

# Results: INF421 2021: Matching under constraints

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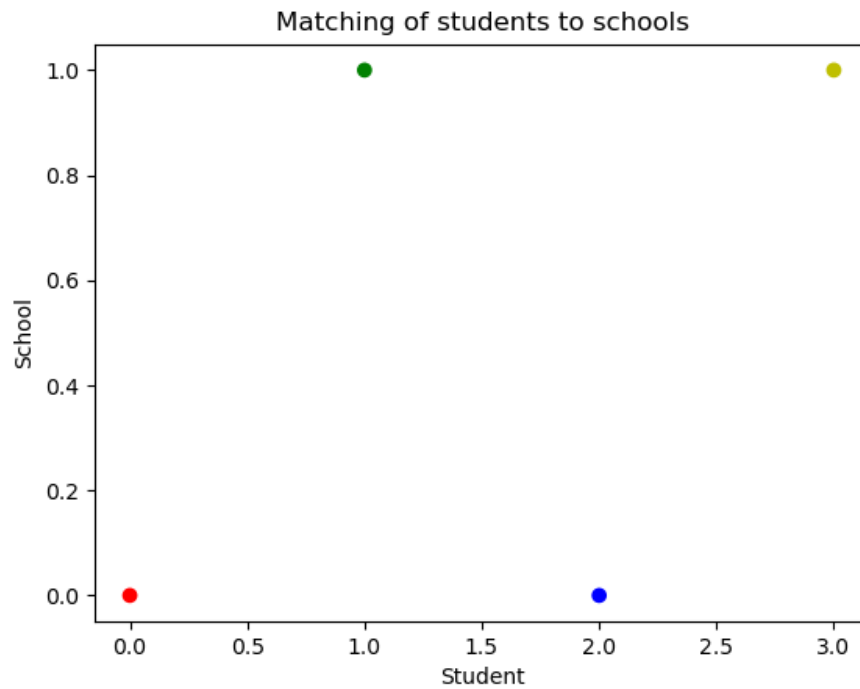
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## 1 Task 4:

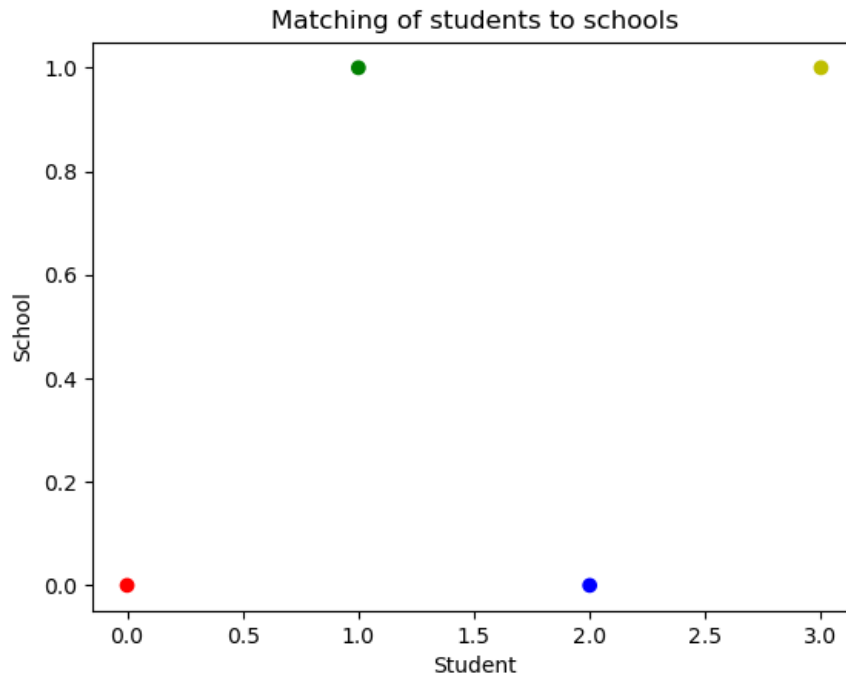
We will present for each instance the results for each task in order to compare the programs for each task.

### 1.1 Test Instance 1:

#### 1.1.1 Test Task 2:



### 1.1.2 Test Task 3:



## 1.2 Test instance 3:

For each value of  $n$ , we show the average number of students from the groups  $A$  and  $B$ . We also show the percentage of students getting their first choice from each group.

### 1.2.1 Test Task 2:

**$n = 1000$  :**

Average number of students from the group  $A$  to get their first choice: 676.115

Percentage of  $A$ 's to get their first choice: 75.12%

Average number of students from the group  $B$  to get their first choice: 74.28

Percentage of  $B$ 's to get their first choice: 74.28%

**$n = 2000$  :**

Average number of students from the group  $A$  to get their first choice: 1352.68

Percentage of  $A$ 's to get their first choice: 75.14%

Average number of students from the group  $B$  to get their first choice: 149.37

Percentage of  $B$ 's to get their first choice: 74.68%

**$n = 3000$  :**

Average number of students from the group  $A$  to get their first choice: 2025.535

Percentage of  $A$ 's to get their first choice: 75.01%

Average number of students from the group  $B$  to get their first choice: 224.595

Percentage of  $B$ 's to get their first choice: 74.86%

### 1.2.2 Test Task 3:

**$n = 1000$  :**

Average number of students from the group  $A$  to get their first choice: 676.365

Percentage of  $A$ 's to get their first choice: 75.15%

Average number of students from the group  $B$  to get their first choice: 74.21

Percentage of  $B$ 's to get their first choice: 74.21%

**n = 2000 :**

Average number of students from the group  $A$  to get their first choice: 1350.555

Percentage of  $A$ 's to get their first choice: 75.03%

Average number of students from the group  $B$  to get their first choice: 150.045

Percentage of  $B$ 's to get their first choice: 75.02%

**n = 3000 :**

Average number of students from the group  $A$  to get their first choice: 2024.795

Percentage of  $A$ 's to get their first choice: 74.99%

Average number of students from the group  $B$  to get their first choice: 224.055

Percentage of  $B$ 's to get their first choice: 74.68%

We see that, for both algorithms, the fraction of students getting their first choice is 75%.

### 1.3 Test instance 4:

#### 1.3.1 Test Task 2:

**n = 1000 :**

Average number of students from the group  $A$  to get their first choice: 376.275

Percentage of  $A$ 's to get their first choice: 75.25%

Average number of students from the group  $B$  to get their first choice: 225.29

Percentage of  $B$ 's to get their first choice: 75.09%

Average number of students from the group  $C$  to get their first choice: 112.47

Percentage of  $C$ 's to get their first choice: 74.98%

Average number of students from the group  $D$  to get their first choice: 36.935

Percentage of  $D$ 's to get their first choice: 73.87%

**n = 2000 :**

Average number of students from the group  $A$  to get their first choice: 751.1

Percentage of  $A$ 's to get their first choice: 75.11%

Average number of students from the group  $B$  to get their first choice: 450.3

Percentage of  $B$ 's to get their first choice: 75.04%

Average number of students from the group  $C$  to get their first choice: 224.69

Percentage of  $C$ 's to get their first choice: 74.89%

Average number of students from the group  $D$  to get their first choice: 74.685

Percentage of  $D$ 's to get their first choice: 74.68%

**n = 3000 :**

Average number of students from the group  $A$  to get their first choice: 1125.305

Percentage of  $A$ 's to get their first choice: 75.02%

Average number of students from the group  $B$  to get their first choice: 675.51

Percentage of  $B$ 's to get their first choice: 75.05%

Average number of students from the group  $C$  to get their first choice: 337.22

Percentage of  $C$ 's to get their first choice: 74.93%

Average number of students from the group  $D$  to get their first choice: 112.285

Percentage of  $D$ 's to get their first choice: 74.85%

#### 1.3.2 Test Task 3:

**n = 1000:**

Average number of students from the group  $A$  to get their first choice: 375.055

Percentage of  $A$ 's to get their first choice: 75.01%

Average number of students from the group  $B$  to get their first choice: 225.015

Percentage of  $B$ 's to get their first choice: 75.0%

Average number of students from the group  $C$  to get their first choice: 112.86

Percentage of  $C$ 's to get their first choice: 75.24%

Average number of students from the group  $D$  to get their first choice: 36.83

Percentage of  $D$ 's to get their first choice: 73.65%

**n = 2000:**

Average number of students from the group  $A$  to get their first choice: 748.81

Percentage of  $A$ 's to get their first choice: 74.88%

Average number of students from the group  $B$  to get their first choice: 449.91

Percentage of  $B$ 's to get their first choice: 74.98%

Average number of students from the group  $C$  to get their first choice: 225.105

Percentage of  $C$ 's to get their first choice: 75.03%

Average number of students from the group  $D$  to get their first choice: 74.605

Percentage of  $D$ 's to get their first choice: 74.6%

**n = 3000:**

Average number of students from the group  $A$  to get their first choice: 1124.955

Percentage of  $A$ 's to get their first choice: 74.99%

Average number of students from the group  $B$  to get their first choice: 675.075

Percentage of  $B$ 's to get their first choice: 75.0% Average number of students from the group  $C$  to get their first choice: 337.655

Percentage of  $C$ 's to get their first choice: 75.03%

Average number of students from the group  $D$  to get their first choice: 111.335

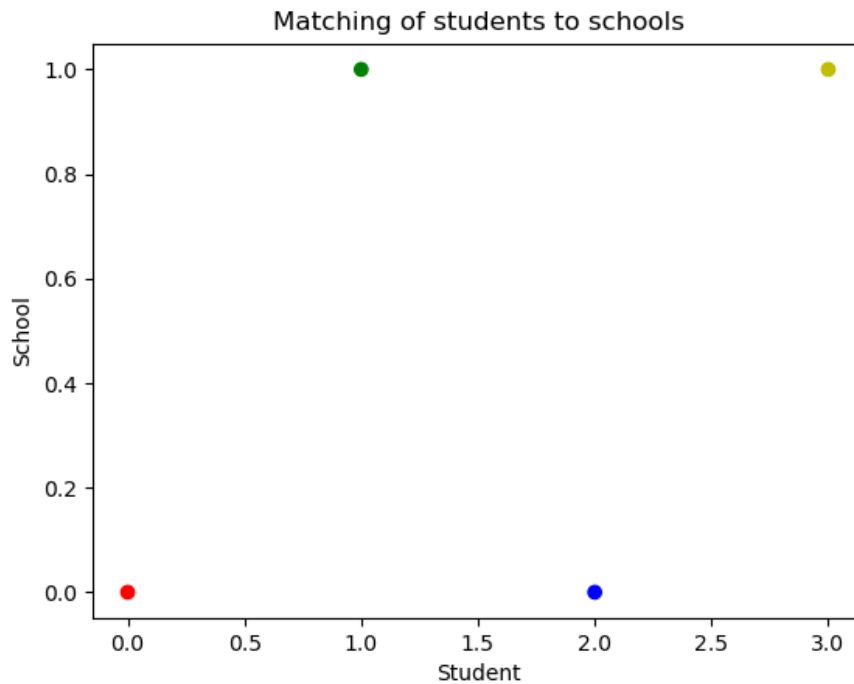
Percentage of  $D$ 's to get their first choice: 74.22%

## 2 Task 6:

### 2.1 Test Instance 1:

Here, we have taken the proportions from the set of students.

We obtain the following result:



In particular, we observe that it is the same matching as previously. The  $\frac{4}{5}$ -rule is not satisfied rigorously because they are not enough candidates.

## 2.2 Test Instance 2:

We obtain the following results:

**n = 1000:**  
 $s_1$  verifies the  $\frac{4}{5}$ -rule: **False**  
 $s_2$  verifies the  $\frac{4}{5}$ -rule: **True**

**n = 1500:**  
 $s_1$  verifies the  $\frac{4}{5}$ -rule: **False**  
 $s_2$  verifies the  $\frac{4}{5}$ -rule: **True**

**n = 2000:**  
 $s_1$  verifies the  $\frac{4}{5}$ -rule: **False**  
 $s_2$  verifies the  $\frac{4}{5}$ -rule: **True**

The schools do not satisfy rigorously the  $4/5$ -rule because there are not enough candidates.

## 2.3 Test Instance 3:

**n = 1000:**  
 $s_1$  verifies the  $\frac{4}{5}$ -rule: **False**  
 $s_2$  verifies the  $\frac{4}{5}$ -rule: **False**

**n = 1500:**  
 $s_1$  verifies the  $\frac{4}{5}$ -rule: **False**  
 $s_2$  verifies the  $\frac{4}{5}$ -rule: **False**

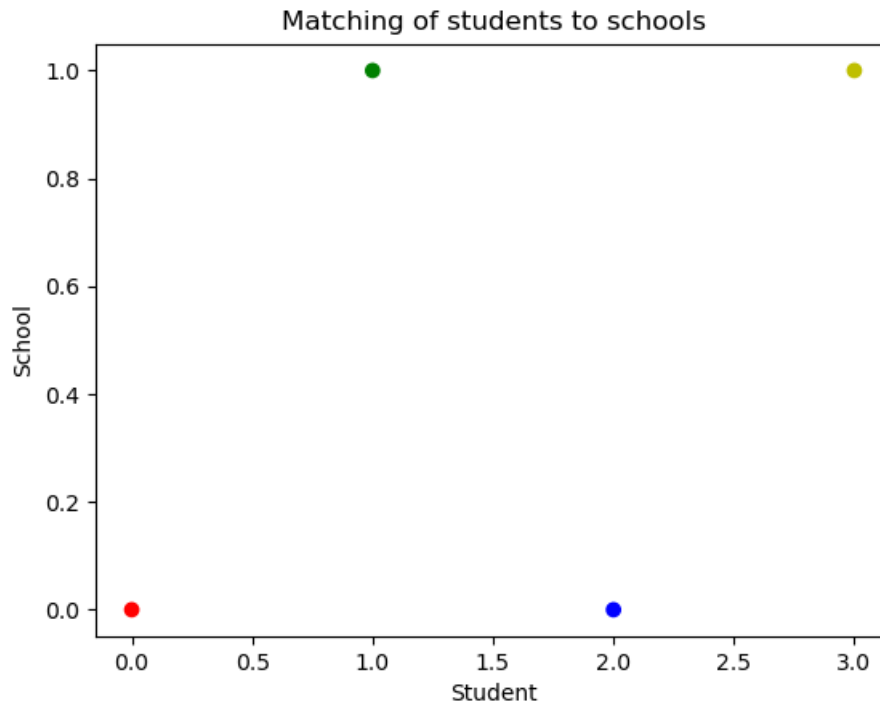
**n = 2000:**  
 $s_1$  verifies the  $\frac{4}{5}$ -rule: **False**  
 $s_2$  verifies the  $\frac{4}{5}$ -rule: **False**

As previously, the schools do not satisfy rigorously the  $4/5$ -rule because there are not enough candidates.

## 3 Task 8:

### 3.1 Test Instance 1:

We get the following result:



which is the same as previously.

### 3.2 Test Instance 2:

**n = 100:**

Average number of students from the group *A*: 22.77

Percentage of *A*'s to get their first choice: 25.3%

Average number of students from the group *B*: 2.23

Percentage of *B*'s to get their first choice: 22.3%

**n = 200:**

Average number of students from the group *A*: 45.25

Percentage of *A*'s to get their first choice: 25.13%

Average number of students from the group *B*: 4.75

Percentage of *B*'s to get their first choice: 23.75%

**n = 300:**

Average number of students from the group *A*: 67.765

Percentage of *A*'s to get their first choice: 25.09%

Average number of students from the group *B*: 7.235

Percentage of *B*'s to get their first choice: 24.11%

### 3.3 Test Instance 3:

**n = 100:**

Average number of students from the group *A*: 12.65

Percentage of *A*'s to get their first choice: 25.3%

Average number of students from the group *B*: 7.495

Percentage of *B*'s to get their first choice: 24.98%

Average number of students from the group *C*: 3.79

Percentage of *C*'s to get their first choice: 25.26%

Average number of students from the group *D*: 1.065

Percentage of *D*'s to get their first choice: 21.3%

**n = 200:**

Average number of students from the group *A*: 25.265

Percentage of A's to get their first choice: 25.26%

Average number of students from the group *B*: 14.91

Percentage of B's to get their first choice: 24.85%

Average number of students from the group *C*: 7.53

Percentage of C's to get their first choice: 25.1%

Average number of students from the group *D*: 2.295

Percentage of D's to get their first choice: 22.95%

**n = 300:**

Average number of students from the group *A*: 37.41

Percentage of A's to get their first choice: 24.94%

Average number of students from the group *B*: 22.9

Percentage of B's to get their first choice: 25.44%

Average number of students from the group *C*: 11.215

Percentage of C's to get their first choice: 24.92%

Average number of students from the group *D*: 3.475

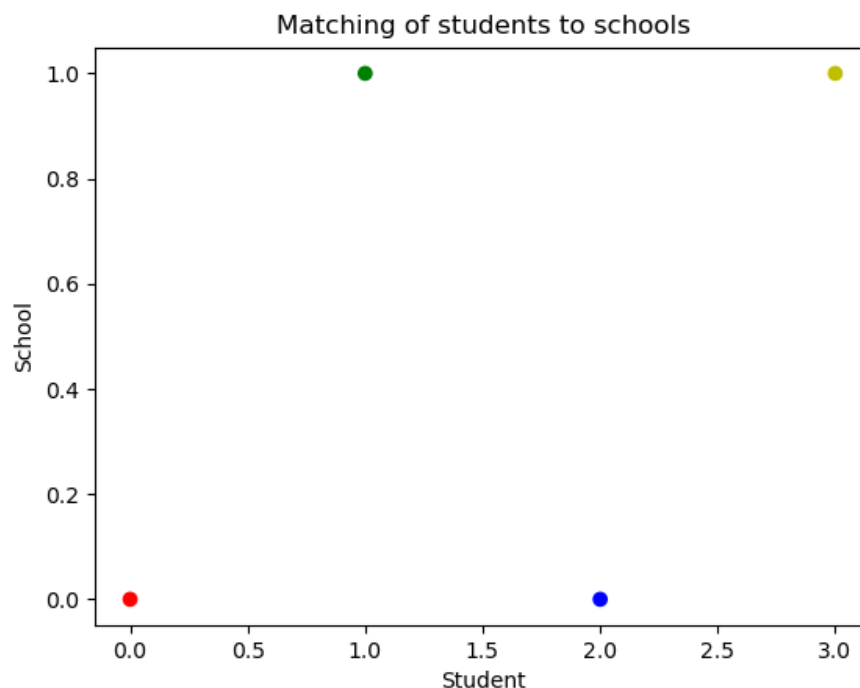
Percentage of D's to get their first choice: 23.16%

We observe a sharp decrease in the number of people getting their first choice since it went from nearly 75% when we used the task 2 implementation to roughly 25%.

## 4 Task 9:

### 4.1 Test Instance 1:

We get the following result:



We see that we get the same result as previously.

### 4.2 Test Instance 2:

**n = 200:**

Average number of students from the group *A*: 44.615

Percentage of A's to get their first choice: 24.78%  
 Average number of students from the group *B*: 4.305  
 Percentage of B's to get their first choice: 21.52%

**n = 300:**

Average number of students from the group *A*: 66.375  
 Percentage of A's to get their first choice: 24.58%  
 Average number of students from the group *B*: 7.135  
 Percentage of B's to get their first choice: 23.78%

**n = 400:**

Average number of students from the group *A*: 89.15  
 Percentage of A's to get their first choice: 24.76%  
 Average number of students from the group *B*: 9.76  
 Percentage of B's to get their first choice: 24.4%

### 4.3 Test Instance 3:

**n = 200:**

Average number of students from the group *A*: 25.065  
 Percentage of A's to get their first choice: 25.06%  
 Average number of students from the group *B*: 15.3  
 Percentage of B's to get their first choice: 25.5%  
 Average number of students from the group *C*: 7.515  
 Percentage of C's to get their first choice: 25.05%  
 Average number of students from the group *D*: 2.12  
 Percentage of D's to get their first choice: 21.2%

**n = 300:**

Average number of students from the group *A*: 37.995  
 Percentage of A's to get their first choice: 25.32%  
 Average number of students from the group *B*: 22.11  
 Percentage of B's to get their first choice: 24.56%  
 Average number of students from the group *C*: 11.435  
 Percentage of C's to get their first choice: 25.41%  
 Average number of students from the group *D*: 3.46  
 Percentage of D's to get their first choice: 23.06%

**n = 400:**

Average number of students from the group *A*: 50.16  
 Percentage of A's to get their first choice: 25.07%  
 Average number of students from the group *B*: 29.935  
 Percentage of B's to get their first choice: 24.94%  
 Average number of students from the group *C*: 14.92  
 Percentage of C's to get their first choice: 24.86%  
 Average number of students from the group *D*: 4.985  
 Percentage of D's to get their first choice: 24.92%

We observe a sharp decrease in the number of people getting their first choice since it went from nearly 75% to roughly 25%.

## 5 Task 12:

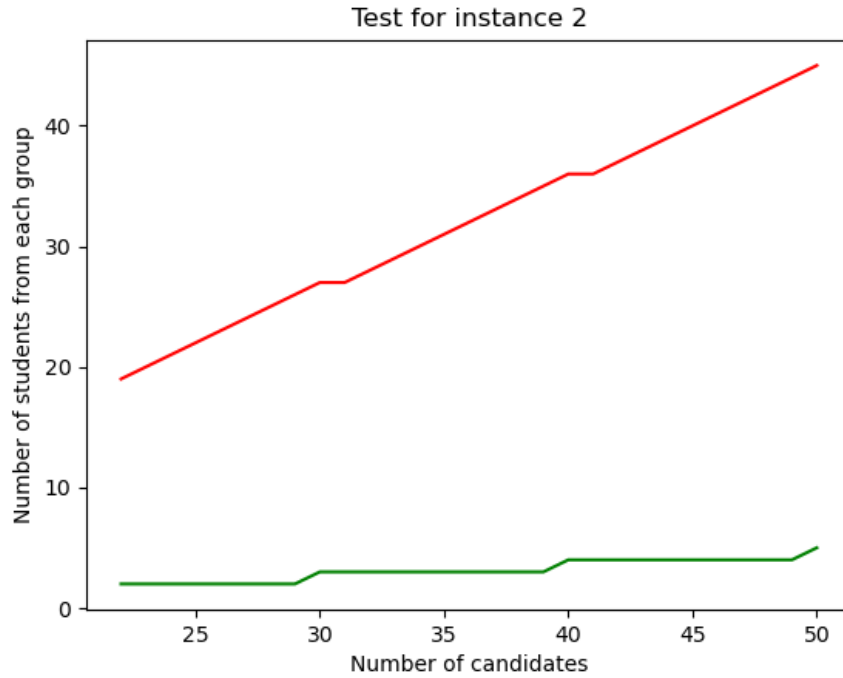
### 5.1 Test Instance 1:

In this case, no school can take any student because of the  $\frac{4}{5}$ -rule and the budget limitation.



## 5.2 Test Instance 2:

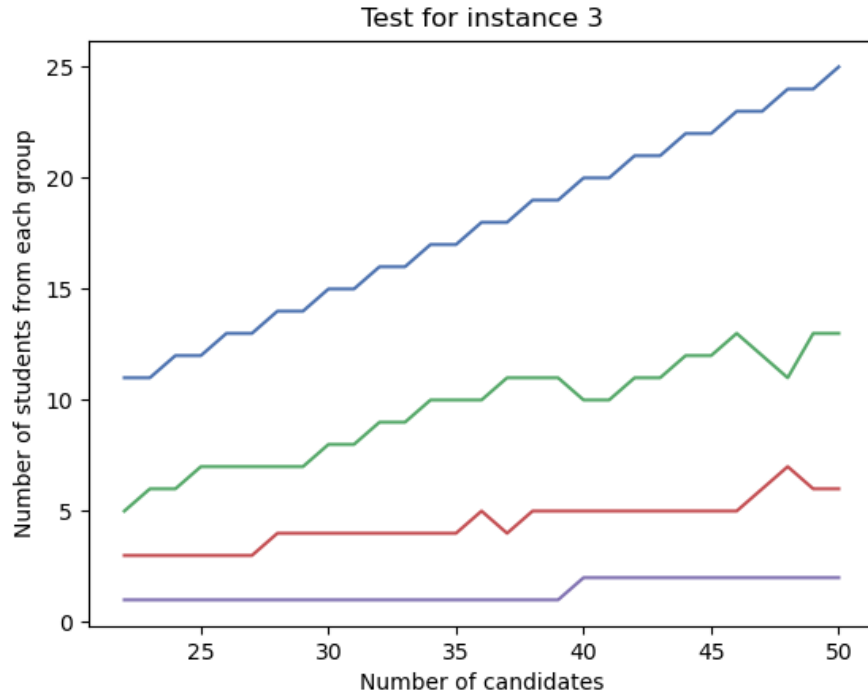
We have represented the evolution of the number of students from each group. The first group is in red ; whereas the second is in green.



We can observe that, since the second group is more costly than the first one, the addition of a student from this group is done when we can no more respect the  $\frac{4}{5}$ -rule.

## 5.3 Test Instance 3:

As before, we represent the evolution of the number of students from each group. The first group is in blue, the second in green, the third in pink and the fourth in purple.

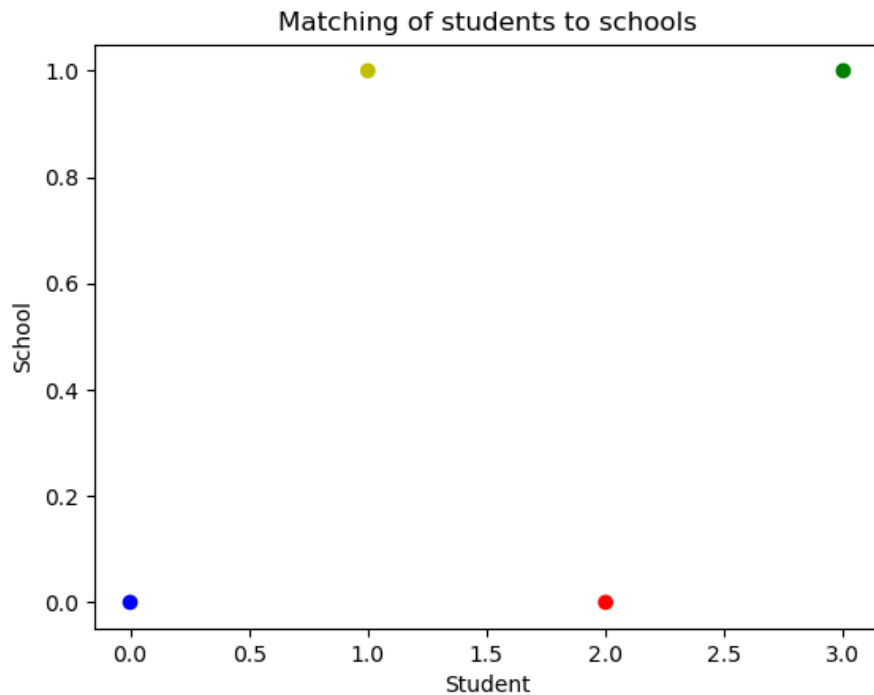


As previously, we observe that the addition of students from the most costly groups is minimal and only done in order to respect the  $\frac{4}{5}$ -rule. We also observe that the number of students from each group does not necessarily increase.

## 6 Task 14:

### 6.1 Test Instance 1:

As usual, we represent the result in the following plot:

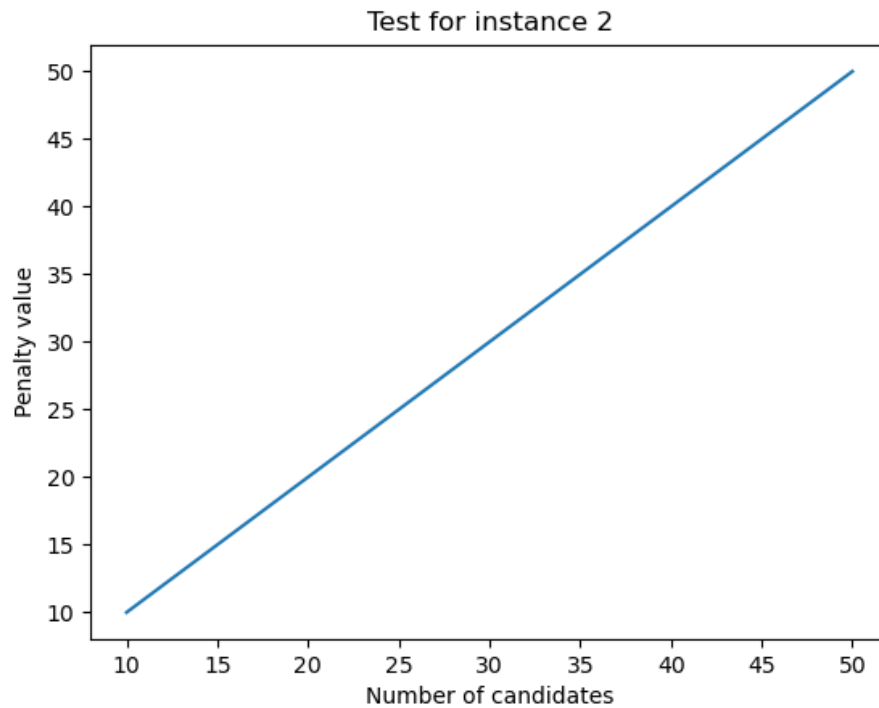


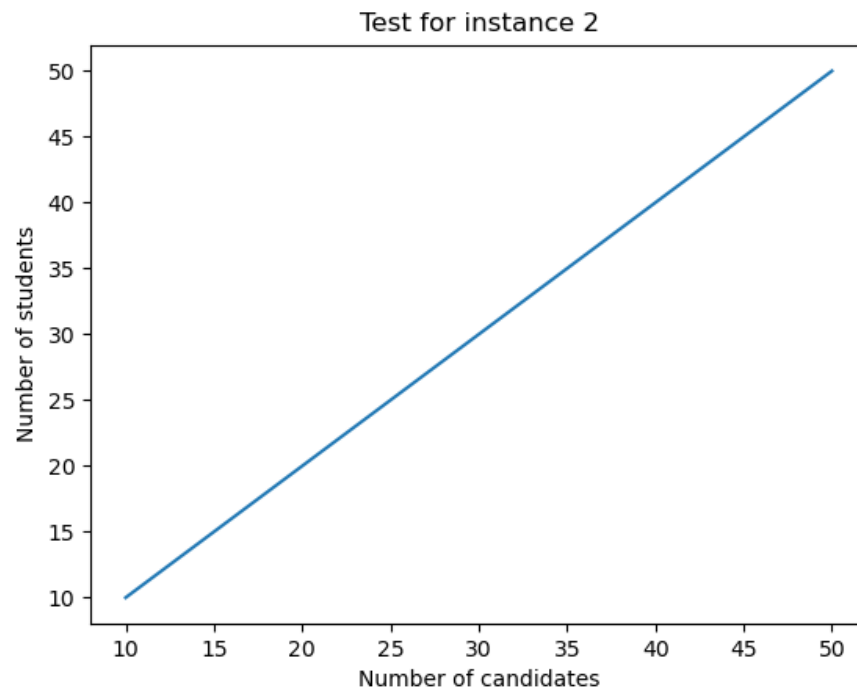
It turns out that the student from the second group goes to  $s_2$ . The penalty is 4, which means that all the students get their first choice.

For the following instances, we represent in function of the number of candidates the evolution of the penalty and the number of students who get a school.

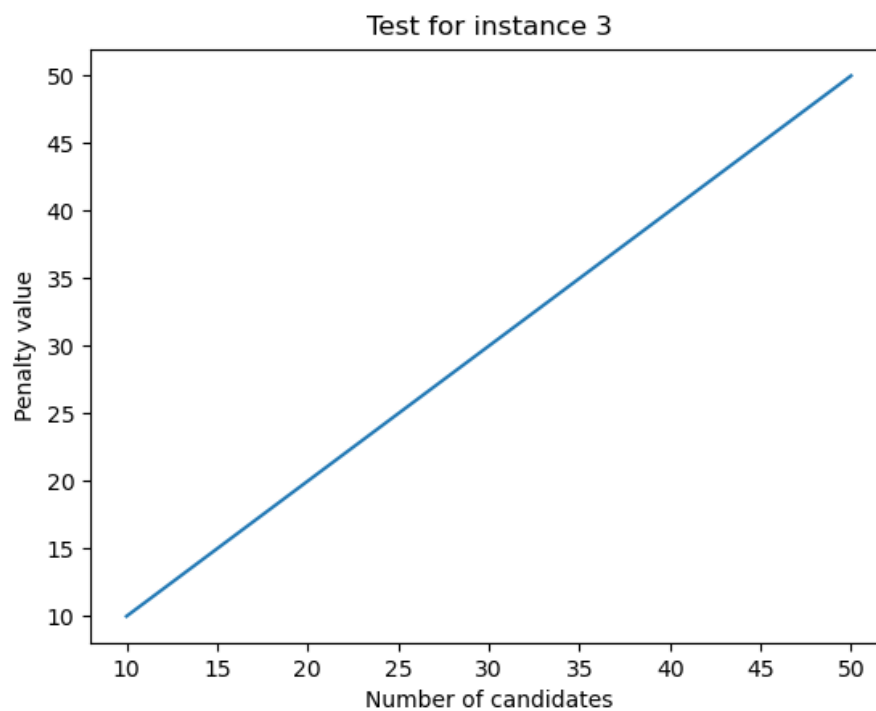
The implementation of `rand_instance` will use two lists that save the students who prefer  $s_1$  over  $s_2$  or the inverse. These lists will make the definition of `s1` and `s2` easier since the  $i^{th}$  list in `studs` contains the students who rank the school at the  $i^{th}$  position in the increasing order of the number of ranked schools.

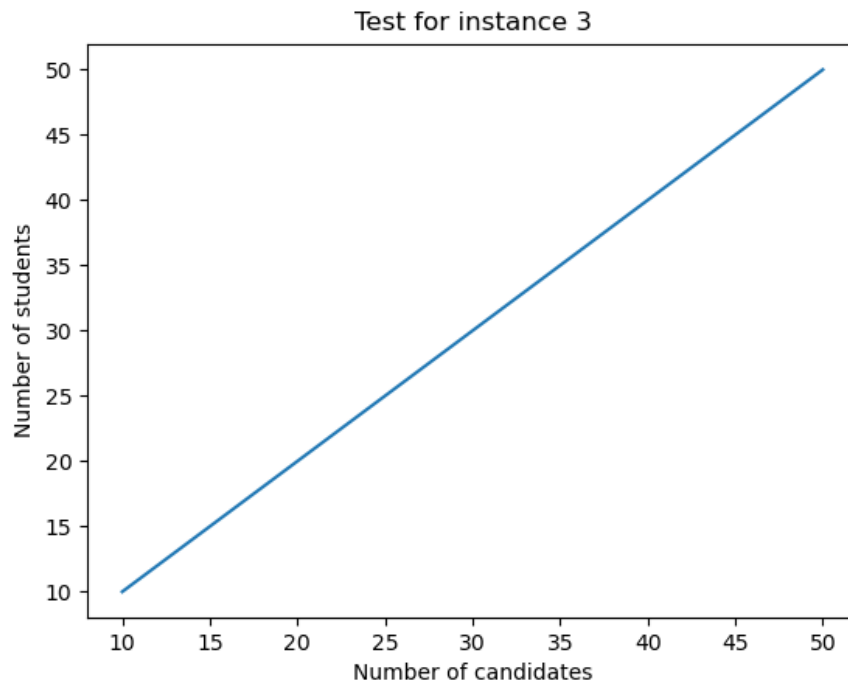
## 6.2 Test Instance 2:





### 6.3 Test Instance 3:





We observe that all the students get their first choices. It seemed for us strangely odd. However, a close analysis of the code has not revealed a bug.