Ch# 25

→ Capacitance → independent (does not depend) of (on) Voltage (V) or charge (Q)
→ depends upon Area of plates (A) and distance b/w plates.

Steps to derive capacitance

1) assume charge as charge enclosed i.e. $q_{enclosed}$
2) find electric field by applying gauss's law,
3) put E.F in potential gradient formula ($V = -\int_{-}^{+} \vec{E} \cdot d\vec{r}$) i.e. find Voltage V
4) then use $q = CV$ or $Q = CV$ formula to find capacitance

Derivations

Parallel Plate Capacitor (capacitance) simple (A)

① $q_{enclose}$

② $\phi_e = \frac{q_{enclosed}}{\varepsilon_0}$

② $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

$\oint E\,dA \cos 0° = \frac{q_{encl}}{\varepsilon_0}$

$E \oint dA = \frac{q_{encl}}{\varepsilon_0}$

$EA = \frac{q_{encl}}{\varepsilon_0}$

$E = \frac{q}{A\varepsilon_0} = \frac{q}{\varepsilon_0 A}$

$E = \frac{q_{encl.}}{\varepsilon_0 A}$

③ $V = -\int_{-}^{+} \vec{E} \cdot d\vec{r}$

$V = -\int_{-}^{+} E\,dr \cos 180°$

$V = E \int_{-}^{+} dr$

$V = Ed$

$V = \frac{qd}{\varepsilon_0 A}$

④ $q = CV$

$\cancel{q} = C \frac{\cancel{q} d}{\varepsilon_0 A}$

$\frac{\varepsilon_0 A}{d} = C$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

we didn't write value of A in it bc in parallel plate capacitor E remain constant.

Cylinderical Capacitor (ds) = -dr

$A = 2\pi r \cdot L$

$ds = -dr$

① $q_{encl.}$

② $\phi_e = \frac{q_{encl.}}{\varepsilon_0}$

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl.}}{\varepsilon_0}$

$\oint E\,dA \cos 0° = \frac{q_{encl.}}{\varepsilon_0}$

$E \int dA = \frac{q_{encl.}}{\varepsilon_0}$

$EA = \frac{q_{encl.}}{\varepsilon_0}$

$E(2\pi r L) = \frac{q_{encl.}}{\varepsilon_0}$

$E = \frac{q_{encl.}}{2\pi r L \cdot \varepsilon_0}$

③ $V = -\int_{-}^{+} \vec{E} \cdot d\vec{s}$

$V = -\int_{-}^{+} E \cdot ds \cos 180°$

$V = \int_{-}^{+} E\,ds$

$V = \int_{-}^{+} \frac{q_{encl.}}{2\pi r L \varepsilon_0} (-dr)$ ($\because ds = -dr$)

$V = -\frac{q_{encl.}}{2\pi L \varepsilon_0} \int_{-}^{+} \frac{1}{r} dr$

$V = -\frac{q_{encl}}{2\pi L \varepsilon_0} \ln r \Big|_b^a$

$V = \frac{-q_{encl}}{2\pi L \varepsilon_0} (\ln a - \ln b)$

$V = \frac{q_{encl.} (\ln b - \ln a)}{2\pi L \varepsilon_0}$

$V = \frac{q_{encl.} (\ln b - \ln a)}{2\pi L \varepsilon_0}$

④ now $q = CV$

$\cancel{q} = C \cdot \frac{\cancel{q_{encl}} (\ln b - \ln a)}{2\pi L \varepsilon_0}$

$\frac{2\pi L \varepsilon_0}{\ln b - \ln a} = C$

$$\boxed{C = \frac{2\pi L \varepsilon_0}{\left(\ln \frac{b}{a}\right)}}$$

we used whole value of A bc E changes with the change of r.

Spherical Capacitor

$A = \pi r^2$

$A = 4\pi r^2$

① $q_{encl.}$

② $\phi_e = \frac{q_{encl.}}{\varepsilon_0}$

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl.}}{\varepsilon_0}$

$\oint E\,dA \cos 0° = \frac{q_{encl.}}{\varepsilon_0}$

$EA = \frac{q_{encl.}}{\varepsilon_0}$

$E = \frac{q}{4\pi r^2 \varepsilon_0}$

③ now $V = -\int_{-}^{+} \frac{q}{4\pi r^2 \varepsilon_0} \cos 180°\, ds$

$V = \int_{-}^{+} \frac{q}{4\pi r^2 \varepsilon_0} (-dr)$ ($ds = -dr$)

$V = \frac{-q}{4\pi\varepsilon_0} \int_{-}^{+} \frac{1}{r^2} dr$

$V = \frac{-q}{4\pi\varepsilon_0} \int_{-}^{+} r^{-2} dr$

$V = \frac{-q}{4\pi\varepsilon_0} \left(\frac{r^{-1}}{-1}\right)\Big|_b^a$

$V = \frac{+q}{4\pi\varepsilon_0} \frac{1}{r}\Big|_b^a$

$V = \frac{+q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$

$V = \frac{+q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$

$V = \frac{+q}{4\pi\varepsilon_0} \left(\frac{b-a}{ab}\right)$

now ④ $q = CV$

$\cancel{q} = C \cdot \frac{\cancel{q}}{4\pi\varepsilon_0} \frac{(b-a)}{ab}$

$\frac{4\pi\varepsilon_0 (ab)}{(a-b)} = C$

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a}}$$

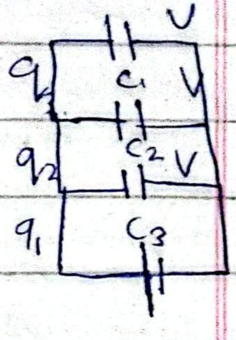
we used value of A bc E changes when r changes.

Note. ∘ pahla (in ϕ_e) $\cos 0° \to (1)$
∘ then in V $\cos 180° \to (-1)$
∘ Parallel plate capacitor → no A value
∘ other's them value of A ...

Concept: charges divide in junction but potential of charges remains same.

Capacitence:

Inparallel:



- Total charge $\Rightarrow q_{total} = q_1+q_2+q_3+\dots$
- total Voltage $\rightarrow V_{total} = V_1 = V_2 = V_3$
- $C_{eq} = C_1+C_2+C_3+\dots$

↳ derivation $\rightarrow q_{total} = q_1+q_2+q_3$

$C_{total}V_{total} = C_1V_1+C_2V_2+C_3V_3$

$C_{total}V_{total} = V(C_1+C_2+C_3)\ (\because V_1=V_2=V_3)$

$$\boxed{C_{eq} = C_1+C_2+C_3}$$

Note
- C_{eq} inparallel will always greater than the largest individual capacitance.

In Series:

charges remain but their potential changes so,

- charge $\rightarrow q_{total} = q_1 = q_2 = q_3$
- potential (Voltage) $\rightarrow V_{total} = V_1+V_2+V_3$
- $C_{eq} \rightarrow \boxed{C_{eq} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$

Note:
- C_{eq} inseries will always smallet than the smallest individual capacitance.

Special Case: (when only exact 2 capacitors inseries):

then

$$\boxed{C_{eq} = \frac{C_1C_2}{C_1+C_2}} \quad \left(\frac{\text{Product}}{\text{sum}}\right)$$

In Presence of Dielectric: for capacitanses → multiply C_{eq} or C with ε_r (dielectric constant)

Shrik & Expand Method:

- Series میں → charges ko pata hota ہے
- Parallel میں → potential (V) ko pata hota ہے
- → so jo pata hai vo find kr lo (kro)

charges & capacitance

(M) mian [illegible]

vakin potential (V) kr sakte

- charge on each capacitor
- voltage on each capacitor

} → diya reverse mai kam کرو

reverse mai calculation کرو

Steps for Calculations:

1. first of all find equivalent capacitance by using respective C_{eq} formula for series and parallel
2. then find total charge Q_{total} by formula $Q_{total} = C_{eq}V_{of\ battery}$
3. Now for charge & V (potential) on each capacitor go back in reverse order i.e. means jis pa (series or parallel) pr jo same ho vo pahla nikal lo.

conservation of charge

$$\boxed{C_1V_{i1} = (C_1+C_2)V_f}$$

$Q_1 = Q_{initial} = C_1V_{i1}$

$Q_{final} = Q_1+Q_2 \Rightarrow C_1V_i + C_2V_f = \boxed{(C_1+C_2)V_f}$