



Chapter #3 (Vectors):

Notes:

- ① north of east \rightarrow start from east to northward. 
- ② east of due north \rightarrow start from N to eastward. 

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad | \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

- ③ Resultant vector:

$$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$$

$$\vec{R}_x = B_x \hat{i} + A_x \hat{i}$$

Similarly others

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$R_x = A_x + B_x$$

$$\vec{R}_x = A_x \hat{i} + B_x \hat{i}$$

$$\text{or } = (A_x + B_x) \hat{i}$$

$$\phi = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

$$\text{or } \phi = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

without sign

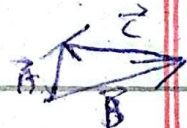
	- +	+ +
row	$\theta = \pi - \phi$	$\theta = \phi$

- -	+ -
$\theta = \pi + \phi$	$\theta = 2\pi - \phi$

④ Minimum dist \Rightarrow find components then add them as scalars (not vectors)

$$\vec{B} + \vec{C} = \vec{A}$$

$$\vec{C} = \vec{A} - \vec{B}$$



② Ground velocity = (v_x, v_y)

② Ground speed = $|v_x, v_y|$

② Position & Displacement:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

② $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

② Always resolve vector in $x, y, (z)$ components.

↳ then add/subtract

② always draw vector diagrams

② always resolve into components.

(bc most mistakes happen if you do not split into x, y .)

② $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

② $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

↳ $(-\vec{B}) =$ negative of vector \vec{B}

② $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$ (unit vector)

② $\vec{A} + (-\vec{A}) = \vec{0}$ null vector.

addition of vectors = head to tail rule.
 $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative)
 $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (associative)
 $\vec{a} \cdot \vec{a} = \text{base}^2 + \text{hyp}^2 / \text{hyp} = \text{longest slope}$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\Rightarrow A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$\Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\Rightarrow \tan \theta = \frac{A_y}{A_x}$$

if 1st Quad $\leftarrow \theta = \tan^{-1} \frac{A_y}{A_x}$

but cases

if other quad

$$\textcircled{\bullet} \text{ find } \phi = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

irrespective of signs

then check and apply suitable relation from given below

2nd	1st
$\theta = \pi - \phi$	$\theta = \phi$
3rd	4th
$\theta = \pi + \phi$	$\theta = 2\pi - \phi$

$$\textcircled{\bullet} \vec{A} \cdot \vec{B} = AB \cos \theta$$

projection of B on A = $B \cos \theta$

projection of A on B = $A \cos \theta$

$\theta = \text{b/w } \vec{A} \text{ \& } \vec{B}$

$$\textcircled{\bullet} \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \text{ (scalar product is commutative)}$$

$$\textcircled{\bullet} \hat{i} \cdot \hat{j} = 0$$

$$\text{bc } \hat{i} \cdot \hat{j} = (1)(1) \cos 90^\circ = 0 (\because \cos 90^\circ = 0)$$

$$\textcircled{\bullet} \hat{i} \cdot \hat{i} = 1$$

$\textcircled{\bullet}$ self product = sq of its magnitude

$$\hookrightarrow \vec{A} \cdot \vec{A} = AA \cos 0^\circ = A^2$$

$$\textcircled{\bullet} \vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= (A_x B_x + A_y B_y + A_z B_z) \rightarrow \text{scalar quantity}$$

$$\textcircled{\bullet} \vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

\hat{n} = unit vector \perp to plane containing A & B

its direction = Right hand rule.

RHR = Finger = first vector

curl in the direction of second vector

Erect thumb = direction of \hat{n}

$$\textcircled{\bullet} \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

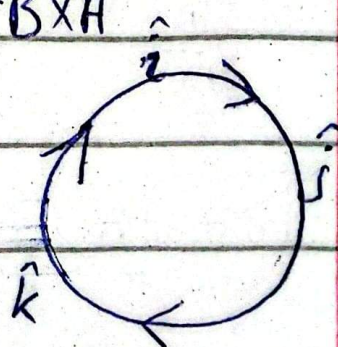
$$\hookrightarrow \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\textcircled{\bullet} \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$



① $F_{net} = +, -$ \sum actual input

② $F_{total} = total +, - \sum$ total forces,

③ Avg velocity / acceleration use

eq of motions $\&$ $\> \sum$ use $\&$ $\> \sum$

④ change in displacement $\Rightarrow \Delta d = d_2 - d_1$

⑤ total disp $\Rightarrow d = d_1 + d_2$

$$\textcircled{\bullet} \hat{i} \times \hat{i} = 0 \quad \because \sin 0^\circ = 0$$

$$\textcircled{\bullet} \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

⑥ $|\vec{A} \times \vec{B}| = \text{area of parallelogram}$