Problem 1. statement 9 is True.

Problem 2.

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + I \qquad \text{——— } (1)$$

$$\frac{dw}{dt} = \varepsilon(\alpha + v)$$

1

$$\alpha + V = 0$$

$$\boxed{V_e = -\alpha}$$

$$V - \frac{V^3}{3} - w + I = 0$$

$$w_e = v - \frac{v^3}{3} + I$$

$$\boxed{w_e = -\alpha + \frac{\alpha^3}{3} + I}$$

2 The stability of fixed pt (v_e, w_e) depend on the sign of trace $(\mathrm{Tr}J)$ and determinant $(\det J)$ of the Jacobian

matrix.

$$F = v - \frac{v^3}{3} - w + I$$

$$G = \varepsilon(\alpha + v)$$

$$\partial f / \partial v = 1 - v^2$$

$$\partial F / \partial w = -1$$

$$\partial G / \partial v = \varepsilon$$

$$\partial G / \partial w = 0$$

$$J(v_e, w_e) = \begin{pmatrix} 1 - v^2 & -1 \\ \varepsilon & 0 \end{pmatrix}$$

$$\operatorname{Tr} J(v_e, w_e) = 1 - v^2 + 0$$

$$\operatorname{Tr} J(v_e, w_e) = 1 - (-\alpha)^2 = 1 - \alpha^2$$

$$\operatorname{Det} J = 0 + (1)(\varepsilon) = \varepsilon$$

For a fixed point (v_e, w_e) to be stable it suffice to show That $\operatorname{Tr} J < 0$ and $\det J > 0$

so

$\Rightarrow \operatorname{Tr} J < 0$

$1 - \alpha^2 < 0$

$-\alpha^2 < -1$

$\boxed{\alpha^2 > 1}$

$|\alpha| > 1$

If α greater than 1 or less than -1.

$\Rightarrow \operatorname{Det} J > 0$

since

$0 < \varepsilon \ll 1$

ε is a small +ve number greater than 0

The fixed pt is stable when $\alpha > 1$ or $\alpha < -1$

The fixed pt is unstable $-1 \leq \alpha \leq 1$

Problem 3.

$$\frac{dx}{dt} = y - ax^3 + bx^2 - z + I$$

$$\frac{dy}{dt} = c - dx^2 - y$$

$$\frac{dz}{dt} = r(s(x - x_{rest}) - z)$$

$$y - ax^3 + bx^2 - z + I = 0 \quad \text{—— } 1$$

$$c - dx^2 - y = 0 \quad \text{—— } 2$$

$$r(s(x - x_{rest}) - z) = 0 \quad \text{—— } 3$$

$$y = c - dx^2$$

$$z = r(s(x - x_{rest})$$

put it in 1

$$c - dx^2 - ax^3 + bx^2 - (sx - sx_{rest}) + I = 0$$

$$-ax^3 + (b - d)x^2 - sx + c + sx_{rest} + I = 0$$

$$-ax^3 + (b-d)x^2 - sx + (c + sx_{rest} + I) = 0$$

$$\frac{ax^3}{a} - \frac{(b-d)x^2}{b} + \frac{sx}{c} - \frac{(c + sx_{rest} + I)}{d} = 0$$

$$t = x + \frac{b}{3a}, \quad x = t - \frac{b}{3a}$$

$$P = \frac{3ac - b^2}{3a^2}$$

$$q = \frac{2b^3 - 9abc + 27a^2 d}{27a^3}$$

$$x^3 + 2x^2 + 5.2x$$
$$-(1 + 5.2 \times (-1.56) + 0) = 0$$

$$\underset{a}{\underline{x^3}} + \underset{b}{\underline{2x^2}} + \underset{c}{\underline{5.2x}} + \underset{d}{\underline{7.112}} = 0 \longrightarrow \star$$

$x = t - \frac{b}{3a}$ putting this in $\star$

$$\left(t - \frac{2}{3}\right)^3 + 2\left(t - \frac{2}{3}\right)^2 + 5.2\left(t - \frac{2}{3}\right)$$

$$+ 7.112 = 0$$

$$t^3 + 3.866t + 4.236 = 0 \quad \text{— (1)}$$

$$\boxed{P = 3.866,\ Q = 4.236}$$

$$u + v = t$$

$$u^3 + v^3 + (3uv + p)(u + v) + q = 0$$

$$u^3 + v^3 = -q$$

$$uv = -p/3$$

$$u^3 + v^3 = -4.236$$

$$uv = -3.866/3 = -1.288$$

$$0 = (x - u^3)(x - v^3)$$

$$= x^2 - (u^3 + v^3)x + u^3v^3$$

$$= x^2 - (u^3 + v^3)x + (uv)^3$$

so

$$x^2 + qx - \frac{p^3}{27} = 0$$

$$\Delta = q^2 + \frac{4p^3}{27} = (4.236)^2 + \frac{4(3.866)^3}{27}$$

$$= 17.94 + 8.560 = 26.50 > 0$$

$$u^3, v^3 = \frac{-q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

$$u, v = \sqrt[3]{\frac{-q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

$$\boxed{\begin{aligned} u &= 0.7686 \\ v &= -1.673 \end{aligned}}$$

$$t = u + v = -0.9052$$

$$\boxed{\begin{aligned} x_e &= -1.5719 \\ y_e &= -11.354 \\ z_e &= -0.0618 \end{aligned}}$$

③ Stability

$$F(x,y,z) = y - ax^3 + bx^2 - z + I$$

$$G(x,y,z) = c - dx^2 - y$$

$$H(x,y,z) = r(s(x - x_{rest}) - z)$$

$$J = \begin{pmatrix} \partial f/\partial x & \partial f/\partial y & \partial f/\partial z \\ \partial g/\partial x & \partial g/\partial y & \partial g/\partial z \\ \partial h/\partial x & \partial h/\partial y & \partial h/\partial z \end{pmatrix}$$

$\partial f/\partial z = -1$, $\partial f/\partial x = -3ax^2 + 2bx$

$\partial f/\partial y = 1$

$$\partial g/\partial y = -1 \;,\; \partial g/\partial x = -2dx$$

$$\partial g/\partial z = 0$$

$$\partial h/\partial x = rs \;,\; \partial h/\partial y = 0 \;,\; \frac{\partial h}{\partial z} = -r$$

$$J = \begin{pmatrix} -3ax^2+2bx & 1 & -1 \\ -2dx & -1 & 0 \\ rs & 0 & -r \end{pmatrix}$$

$$J = \begin{pmatrix} -3(1)(-1.571)^2+2(3)(-1.571) & 1 & -1 \\ -2(5)(-1.571) & -1 & 0 \\ 0.006\times 5.2 & 0 & -0.006 \end{pmatrix}$$

$$J = \begin{pmatrix} -16.84 & 1 & -1 \\ -15.71 & -1 & 0 \\ 0.0312 & 0 & -0.006 \end{pmatrix}$$

$$J - \lambda I = \begin{pmatrix} -16.84-\lambda & 1 & -1 \\ -15.71 & -1-\lambda & 0 \\ 0.0312 & 0 & -0.006-\lambda \end{pmatrix}$$

$$\det(J-\lambda I) = (-16.84-\lambda)\begin{vmatrix} -1-\lambda & 0 \\ 0 & -0.006-\lambda \end{vmatrix}$$

$$-1\begin{vmatrix} -15.71 & 0 \\ 0.0312 & -0.006-\lambda \end{vmatrix} + (-1)$$

$$\begin{vmatrix} -15.71 & -1-\lambda \\ 0.0312 & 0 \end{vmatrix}$$

$det = -0.0379$

$Tr J = -17.85$

$\lambda_1 = -17.77$, $\lambda_2 = -0.035$

$\lambda_3 = -0.035$

For $0 \leq I \leq 1.5$ (Stable

For $I > 1.5$ unstable.