

MULTI-EDGES AND LOOPS IN $\mathcal{G}_\ell(\mathbb{F}_p)$ FOR $2 \leq \ell < 100$ AND $p > \ell$

An entry (ℓ, p) in column 2 means that $\mathcal{G}_\ell(\mathbb{F}_p)$ has double directed edges that are not loops. An entry (ℓ, p) in column 3 means that $\mathcal{G}_\ell(\mathbb{F}_p)$ has loops, but no other multi-edges.

Except for the entry for $(\ell, p) = (2, 3)$ which has a triple directed edge arising from extra automorphisms of the supersingular elliptic curve $E/\mathbb{F}_3 : y^2 = x^3 - x$, columns 2 and 3 only contain primes $p \equiv 1 \pmod{4}$ and $p \equiv 3 \pmod{4}$, respectively.

ℓ	p with non-loop double edges	p with loops
2	3	7
3	5	11
5	—	11, 19
7	13	19
11	13	19, 43
13	17	43
17	—	19, 43, 59, 67
19	29, 37	67
23	37	43, 67, 83
29	—	67, 107
31	37, 53, 61	43
37	73	67, 139
41	73	43, 83, 139, 163
43	61	163
47	—	67, 107, 139, 163, 179
53	97	131, 163, 211
59	109	67, 211, 227
61	73, 97, 113	163
67	109	—
71	—	163, 283
73	97, 137	211, 283
79	109, 149, 157	307
83	157	107, 163, 211, 251, 283, 207, 311
89	97	131, 307, 331, 347
97	113, 193	163, 307, 379