Lecture-5

Analog Communications

Abstract:

Pre-envelope

For the real valued signal $g(t)$, the pre-envelope is defined as

$$g_p(t) = g(t) + j\hat{g}(t) \tag{1}$$

Summary for the Pre-Envelope Calculation of a Signal

1) The pre-envelope of $g(t)$ can be calculated using

$$g_p(t) = g(t) + j\hat{g}(t).$$

where $\hat{g}(t)$ is the Hilbert transform of $g(t)$.

2) The pre-envelope of $g(t)$ can be calculated first calculating the Fourier transform of $g(t)$, i.e., finding $G(f)$, and next evaluating

$$G_p(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0. \end{cases}$$

and lastly taking the inverse Fourier transform of $G_p(f)$, we get $g_p(t)$, i.e.,

$$g_p(t) = 2\int_0^{\infty} G(f)e^{j2\pi ft}df.$$

Complex Envelope

The complex envelope of $g(t)$ is calculated using

$$g_c(t) = g_p(t)e^{-j2\pi f_c t}.$$

where we have

$$g_p(t) = g(t) + j\hat{g}(t)$$

If $g_c(t)$ is a low-pass complex signal such that

$$g_c(t) = g_I(t) + jg_Q(t) \tag{2}$$

then

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

is a band-pass signal which can also be written as

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t).$$

The complex envelope signal

$$g_c(t) = g_I(t) + jg_Q(t) \tag{3}$$

can also be written as

$$g_c(t) = |g_c(t)|e^{j\theta(t)} \tag{4}$$

where

$$|g_c(t)| = \sqrt{g_I^2(t) + g_Q^2(t)} \text{ and } \theta(t) = \arctan\frac{g_Q(t)}{g_I(t)}.$$

The equation in (4) can be written as

$$g_c(t) = |g_c(t)|\cos\theta(t) + j|g_c(t)|\sin\theta(t) \tag{5}$$

Comparing (3) and (4), we can write that

$$g_I(t) = |g_c(t)|\cos\theta(t) \quad g_Q(t) = |g_c(t)|\sin\theta(t).$$

Besides employing using (4) in

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

we obtain

$$g(t) = |g_c(t)| \cos\big(2\pi f_c t + \theta(t)\big).$$

Example: For the low-pass, baseband signal $g_c(t)$, the pass-band signal is calculated as

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

Find the Fourier transform of $g(t)$ in terms of the Fourier transform of $g_c(t)$.

Solution: Real part of a complex signal $x(t)$ is calculated using

$$Re\{x(t)\} = \frac{1}{2}[x(t) + x^*(t)]$$

where $x^*(t)$ is the conjugate of $x(t)$.

In addition, we know that

$$x^*(t) \overset{FT}{\leftrightarrow} X^*(-f).$$

Then, we can write that

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\} \rightarrow g(t) = \frac{1}{2}\big[g_c(t)e^{j2\pi f_c t} + g_c^*(t)e^{-j2\pi f_c t}\big]$$

The Fourier transform of

$$g(t) = \frac{1}{2}\big[g_c(t)e^{j2\pi f_c t} + g_c^*(t)e^{-j2\pi f_c t}\big]$$

can be obtained as

$$G(f) = \frac{1}{2}\big[G_c(f - f_c) + G_c^*\big(-(f + f_c)\big)\big]$$

Low-pass and Pass-band Systems

$$G(f) = \frac{1}{2}\big[G_c(f - f_c) + G_c^*\big(-(f + f_c)\big)\big]$$

$$g(t) = Re\{g_c(t)e^{j2\pi f_c t}\}$$

$x(t)$ is the input of a band-pass system, $h(t)$ is the impulse response of the band-pass system and $y(t)$ is the output of the band-pass system. We have the relations

$$y(t) = x(t) * h(t) \rightarrow Y(f) = X(f)H(f)$$

The band-pass signals $x(t), y(t)$, and $h(t)$ can be written as

$$x(t) = Re\{x_c(t)e^{j2\pi f_c t}\}$$

$$y(t) = Re\{y_c(t)e^{j2\pi f_c t}\}$$

$$h(t) = Re\{h_c(t)e^{j2\pi f_c t}\}$$

In frequency domain, we have the relations

$$X(f) = \frac{1}{2}\left[X_c(f - f_c) + X_c^*\left(-(f + f_c)\right)\right]$$

$$Y(f) = \frac{1}{2}\left[Y_c(f - f_c) + Y_c^*\left(-(f + f_c)\right)\right]$$

$$H(f) = \frac{1}{2}\left[H_c(f - f_c) + H_c^*\left(-(f + f_c)\right)\right]$$

For the passband system we have,

$$Y(f) = X(f)H(f)$$

for which we can write

$$\frac{1}{2}\left[Y_c(f - f_c) + Y_c^*\left(-(f + f_c)\right)\right] = \frac{1}{2}\left[X_c(f - f_c) + X_c^*\left(-(f + f_c)\right)\right]\frac{1}{2}\left[H_c(f - f_c) + H_c^*\left(-(f + f_c)\right)\right]$$

$$\frac{1}{2}\left[Y_c(f - f_c) + Y_c^*\left(-(f + f_c)\right)\right] = \frac{1}{4}X_c(f - f_c)H_c(f - f_c) + \frac{1}{4}X_c^*\left(-(f + f_c)\right)H_c^*\left(-(f + f_c)\right)$$

where using

$$X_c^*\left(-(f + f_c)\right)H_c(f - f_c) = 0 \qquad X_c(f - f_c)H_c^*\left(-(f + f_c)\right) = 0$$

we obtain

$$\frac{1}{2}Y_c(f - f_c) = \frac{1}{4}X_c(f - f_c)H_c(f - f_c)$$

which implies that

$$Y_c(f) = \frac{1}{2} X_c(f) H_c(f)$$

from which, we obtain

$$y_c(t) = \frac{1}{2} x_c(t) * h_c(t)$$

Continuous Wave Modulation

Information bearing signals are usually low-pass, i.e., baseband, signals. They include low frequency harmonics.

For this reason, for the transmission of these signals we need a very long antenna.

Human speech fundamental frequency is limited to 300Hz. The antenna length of the transmission frequency f_c is approximately calculated using

$$l = \frac{\lambda}{2} \rightarrow l = \frac{c}{2f_c}$$

where c is the speed of light.

For 300Hz, we need an antenna of length

$$l = \frac{3 \times 10^8}{2 \times 300} \; meter$$

which is a very large number.

For 1800MHz, we need an antenna of length

$$l = \frac{3 \times 10^8}{2 \times 1800 \times 10^6} \rightarrow l = \frac{1}{12} meter \rightarrow l \approx 10cm$$

which is a reasonable length.

Modulation is the process of shifting the range of frequencies of the data signals, i.e., information bearing signals, to high frequency ranges so that they can be transmitted via antennas.

Modulation can be considered as obtaining a band-pass signal from a low-pass signal and band-pass signal is used for the transmission.

In modulation operation we have two signals, and these two signals are:

message signal: $m(t)$ and carrier signal: $A_c \cos 2\pi f_c\, t$.

Amplitude Modulation

Message signal is $m(t)$ and carrier signal is $A_c \cos 2\pi f_c\, t$.

In amplitude modulation the pass-band signal that carries the information of the baseband signal to be transmitted is generated as follows.

The complex envelope signal is obtained from message and carrier signal as

$$s_c(t) = A_c[1 + k_a m(t)]$$

where k_a is a constant.

The pass-band, i.e., amplitude modulated signal is generated using

$$s(t) = Re\{s_c(t)e^{j2\pi f_c t}\}$$

as

$$s(t) = Re\{A_c[1 + k_a m(t)][\cos 2\pi f_c t + \sin 2\pi f_c t]\}$$

resulting in

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

which is the amplitude modulated signal.

Information signal $m(t)$ is called modulating signal, and band-pass signal $s(t)$ is called modulated signal.

The constant terms A_c and k_a can be chosen as 1 for the simplicity of illustration.

For the AM Signal

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

the term

$$\mu = k_a \max\ |m(t)|$$

is called the modulation factor. Assuming that

$$\max\ |m(t)| = -\min\ |m(t)|$$

and defining

$$S_{max} = \max s(t) \qquad S_{min} = \min s(t)$$

we have

$$\frac{S_{max}}{S_{min}} = \frac{A_c[1 + k_a m(t)]}{A_c[1 + k_a m(t)]} \to \frac{S_{max}}{S_{min}} = \frac{1+\mu}{1-\mu}$$

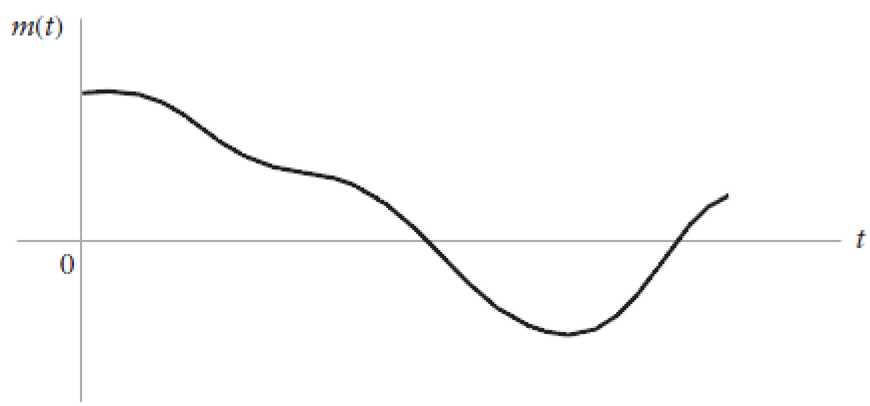
from which we get

$$\mu = \frac{S_{max} - S_{min}}{S_{max} + S_{min}}$$

If $\mu = 1$, then we get

$$S_{min} = 0.$$

Example: The message signal $m(t)$ is shown in Figure-1.



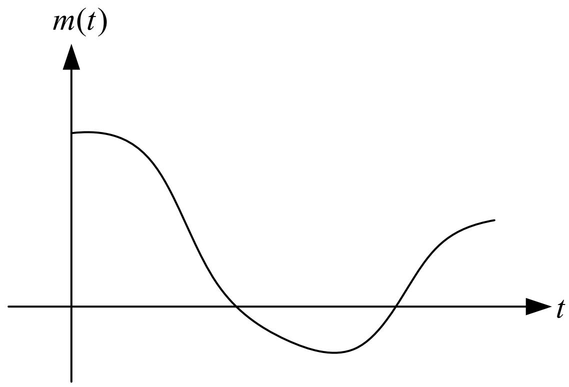


Figure-1

Assume that k_a is chosen such that $|k_a m(t)| \leq 1$, this means that

$$[1 + k_a m(t)] \geq 0$$

Assume that the graph of $[1 + k_a m(t)]$ is something like in Figure-2

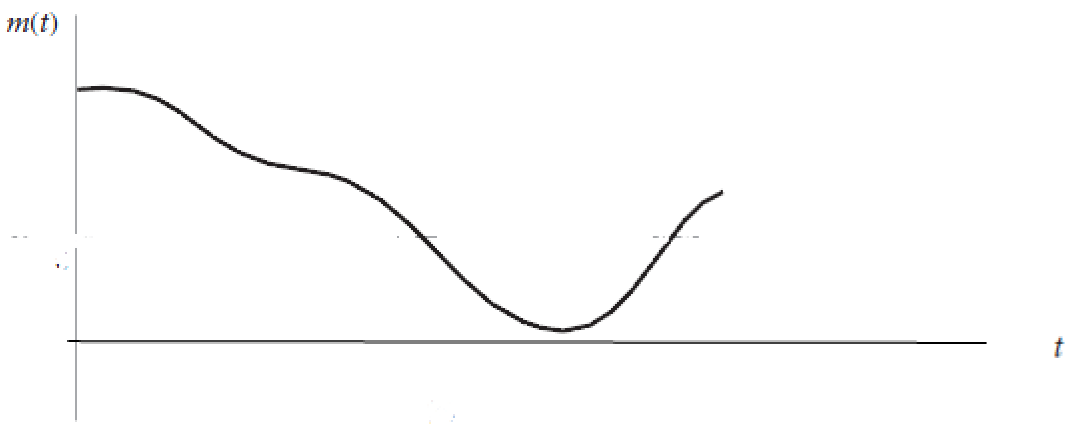


Figure-2

Then, the graph of

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

happens to be as in Figure-3 where the envelope of the modulated signal carries the information content of the message signal.

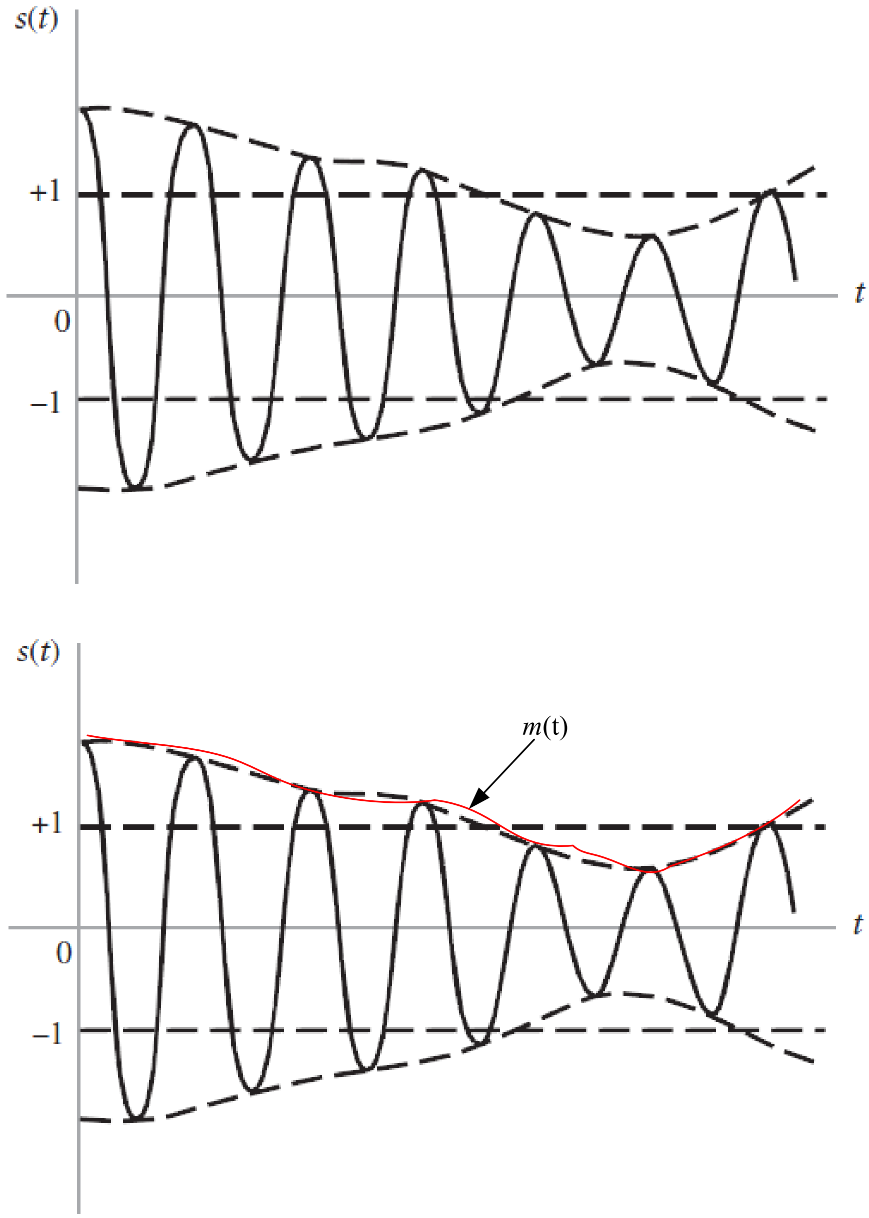


Figure-3

If the message signal is normalized before the modulation operation such that

$$|m(t)| \leq 1$$

Then the amplitude modulated signal can be indicated as

$$s(t) = A_c[1 + m(t)] \cos 2\pi f_c t.$$

Example: The message signal $m(t)$ is shown in Figure-4.

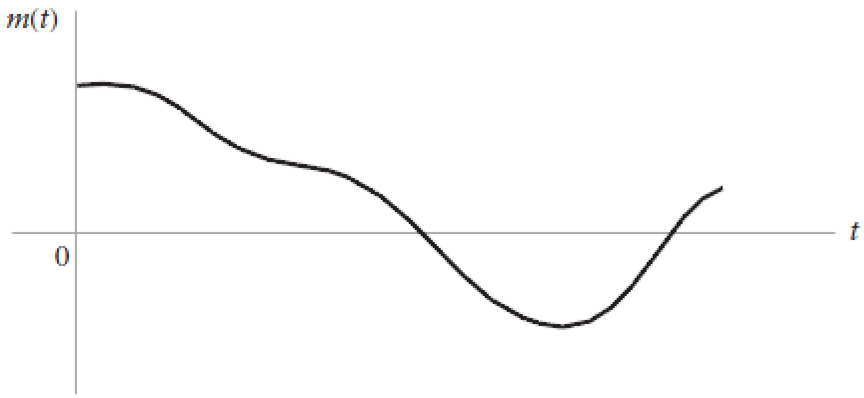


Figure-4

Assume that k_a is chosen such that $|k_a m(t)| > 1$, this means that

$$[1 + k_a m(t)]$$

can be greater than or smaller than 0.

Assume that the graph of $[1 + k_a m(t)]$ is something like in Figure-5

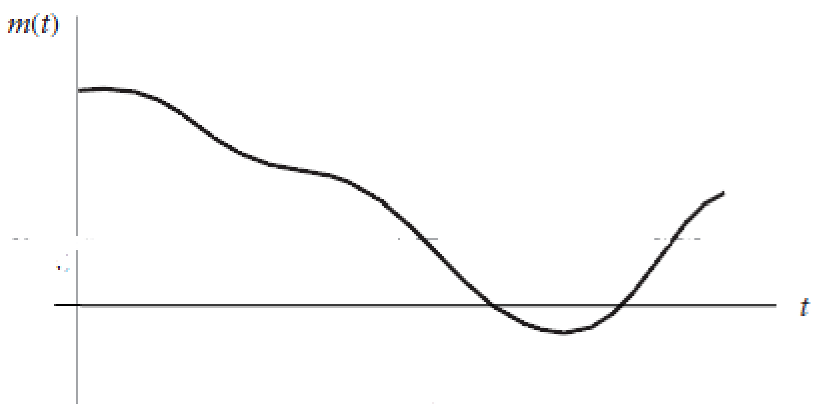


Figure-5

Then, the graph of

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

happens to be as in Figure-6 where the envelope of the modulated signal carries the information content of the message signal.

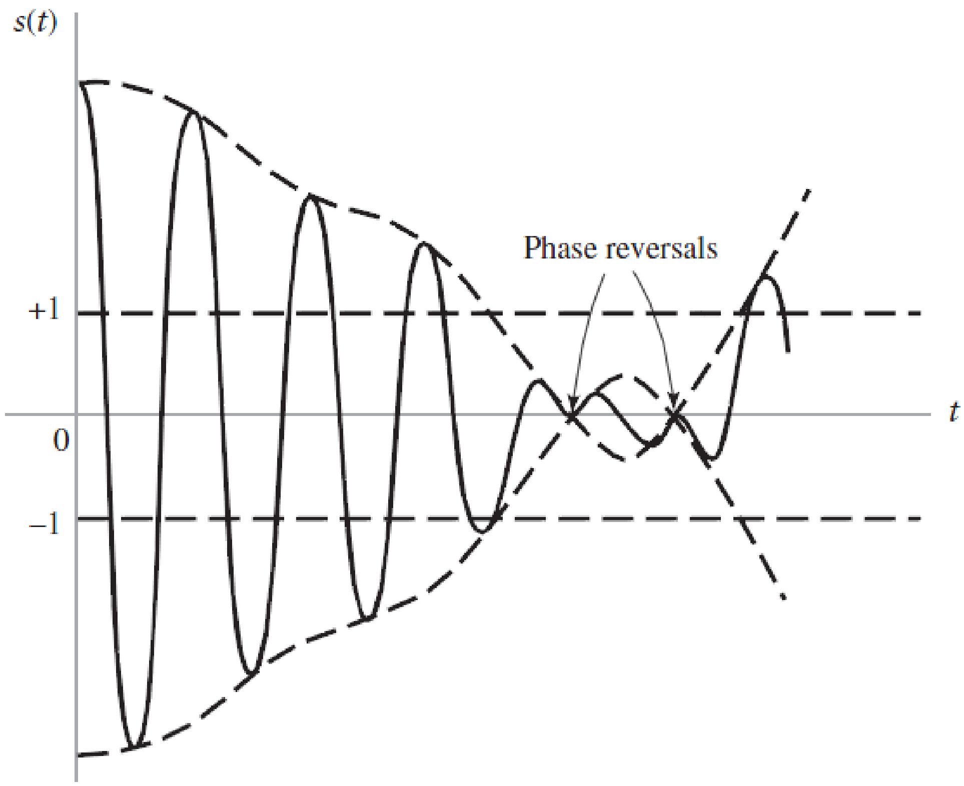


Figure-6

In Figure-6 we have envelope distortion.