

Chapter #3 (Vectors):Notes:

- ① north of east → start from east to northward.
- ② east of due north → start from N to eastward

③  $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$  |  $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

④ Resultant vector:

$$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$$

⑤  $R_x = B_x + A_x$

Similarly others.

⑥  $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

⑦  $|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$

⑧  $R_x = A_x + B_x$

⑨  $\vec{R}_n = A_n \hat{i} + B_n \hat{i}$

or  $= (A_n + B_n) \hat{i}$

⑩  $\phi = \tan^{-1} \left( \frac{A_y}{A_x} \right)$

⑪  $\phi = \tan^{-1} \left( \frac{R_y}{R_x} \right)$

without sign

now  $\theta = \pi - \phi$  |  $\theta = \phi$

$\theta = \pi + \phi$  |  $\theta = 2\pi - \phi$

⑪ minimum dist  $\Rightarrow$  find components then add them as scalars (not vectors)

⑫  $\vec{B} + \vec{C} = \vec{A}$   
 $\vec{C} = \vec{A} - \vec{B}$



- ④ Ground velocity =  $(v_x, v_y)$
- ④ Ground speed =  $|v_x, v_y|$

## ⑤ Position & Displacement:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$④ \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

- ④ Always resolve vector in  $x, y, (z)$  components.

↳ then add / subtract

- ④ always draw vector diagrams
- ④ always resolve into components.

(bc most mistakes happen if you do not split into  $x, y$ .)

$$④ \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

$$④ \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

↳  $(-\vec{B})$  = negative of vector  $\vec{B}$

$$④ \hat{\vec{A}} = \frac{\vec{A}}{|\vec{A}|} \quad (\text{unit vector})$$

$$④ \vec{A} + (-\vec{A}) = \vec{0} \text{ null vector.}$$

Addition of vectors = head to tail rule.

$$\vec{0} + \vec{A} + \vec{B} = \vec{A} + \vec{B}$$

(commutative)

$$(\vec{0} + (\vec{A} + \vec{B})) + \vec{C} = (\vec{A} + \vec{B}) + \vec{C}$$

(associative)

$\vec{0} + \vec{A}$  is the base & hypotenuse of a right-angled triangle.

$$\textcircled{O} \quad \vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\Rightarrow A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$\rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\rightarrow \tan \theta = \frac{A_y}{A_x}$$

$$\text{if 1st quad} \leftarrow \theta = \tan^{-1} \frac{A_y}{A_x}$$

but cases

if other quads

$$\textcircled{O} \quad \text{find } \phi = \tan^{-1} \left( \frac{A_y}{A_x} \right)$$

irrespective of signs

then check and apply suitable relation from given below

$\theta = \pi - \phi$	$\text{2nd}$	$\theta = \phi$	$\text{1st}$
$\theta = \pi + \phi$	$\text{3rd}$	$\theta = 2\pi - \phi$	$\text{4th}$

$$\textcircled{O} \quad \vec{A} \cdot \vec{B} = AB \cos \theta$$

projection of  $\vec{B}$  on  $\vec{A}$  =  $B \cos \theta$

projection of  $\vec{A}$  on  $\vec{B}$  =  $A \cos \theta$

$$\theta = b/w \vec{A} \cdot \vec{B}$$

$$\textcircled{O} \quad \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad (\text{scalar product is commutative.})$$

$$\textcircled{O} \quad \hat{i} \cdot \hat{j} = 0$$

$$\text{bc } \hat{i} \cdot \hat{j} = (\hat{1})(\hat{1}) \cos 90^\circ = 0 \quad (\because \cos 90^\circ = 0)$$

$$\textcircled{O} \quad \hat{i} \cdot \hat{i} = 1$$

$\textcircled{O}$  self product = sq. of its magnitude

$$\rightarrow \vec{A} \cdot \vec{A} = AA \cos 90^\circ = A^2$$

$$\textcircled{O} \quad \vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= (A_x B_x + A_y B_y + A_z B_z) \rightarrow \text{scalar quantity}$$

$$\textcircled{O} \quad \vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$\hat{n}$  = unit vector  $\perp$  to plane containing  $\vec{A}$  &  $\vec{B}$

its direction = Right hand rule.

R.H.R = Finger = first vector

curl in the direction of second vector

erect thumb = direction of  $\hat{n}$

$$\textcircled{O} \quad \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

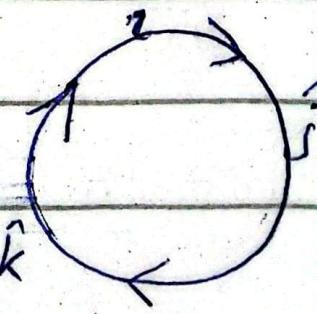
$$\rightarrow \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\textcircled{O} \quad \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$



- ①  $F_{\text{net}} = +, - \neq \Sigma$  actual impact
- ②  $F_{\text{total}} = \text{total} +, + \neq \Sigma$  total forces.
- ③ Avg velocity / acceleration vs.  
eq of motions &  $\Rightarrow \Sigma m \epsilon f$
- ④ change in displacement  $\Rightarrow d = d_2 - d_1$   
~~total disp~~  $\Rightarrow d = d_1 + d_2$ .

⑤  $\vec{i} \times \vec{i} = 0 \because \sin 0^\circ = 0$

⑥  $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

⑦  $|\vec{A} \times \vec{B}| = \text{area of parallelogram}$