CIRCUIT THEORY I

Lecture 3

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RESISTORS IN SERIES

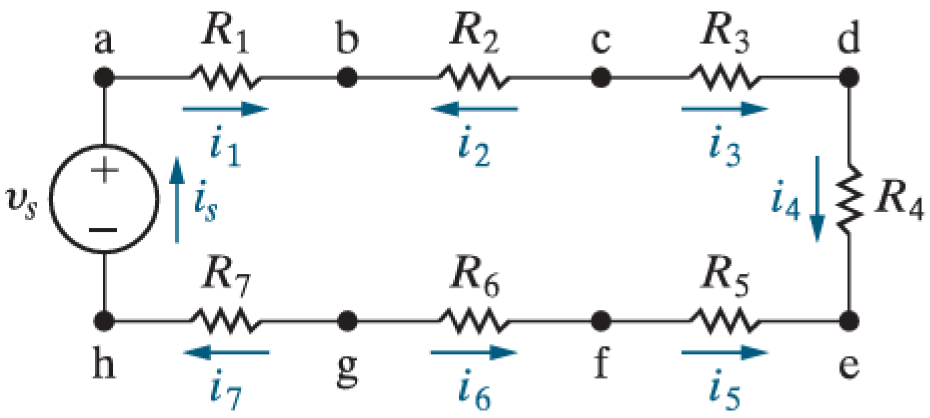
Series connection: Elements that share the same single current path (no branching) are *in series*. Therefore, the current is identical through every series resistor:

$$i_s = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7,$$

Equivalent resistance (series)

COMBINING RESISTORS IN SERIES

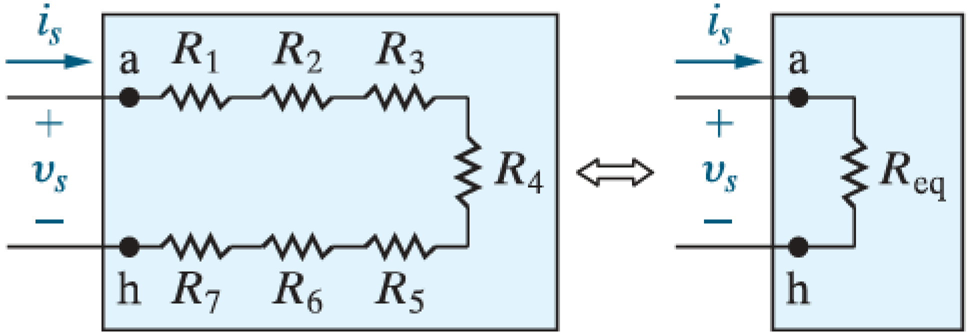
$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k.$$



Resistors connected in series.

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

RESISTORS IN SERIES



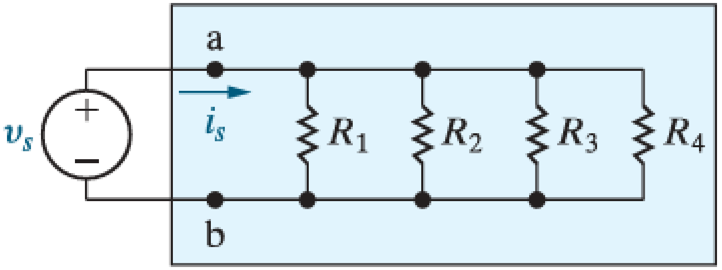
The black box equivalent of the circuit

RESISTORS IN PARALLEL

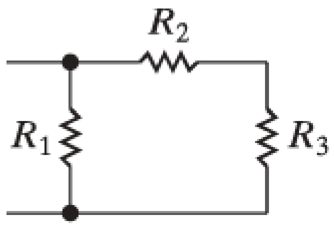
Core ideas

Parallel connection: Two elements are in parallel if **both of their terminals are connected to the same two nodes.** The defining feature is they all share the **same voltage** across their terminals:

$$v_1 = v_2 = \cdots = v_k = v_s.$$



Resistors in parallel.



Nonparallel resistors.

Don't assume elements are parallel just because they're drawn side-by-side; check the **node pairs.** If any other element sits between the terminal pair, those two aren't directly in parallel.

RESISTORS IN PARALLEL

From Kirchhoff's current law

$$i_s = i_1 + i_2 + i_3 + i_4.$$

From Ohm's law,

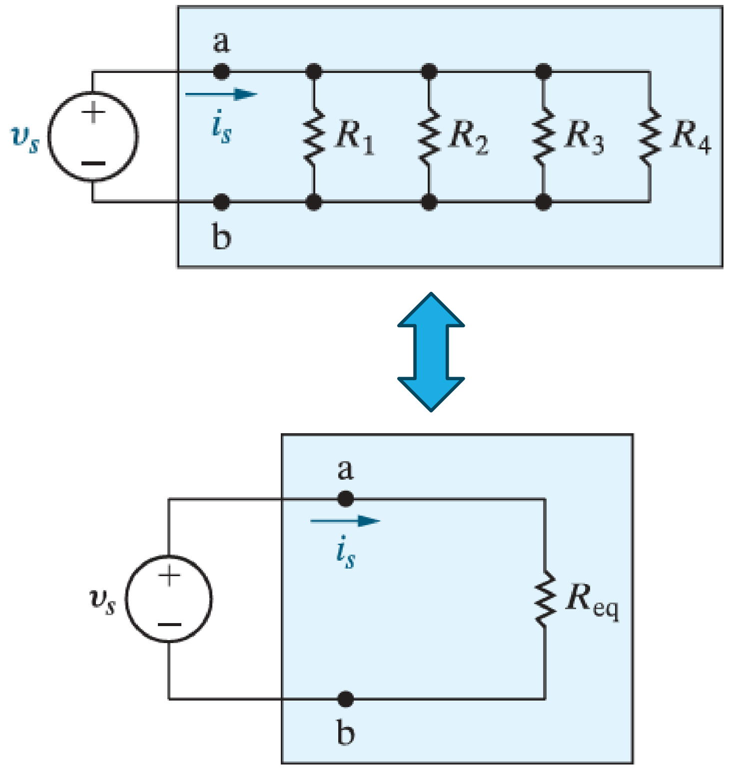
$$i_1 R_1 = i_2 R_2 = i_3 R_3 = i_4 R_4 = v_s.$$

$$\Rightarrow \quad i_1 = \frac{v_s}{R_1}, \quad i_2 = \frac{v_s}{R_2}, \quad i_3 = \frac{v_s}{R_3}, \quad \text{and} \quad i_4 = \frac{v_s}{R_4}.$$

Substituting into the KCL equation

$$i_s = v_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right),$$

$$\Rightarrow \quad \frac{i_s}{v_s} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}.$$



$\Rightarrow$

COMBINING RESISTORS IN PARALLEL

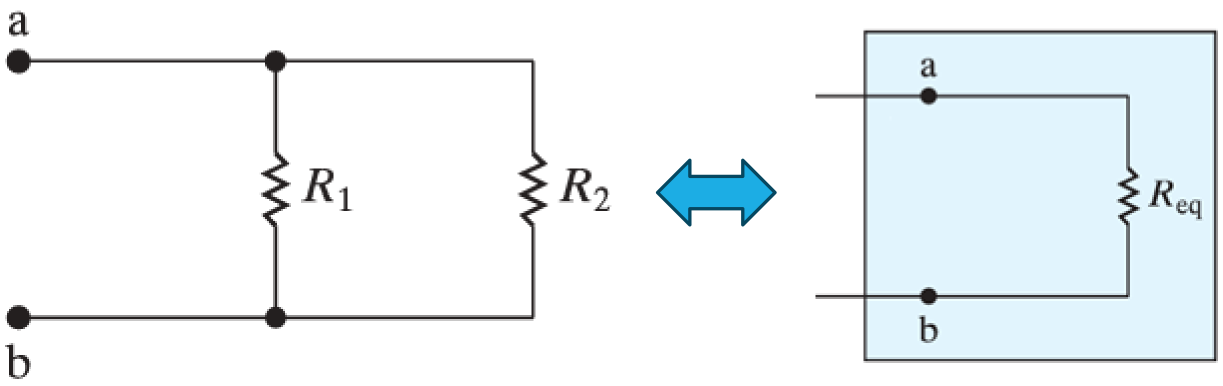
$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}.$$

RESISTORS IN PARALLEL

Using conductance when dealing with resistors connected in parallel is sometimes more convenient.

$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \cdots + G_k.$$

Two resistors connected in parallel



Two resistors connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2},$$

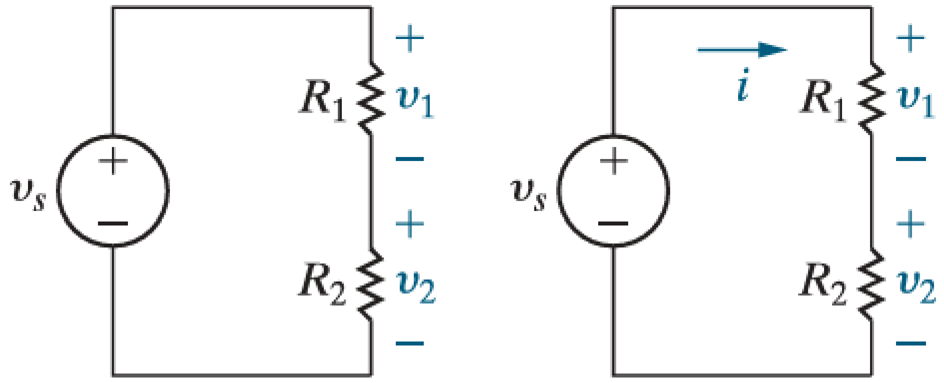
COMBINING TWO RESISTORS IN PARALLEL

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

THE VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

The Voltage-Divider Circuit

A voltage-divider circuit produces two or more smaller voltages from a single voltage supply. This is especially useful in electronic circuits, where a single circuit may require voltages of +15 V, −15 V, and +5 V.



(a) (b)

$$v_s = iR_1 + iR_2,$$

$$i = \frac{v_s}{R_1 + R_2}.$$

$$v_1 = iR_1 = v_s \frac{R_1}{R_1 + R_2},$$

$$v_2 = iR_2 = v_s \frac{R_2}{R_1 + R_2}.$$

v1 and v2 are fractions of vs , always less then source voltage

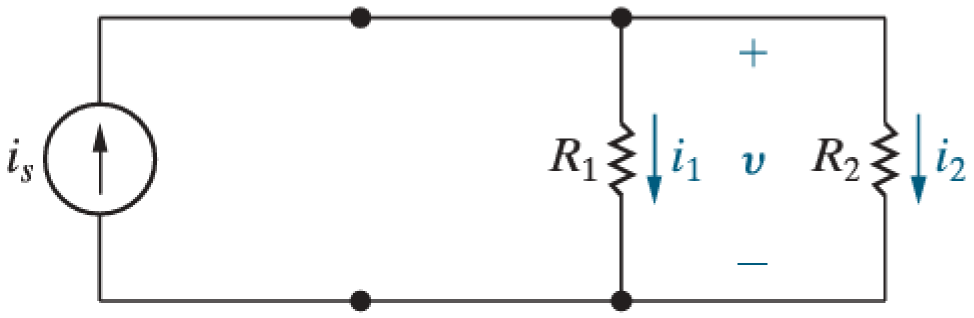
Each fraction is:

$$\boxed{\frac{\text{the resistance across which the divided voltage is defined}}{\text{the sum of the two resistances}}.}$$

THE VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

The Current-Divider Circuit

The current-divider circuit shown in figure below consists of two resistors connected in parallel across a current source. It divides the current is between R_1 and R_2 .



$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s .$$

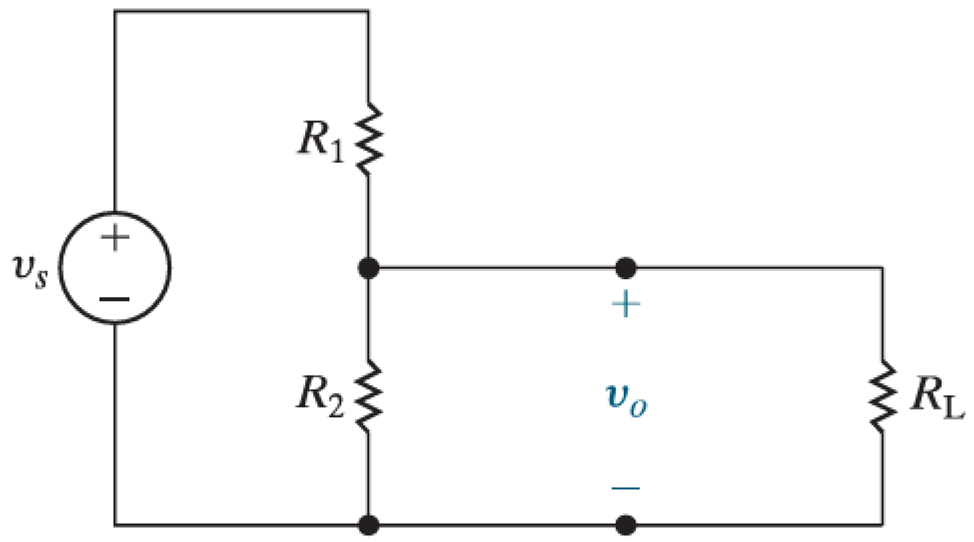
$$i_1 = \frac{R_2}{R_1 + R_2} i_s ,$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s .$$

When the current divides between two resistors in parallel, the current in one resistor equals the current entering the parallel pair multiplied by the other resistance and divided by the sum of the resistors.

THE VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

A voltage divider connected to a load R_L.



The expression for the output voltage is $$v_o = \frac{R_{eq}}{R_1 + R_{eq}} v_s,$$

$$R_{eq} = \frac{R_2 R_L}{R_2 + R_L}.$$

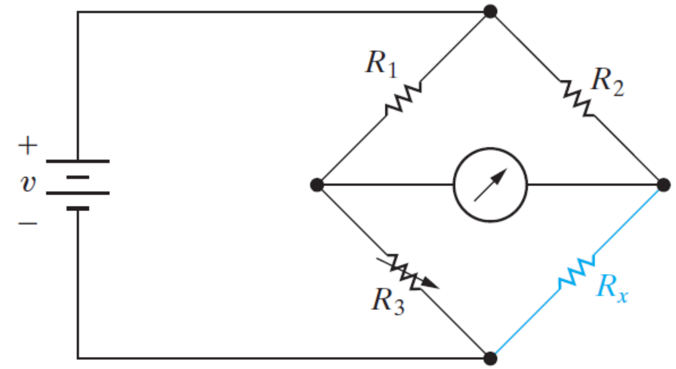
$$v_o = \frac{R_2}{R_1[1 + (R_2/R_L)] + R_2} v_s. \quad (2)$$

$$v_2 = iR_2 = v_s \frac{R_2}{R_1 + R_2}. \quad (1)$$

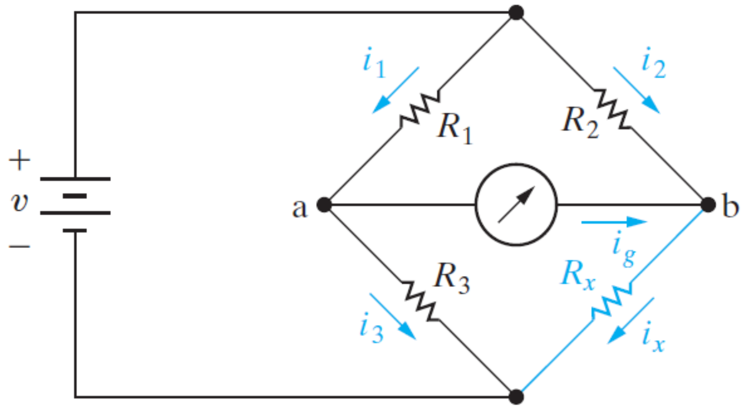
Note (2) reduces to (1) as RL, as it should. Equation (2) shows that, as long as RL >> R2 , the voltage ratio vo/vs is essentially undisturbed by adding a load to the divider.

MEASURING RESISTANCE—THE WHEATSTONE BRIDGE

The Wheatstone Bridge is used to measure an unknown resistance. The Wheatstone Bridge is shown in the Figure below:



The currents are shown on the Wheatstone Bridge in the Figure below



MEASURING RESISTANCE—THE WHEATSTONE BRIDGE

When the voltages at a and b are equal to each other then

$$i_g = 0 \text{ (the bridge is balanced)}$$

Since

$$i_g = \frac{v_a - v_b}{R}$$

If $v_a = v_b$ then

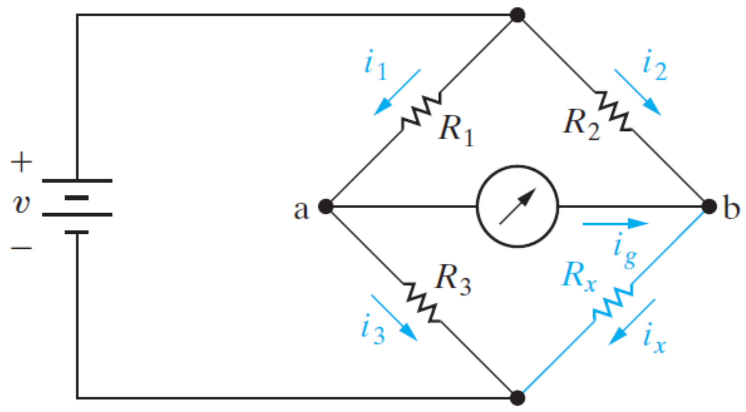
$$v - i_1R_1 = v - i_2R_2 \rightarrow i_1R_1 = i_2R_2$$

$$i_3R_3 = i_xR_x$$

Thus, we have

$$i_1R_1 = i_2R_2$$

$$i_3R_3 = i_xR_x$$



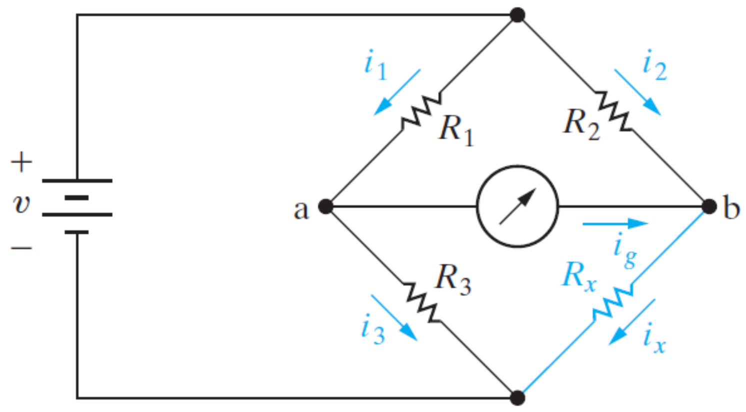
MEASURING RESISTANCE—THE WHEATSTONE BRIDGE

Dividing both equations side by side we obtain

$$\frac{i_1 R_1}{i_3 R_3} = \frac{i_2 R_2}{i_x R_x} \qquad (Eq1)$$

When $v_a = v_b$

$$i_1 = i_3 \qquad i_g = 0 \qquad i_2 = i_x$$



then, $Eq1$ is simplified as

$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

from which R_x is solved as

$$R_x = \frac{R_2 R_3}{R_1}$$

MEASURING RESISTANCE—THE WHEATSTONE BRIDGE

Key points for practice

- **You must be able to vary the ratio R_2/R_1 ,not just R_3.**
  If $R_2/R_1 = 1$,then balance requires $R_x = R_3$ .So R_3must cover the expected range of R_x .Example: if $R_x$might be $1\ k\Omega$but R_3only spans $0 - 100\ \Omega$, the bridge can never balance. Adjustability of R_2/R_1lets a single bridge cover a **wide range** of R_x.

- **Commercial/decade arrangement.**
  In practice, $R_1$and $R_2$are switchable **decade resistors** (e.g., 1, 10, 100, 1000 Ω, ...) so the ratio R_2/R_1can sweep roughly **0.001 to 1000** in convenient steps.
  $R_3$is a continuously adjustable resistor (e.g., about **1 to 11 kΩ**) used to fine-tune the final balance.

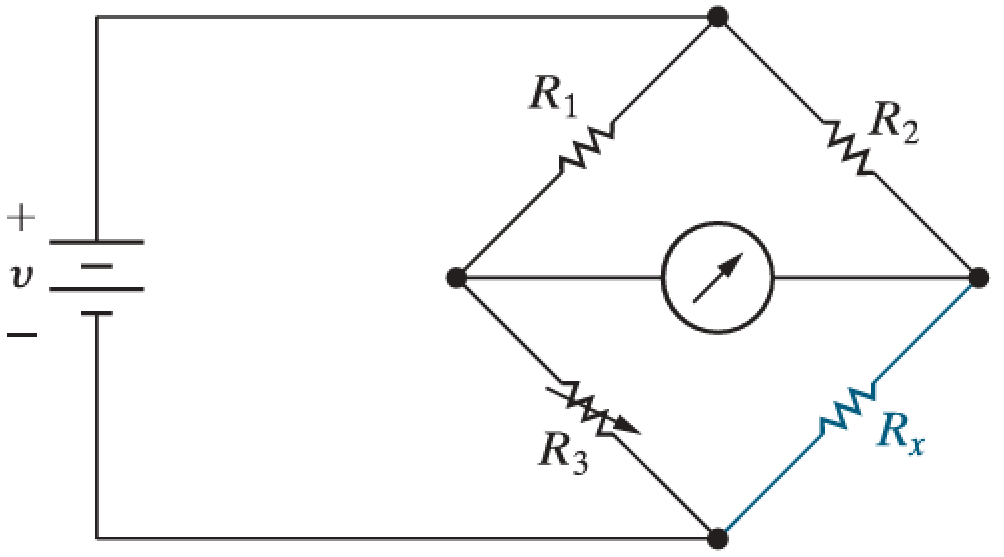
- **Practical measurement range of R_x.**
  Although the equation is valid for any positive R_x ,a standard Wheatstone bridge measures **about** 1Ω **to** $1\ M\Omega$ with good accuracy.
    - **Below ~1 Ω** is difficult because tiny thermoelectric (Seebeck) voltages at junctions and i^2R**self-heating** distort readings relative to the very small bridge voltages you're trying to sense.
    - **Above ~1 MΩ** is difficult because **leakage currents** in wiring/insulation become comparable to branch currents, upsetting the balance.

MEASURING RESISTANCE—THE WHEATSTONE BRIDGE

The bridge circuit shown is balanced when $R_1 = 1\ \text{k}\Omega$, $R_2 = 500\ \Omega$, and $R_3 = 2\ \text{k}\Omega$. The bridge is energized from a 30 V dc source.

a) What is the value of R_x?

b) If the bridge is left in the balanced state, which resistor must dissipate the most power?



MEASURING RESISTANCE—THE WHEATSTONE BRIDGE

Bridge is balanced when $R_1 = 1\ k\Omega$, $R_2 = 500\ \Omega$, $R_3 = 2\ k\Omega$. Source $= 30\ V dc$.

(a) Find R_x

At balance (no current through the detector), the Wheatstone condition is

$$\frac{R_x}{R_3} = \frac{R_2}{R_1} \quad \Rightarrow \quad R_x = R_3 \frac{R_2}{R_1} = (2000)\frac{500}{1000} = 1000\ \Omega = 1\ \mathrm{k}\Omega.$$

(b) Which resistor dissipates the most power at balance?

With no bridge current, the two series legs are independent:

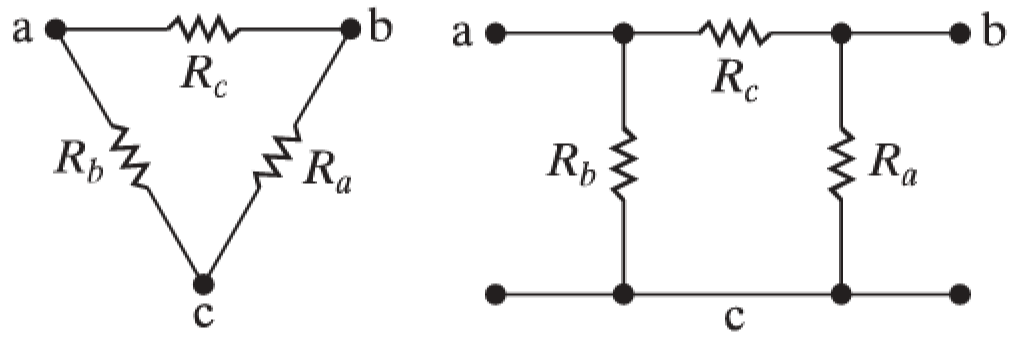
- Leg A: $R_1 + R_3 = 1000 + 2000 = 3000\ \Omega \Rightarrow I_A = \frac{30}{3000} = 10$ mA.
- Leg B: $R_2 + R_x = 500 + 1000 = 1500\ \Omega \Rightarrow I_B = \frac{30}{1500} = 20$ mA.

Power in each element $P = I^2R$:

- $P_{R_1} = (0.01)^2 \cdot 1000 = 0.10$ W
- $P_{R_3} = (0.01)^2 \cdot 2000 = 0.20$ W
- $P_{R_2} = (0.02)^2 \cdot 500 = 0.20$ W
- $P_{R_x} = (0.02)^2 \cdot 1000 = 0.40$ W

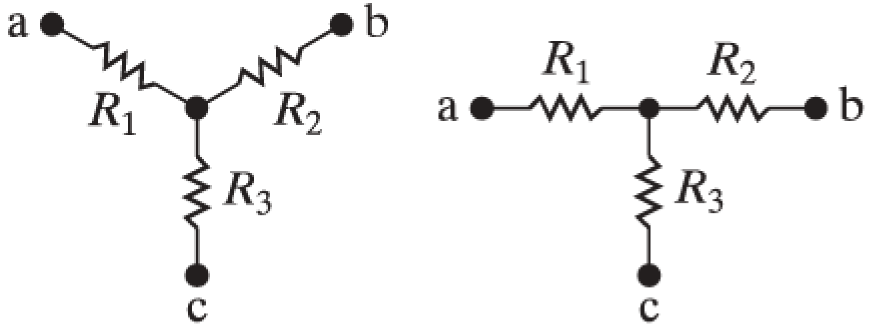
The **largest power** is dissipated by R_x.

DELTA-TO-WYE (PI-TO-TEE) EQUIVALENT CIRCUITS



Δ configuration viewed as a π configuration.

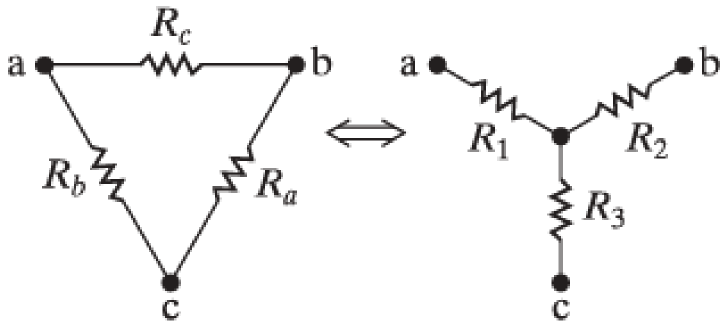
DELTA-TO-WYE (PI-TO-TEE) EQUIVALENT CIRCUITS



A Y structure viewed as a T structure.

IDEAL SOURCE CONCEPT

Saying the Δ-connected circuit is equivalent to the Y-connected circuit means that the Δ configuration can be replaced with a Y configuration without changing the terminal behavior.



This condition is true only if the resistance between corresponding terminal pairs is the same for each configuration.

For each pair of terminals in the Δ-connected circuit, compute the equivalent resistance using series and parallel simplifications.

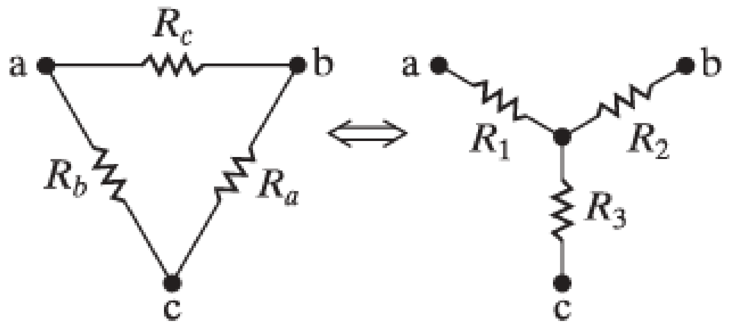
IDEAL SOURCE CONCEPT

For Delta circuit the terminal resistance values are

$$R_{ac} = R_b \,||\, (R_c + R_b)$$

$$R_{ab} = R_c \,||\, (R_a + R_b)$$

$$R_{bc} = R_a \,||\, (R_b + R_c)$$



For Y circuit the terminal resistance values are

$$R_{ac} = R_1 + R_3$$

$$R_{ab} = R_1 + R_2$$

$$R_{bc} = R_2 + R_3$$

$$R_1 + R_3 = R_b \,||\, (R_c + R_b \quad (EQA)$$

$$R_1 + R_2 = R_c \,||\, (R_a + R_b) \quad (EQB)$$

$$R_2 + R_3 = R_a \,||\, (R_b + R_c) \quad (EQC)$$

If the terminal resistance values are the same for Delta and Y circuits, then we can write the following equations

IDEAL SOURCE CONCEPT

When these equations are used to solve R_1, R_2, and R_3 we get

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y-connected resistors in terms of the Δ-connected resistors.

Reversing the Δ-to-Y transformation

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

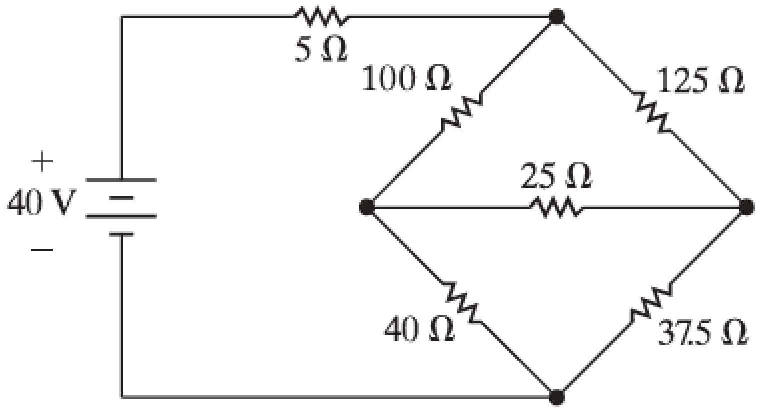
$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

Δ-connected resistors as functions Y-connected resistors

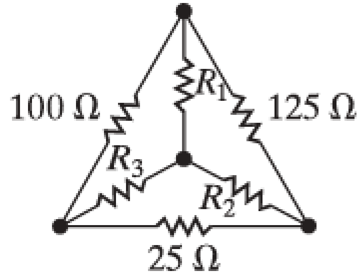
APPLYING A DELTA-TO-WYE TRANSFORM

Find the current and power supplied by the 40 V source in the circuit shown in figure below.



APPLYING A DELTA-TO-WYE TRANSFORM

Replace the upper Δ by computing the three Y resistances,

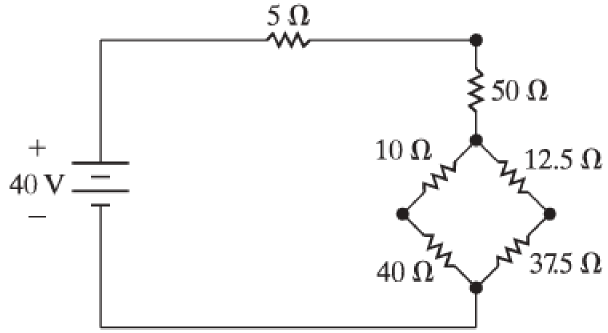


$$R_1 = \frac{100 \times 125}{250} = 50\ \Omega,$$

$$R_2 = \frac{125 \times 25}{250} = 12.5\ \Omega,$$

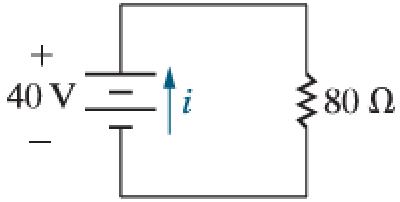
$$R_3 = \frac{100 \times 25}{250} = 10\ \Omega.$$

Substituting the Y resistors into the initial circuit produces the circuit shown in below:



The resistance seen by the 40 V source can be calculated easily by using seriesparallel simplifications:

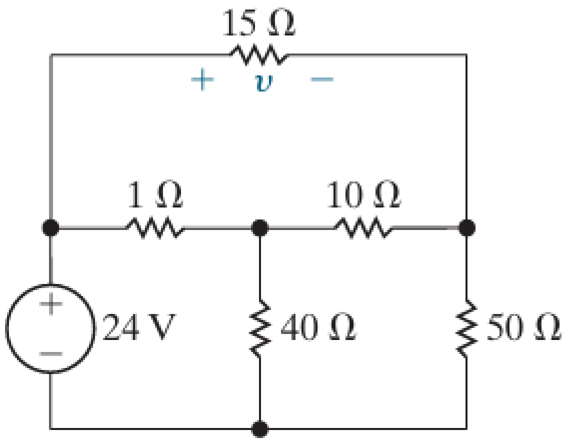
$$R_{eq} = 5 + 50 + (10 + 40)||(12.5 + 37.5)$$
$$= 55 + \frac{(50)(50)}{50 + 50} = 80\ \Omega.$$



i = 40/80 = 0.5 A
Power *p* = 40x0.5 = 20 W is delivered to the circuit.

APPLYING A DELTA-TO-WYE TRANSFORM

Use a Δ-to-Y transformation to find the voltage v in the circuit shown.



APPLYING A DELTA-TO-WYE TRANSFORM

Let's solve for v(across the 15 Ω, left-to-right).

Label the top-left, top-middle, and top-right nodes A, B, C. The bottom node is reference (0 V).
The source fixes $V_A = +24\ V$.

KCL at B:

$$\frac{V_B - V_A}{1} + \frac{V_B - V_C}{10} + \frac{V_B - 0}{40} = 0$$

KCL at C:

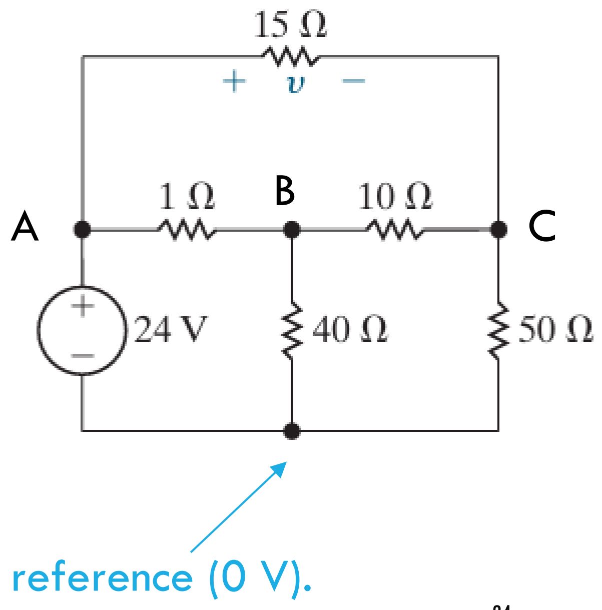
$$\frac{V_C - V_A}{15} + \frac{V_C - V_B}{10} + \frac{V_C - 0}{50} = 0$$

Solve:

$$V_B = 23.2\ V, \qquad V_C = 21\ V.$$

The requested voltage is the drop across the 15 Ω from left to right:

$$v = V_A - V_C = 24 - 21 = \boxed{3\ V}.$$



APPLYING A DELTA-TO-WYE TRANSFORM

Identify the Δ.

The three top resistors form a delta between the three top nodes: $R_{AB} = 1\ \Omega,\ \ R_{BC} = 10\ \Omega,\ \ R_{CA} = 15\ \Omega.$

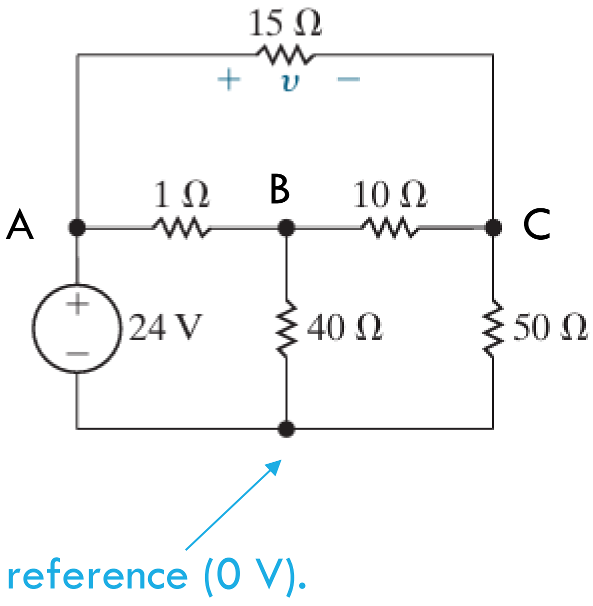
We want $v = V_A - V_C)$ across the original $15\,\Omega$), and the source fixes $V_A = +24\ V.$

Convert Δ to Y.

With the usual formulas $R_1 = \frac{R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}}, R_2 = \frac{R_{AB}R_{BC}}{R_{AB}+R_{BC}+R_{CA}}, R_3 = \frac{R_{BC}R_{CA}}{R_{AB}+R_{BC}+R_{CA}},$

we get

$$R_1 = \frac{1 \cdot 15}{1 + 10 + 15} = \frac{15}{26} \approx 0.5769\ \Omega,$$
$$R_2 = \frac{1 \cdot 10}{26} = \frac{10}{26} \approx 0.3846\ \Omega,$$
$$R_3 = \frac{10 \cdot 15}{26} = \frac{150}{26} \approx 5.7692\ \Omega.$$



APPLYING A DELTA-TO-WYE TRANSFORM

Solve the simple Y network (two-node KCL + center KCL). Unknowns: $V_B,\ V_C,\ V_o$.

$$\text{At } B: \qquad \frac{V_B - V_o}{R_2} + \frac{V_B}{40} = 0,$$

$$At\,C: \qquad \frac{V_C - V_o}{R_3} + \frac{V_C}{50} = 0,$$

$$At\ 0: \quad \frac{V_o - 24}{R_1} + \frac{V_o - V_B}{R_2} + \frac{V_o - V_C}{R_3} = 0.$$

Solving gives

$$V_B = 23.2\ \ V, \qquad V_C = 21.0\ \ V, \qquad V_o \approx 23.423\ \ V.$$

Convert back to the requested voltage.

v is the drop across the original 15-Ω branch, i.e.

$$v = V_A - V_C = 24 - 21 = \boxed{3\ \ V}.$$