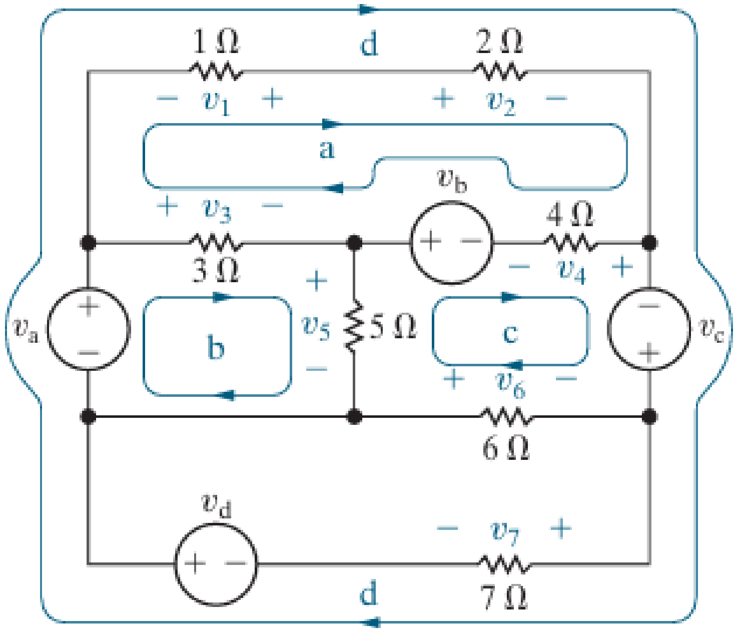
Sum the voltages around each designated path in the circuit shown in the figure.



KIRCHHOFF'S LAWS

EXAMPLE: Using Kirchhoff's Voltage Law

Sum the voltages around each designated path in the circuit shown in the figure.



Solution

In writing the equations, we use a positive sign for a voltage drop. The four equations are:

$$\text{path a} \quad -v_1 + v_2 + v_4 - v_b - v_3 = 0,$$

$$\text{path b} \quad -v_a + v_3 + v_5 = 0,$$

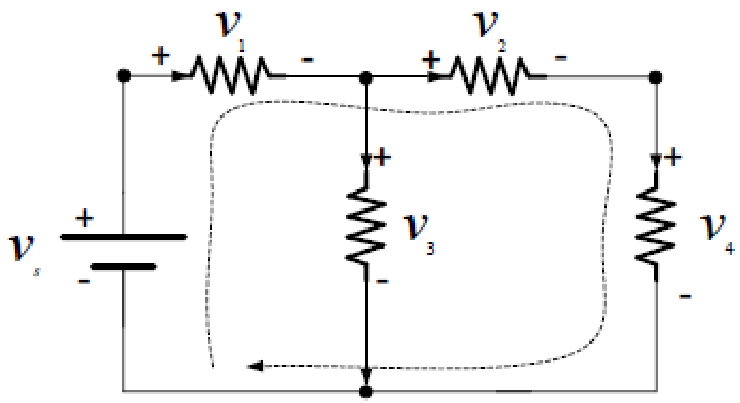
$$\text{path c} \quad v_b - v_4 - v_c - v_6 - v_5 = 0,$$

$$\text{path d} \quad -v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$$

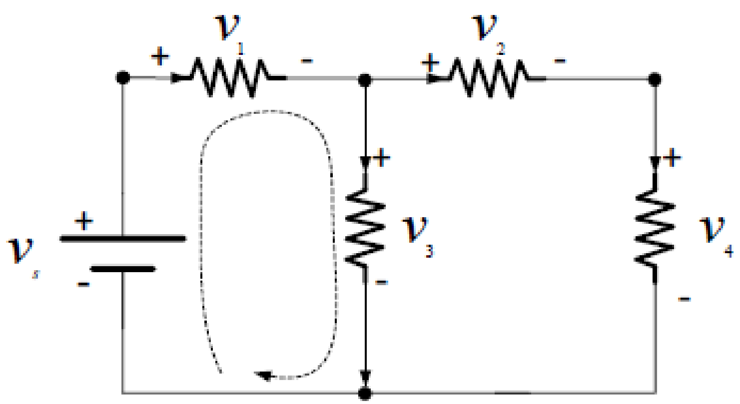
KIRCHHOFF'S LAWS

EXAMPLE

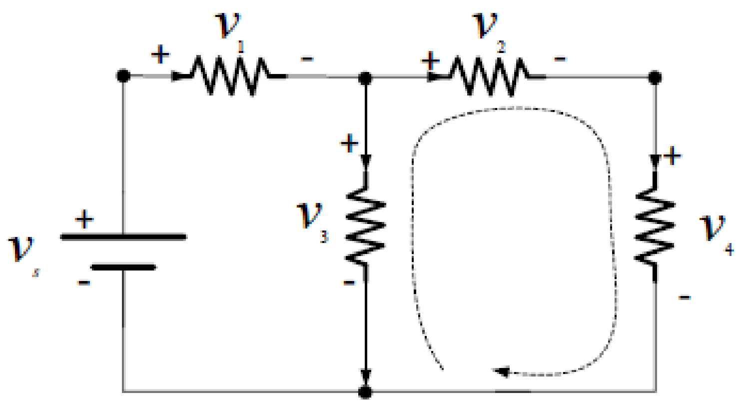
$$-v_s + v_1 + v_2 + v_4 = 0$$



$$-v_s + v_1 + v_3 = 0$$



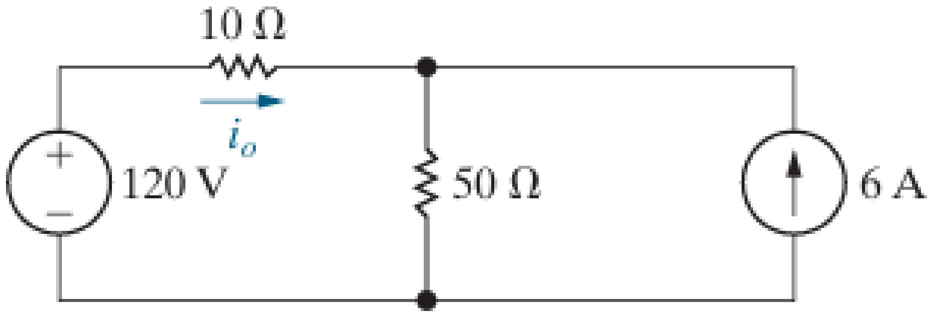
$$-v_3 + v_2 + v_4 = 0$$



KIRCHHOFF’S LAWS

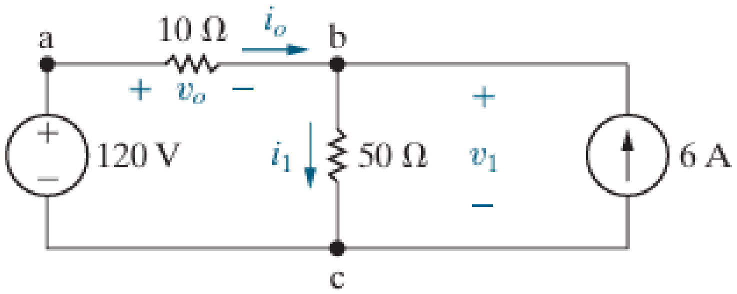
EXAMPLE

Use Kirchhoff’s laws and Ohm’s law to find io in the circuit shown in the figure below:



KIRCHHOFF'S LAWS

We begin by redrawing the circuit and assigning an unknown current to the 50 Ωresistor and unknown voltages across the 10 Ωand 50 Ω resistors. The nodes are labeled a, b, and c to aid the discussion.



Kirchhoff's Current Law at node b:

$$i_1 - i_o - 6 = 0.$$

Kirchhoff's Voltage Law

$$-120 + \underbrace{10i_o}_{v_o} + \underbrace{50i_1}_{v_1} = 0.$$

$$i_o = -3 \text{ A} \quad \text{and} \quad i_1 = 3 \text{ A}.$$

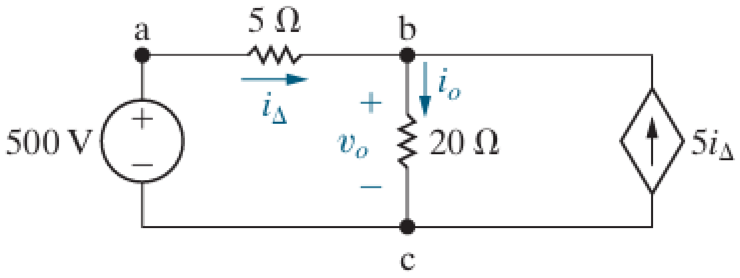
In writing this equation, a positive sign is assigned to voltage drops in the clockwise direction.

KIRCHHOFF'S LAWS

Circuit Containing Dependent Sources

What to notice first

- Once you know i_o ,**Ohm's law** gives $v_o = (20\ \Omega)\ i_o$.
- Once you know i_Δ ,you also know the **dependent source current** $5i_\Delta$.
- By **KCL at node** a, the current in the 500 V source is i_Δ.

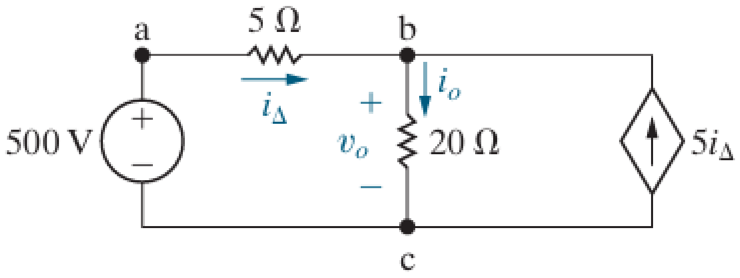


There are thus two unknown currents, i_Δ and i_o. We need to construct and solve two independent equations involving these two currents to produce a value for v_o.

KIRCHHOFF'S LAWS

Unknowns and equations

Unknowns: i_Δ and i_o.



1. **KCL at node** b (currents leaving positive):

$$i_o = i_\Delta + 5i_\Delta = 6\, i_\Delta$$

2. **KVL around** $c \to a \to b \to c$ (treat drops as positive):

$$-500 + 5\, i_\Delta + 20\, i_o = 0$$

From (1) in (2):

$$-500 + 5i_\Delta + 20(6i_\Delta) = 0 \;\; \Rightarrow \;\; i_\Delta = 4\ A, \qquad i_o = 24\ A$$

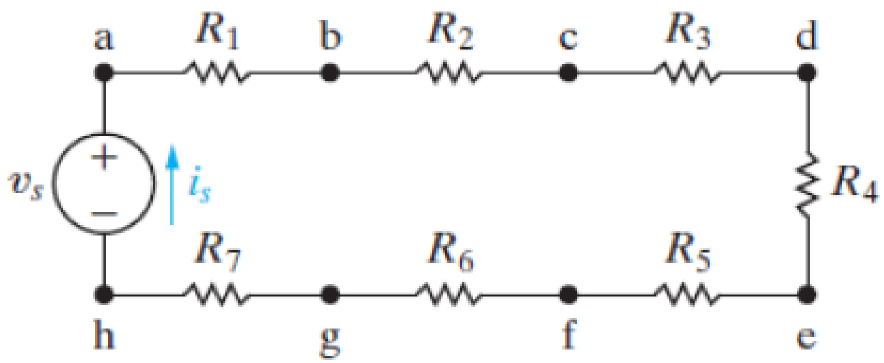
Then

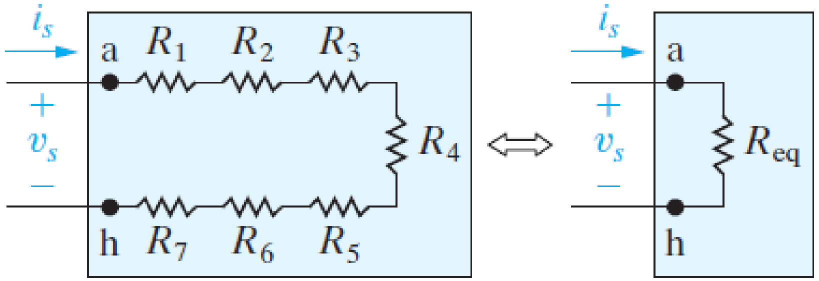
$$\boxed{v_o = 20\, i_o = 480\ V}$$

KIRCHHOFF'S LAWS

Simple Resistive Circuits

Resistors in Series





Applying KVL, we get

$$-v_s + i_sR_1 + i_sR_2 + i_sR_3 + i_sR_4 + i_sR_5 + i_sR_6 + i_sR_7 = 0 \rightarrow$$

$$v_s = i_s(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7)$$

where

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

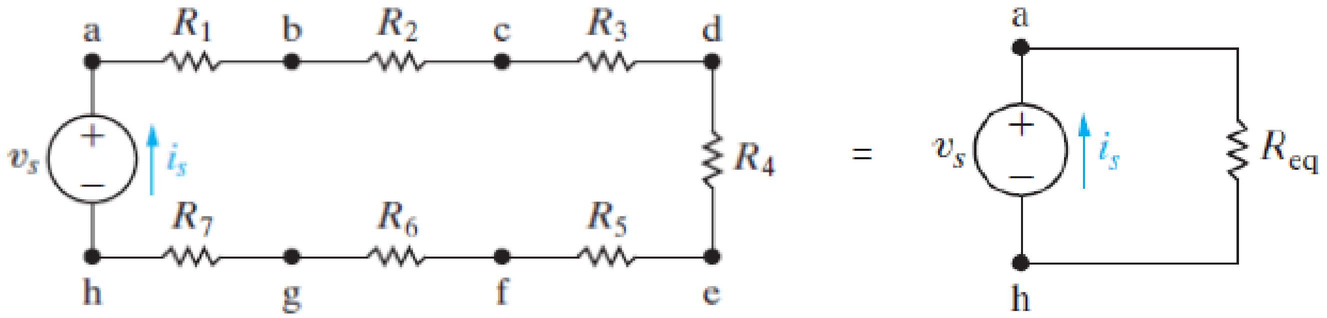
That is

$$R_{eq} = \frac{v_s}{i_s} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

KIRCHHOFF'S LAWS

In general, if k resistors are connected in series, the equivalent single resistor has a resistance equal to the sum of the k resistances, or

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k$$



KIRCHHOFF'S LAWS

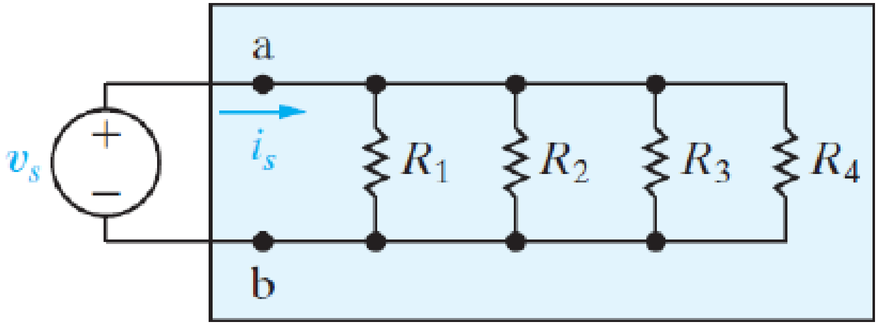
Simple Resistive Circuits

Resistors in Parallel

Let the currents i_1,i_2,i_3 and i_4 be the currents in the resistors through respectively.

Applying KCL, we get

$$i_s = i_1 + i_2 + i_3 + i_4$$



The parallel connection of the resistors means that the voltage across each resistor must be the same.

Hence, from Ohm's law

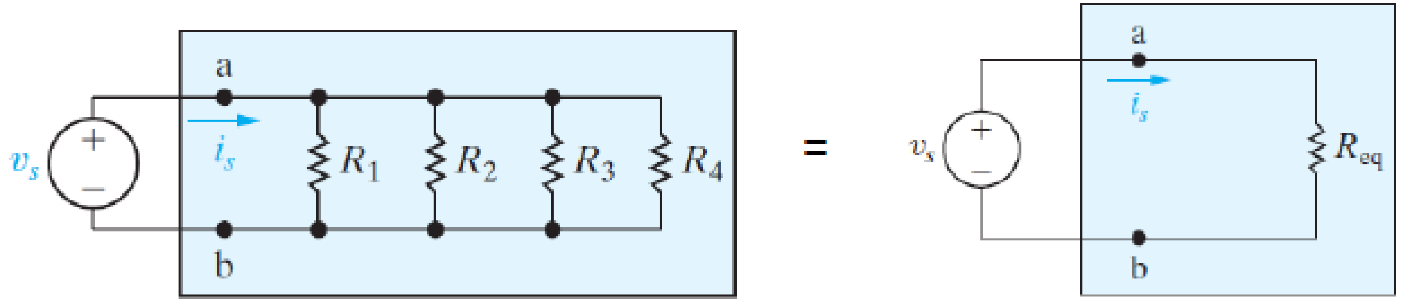
$$v_s = i_1 R_1 = i_2 R_2 = i_3 R_3 = i_4 R_4 \quad \Rightarrow \quad i_1 = \frac{v_s}{R_1} \quad i_2 = \frac{v_s}{R_2} \quad i_3 = \frac{v_s}{R_3} \quad i_4 = \frac{v_s}{R_4}$$

KIRCHHOFF'S LAWS

Substituting the above equations into $i_s = i_1 + i_2 + i_3 + i_4$

we get $$i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3} + \frac{v_s}{R_4}$$

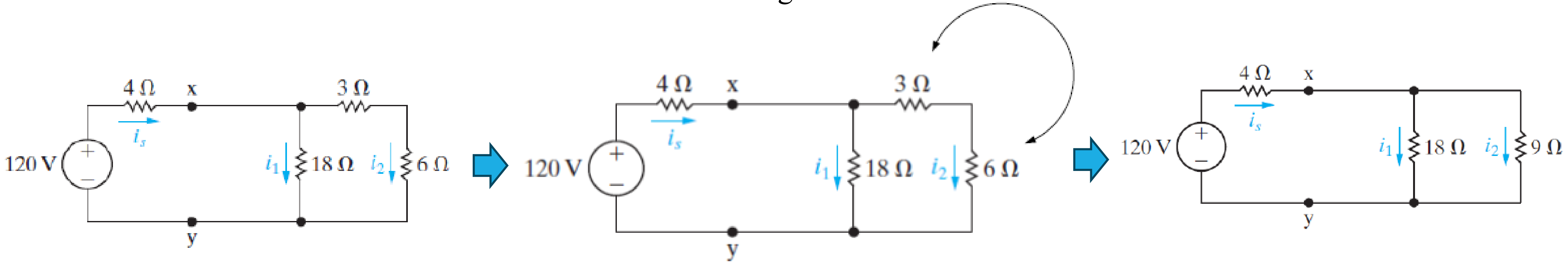
leading to $$\frac{i_s}{v_s} = \frac{1}{R_{eq}} \rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$



KIRCHHOFF’S LAWS

EXAMPLE Find i_s, i_1 and i_2 in the circuit shown

The series resistors can be summed as shown in figure on the left.

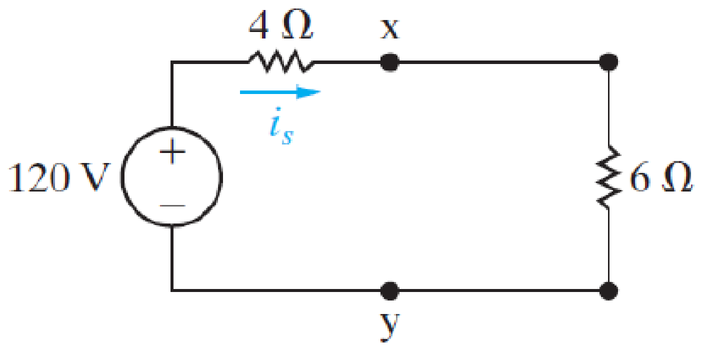


The equivalent of the parallel resistors 18Ω and 9Ω can be calculated as

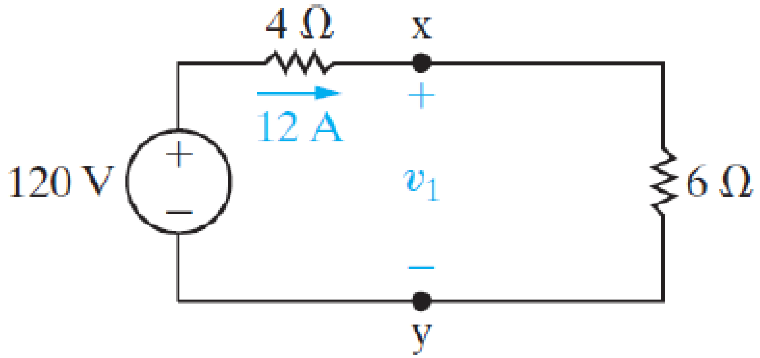
$$\frac{1}{R} = \frac{1}{18} + \frac{1}{9} \quad \rightarrow \quad R = 6\Omega$$

KIRCHHOFF'S LAWS

Then, we get



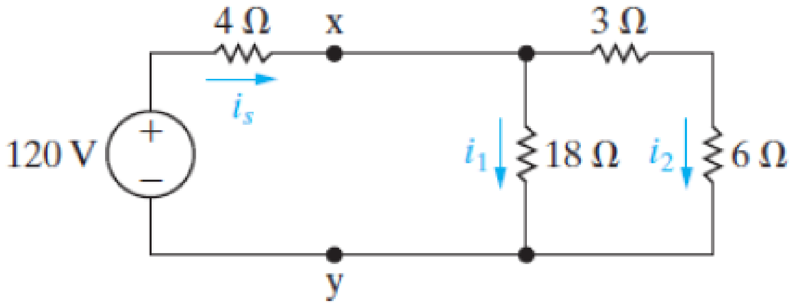
The current i_s can be calculated using KVL as $-120 + 4i_s + 6i_s = 0 \;\rightarrow\; i_s = 12\text{A}$



The voltage drop v_1 can be calculated as

$$-v_1 + 6 \times 12 = 0 \rightarrow v_1 = 72V$$

KIRCHHOFF'S LAWS



Note that $v_1 = v_x - v_y$

The voltage

$v_1 = v_x - v_y = v_{xy}$ is found as $72V$.

The voltage drop across parallel connections is the same. Then

$$i_1 = \frac{v_{xy}}{18} \rightarrow i_1 = \frac{72}{18} = 4A$$

and $$i_2 = \frac{72}{9} = 8A$$