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Q6)

Sol) $V_1 = (1,6,4)$, $V_2 = (2,4,-1)$, $V_3 = (-1,2,5)$,
$W_1 = (1,-2,-5)$ and $W_2 = (0,2,9)$
$\text{Span}(V_1, V_2, V_3) = \text{Span}(W_1, W_2)$
$\therefore$ If W_1 and W_2 span of $\{V_1, V_2, V_3\}$
$k_1V_1 + k_2V_2 + k_3V_3 = W_1$
$k_1(1,6,4) + k_2(2,4,-1) + k_3(-1,2,5) = (1,-2,-5)$

$$A_b = \left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 6 & 4 & 2 & -2 \\ 4 & -1 & 5 & -5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 0 & -8 & 8 & -8 \\ 0 & -9 & 9 & -9 \end{array}\right] \begin{array}{l} R_2 - 6R_1, \\ R_3 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 \end{array}\right] \frac{1}{8}R_2, \frac{1}{9}R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 - R_2$$

$\text{Rank}(A) = 2$, $\text{Rank}(A_b) = 2$
$\therefore \text{Rank}(A) = \text{Rank}(A_b)$
$\therefore$ System is Consistent and will Span

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$k_1V_1 + k_2V_2 + k_3V_3 = W_2$

$k_1(1,6,4) + k_2(2,4,-1) + k_3(-1,2,5) = (0,8,9)$

$$A_b = \left[\begin{array}{ccc:c} 1 & 2 & -1 & 0 \\ 6 & 4 & 2 & 8 \\ 4 & -1 & 5 & 9 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 0 \\ 0 & -8 & 8 & 8 \\ 0 & -9 & 9 & 9 \end{array}\right] \begin{array}{l} R_2 - 6R_1, \\ R_3 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 1 \end{array}\right] \frac{1}{8}R_2, \frac{1}{9}R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 - R_2$$

Rank(A) = 2, Rank(A_b) = 2

Rank(A) = Rank(A_b)

$\therefore$ System is Consistent and will span.

$\therefore$ Both W_1 and W_2 are Span.

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$\therefore$ If $\{V_1, V_2, V_3\}$ span of $\{W_1, W_2\}$

$\therefore K_1W_1 + K_2W_2 = V_1$

$K_1(1,-2,-5) + K_2(0,8,9) = (1,6,4)$

$$A_b = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ -2 & 8 & 6 \\ -5 & 9 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 8 & 8 \\ 0 & 9 & 9 \end{array}\right] \begin{array}{l} R_2 + 2R_1, \\ R_3 + 5R_1 \end{array}$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right] \frac{1}{8}R_2,\ \frac{1}{9}R_3$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] R_3 - R_2$$

$Rank(A) = 2$, $Rank(A_b) = (2)$

$\therefore Rank(A) = Rank(A_b)$

$\therefore$ System is Consistant and will span.

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$k_1 w_1 + k_2 w_2 = v_2$

$k_1(1,-2,-5) + k_2(0,8,9) = (2,4,-1)$

$$A_b = \left[\begin{array}{cc:c} 1 & 0 & 2 \\ -2 & 8 & 4 \\ -5 & 9 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & 2 \\ 0 & 8 & 8 \\ 0 & 9 & 9 \end{array}\right] \begin{array}{l} R_2 + 2R_1, \\ R_3 + 5R_1 \end{array}$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right] \frac{1}{8}R_2, \frac{1}{9}R_3$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] R_3 - R_2$$

$\text{Rank}(A) = 2$, $\text{Rank}(A_b) = 2$

$\therefore \text{Rank}(A) = \text{Rank}(A_b)$

$\therefore$ System is Consistent and will span.

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$K_1 W_1 + K_2 W_2 = V_3$

$K_1(1,-2,-5) + K_2(0,8,9) = (-1,2,5)$

$$A_b = \left[\begin{array}{cc:c} 1 & 0 & -1 \\ -2 & 8 & 2 \\ -5 & 9 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & -1 \\ 0 & 8 & 0 \\ 0 & 9 & 0 \end{array}\right] \begin{array}{l} R_2+2R_1, \\ R_3+5R_1 \end{array}$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right] \frac{1}{8}R_2, \frac{1}{9}R_3$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] R_3 - R_2$$

$\text{Rank}(A) = 2$, $\text{Rank}(A_b) = 2$

$\therefore \text{Rank}(A) = \text{Rank}(A_b)$

$\therefore$ System is Consistent and will Span

$\therefore$ All the three vectors (V_1, V_2, V_3) are Span.

proved

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Q7)

Sol) $2x_1+5x_2+x_3=0$

$x_1+3x_2+2x_3=0$

$3x_1+4x_2-9x_3=0$

$$A_b=\begin{bmatrix}2&5&1\\1&3&2\\3&4&-9\end{bmatrix}$$

$$A_b=\begin{bmatrix}1&3&2\\2&5&1\\3&4&-9\end{bmatrix}R_{12}$$

$$\cancel{A_b=\begin{bmatrix}1&-1&3\\0&&\\0&&\end{bmatrix}R_2-2R_1,\ R_3-3R_1}$$

$$A_b=\begin{bmatrix}1&3&2\\0&-1&-3\\0&-5&-15\end{bmatrix}\begin{matrix}R_2-2R_1,\\R_3-3R_1\end{matrix}$$

$$A_b=\begin{bmatrix}1&3&2\\0&1&3\\0&-5&-15\end{bmatrix}(-1)R_2$$

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$$A_b = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 + 5R_2$$

$$x_1 + 3x_2 + 2x_3 = 0$$
$$x_2 + 3x_3 = 0$$

let $x_3 = t$

$\Rightarrow x_2 + 3t = 0$

$$\boxed{x_2 = -3t}$$

$\Rightarrow x_1 + 3(-3t) + 2t = 0$

$x_1 - 9t + 2t = 0$

$x_1 - 7t = 0$

$$\boxed{x_1 = 7t}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix}$$

$$\text{Basis} = \left\{ \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

$\therefore$ The dimension of solution space is 1.

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Q8)

Sol) $\{1, (t-1), (t-1)^2, (t-1)^3, (t-1)^4\}$

Let $V_1 = 1$

$V_2 = t-1$, $V_3 = (t-1)^2$,

$V_4 = (t-1)^3$, $V_5 = (t-1)^4$

$\Rightarrow k_1V_1 + k_2V_2 + k_3V_3 + k_4V_4 + k_5V_5 = 0$

$\Rightarrow (t-1)^2 = t^2 - 2t + 1$

$\Rightarrow (t-1)^3 = t^3 - 3(t)^2(1) + 3(t)(1) - (1)^2$

$\Rightarrow ((t-1)^2)^2 = (t^2 - 2t + 1)^2$

$(t^2 + (-2t) + (1)) = (t^2)^2 + (-2t)^2 + (1)^2 + 2(t^2)(-2t) + 2(-2t)(1) + 2(t^2)(1)$

$\Rightarrow (t^2 + (-2t) + (1)) = t^4 + 4t^2 + 1 - 4t^3 - 4t + 2t^2$

$(t^2 + (-2t) + (1)) = t^4 - 4t^3 + 6t^2 - 4t + 1$

$k_1(1) + k_2(t-1) + k_3(t^2 - 2t + 1) + k_4(t^3 - 3t^2 + 3t - 1) + k_5(t^4 - 4t^3 + 6t^2 - 4t + 1) = (0, 0, 0, 0, 0)$

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Comparing Eqⁿ:

$k_5 = 0$

$k_4 - 4k_5 = 0$

$k_3 - 3k_4 + 6k_5 = 0$

$k_2 - 2k_3 + 3k_4 - 4k_5 = 0$

$k_1 - k_2 + k_3 - k_4 + k_5 = 0$

$$A_b = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -2 & 3 & -4 & 0 \\ 0 & 0 & 1 & -3 & 6 & 0 \\ 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|A_b| = \begin{vmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 1 & -2 & 3 & -4 \\ 0 & 0 & 1 & -3 & 6 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

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$|A_b| = 1 \times 1 \times 1 \times 1 \times 1$

$1 \neq 0$

$\therefore$ Hence the matrix A is Linearly independent.

Span:

Let $u = (a, b, c, d, e)$

$k_1 V_1 + k_2 V_2 + k_3 V_3 + k_4 V_4 + k_5 V_5 = u$

$\cancel{\ } \; k_1(1) + k_2(t-1) + k_3(t^2 - 2t + 1) + k_4(t^3 - 3t^2 + 3t - 1) + k_5(t^4 - 4t^3 + 6t^2 - 4t + 1) = (a, b, c, d, e)$

Comparing Eq^n:

$$\begin{aligned}
k_1 - k_2 + k_3 - k_4 + k_5 &= a \\
k_2 - 2k_3 + 3k_4 - 4k_5 &= b \\
k_3 - 3k_4 + 6k_5 &= c \\
k_4 - 4k_5 &= d \\
k_5 &= e
\end{aligned}$$

k_4

$$A_b = \left[\begin{array}{ccccc|c}
1 & -1 & 1 & -1 & 1 & a \\
0 & 1 & -2 & 3 & -4 & b \\
0 & 0 & 1 & -3 & 6 & c \\
0 & 0 & 0 & 1 & -4 & d \\
0 & 0 & 0 & 0 & 1 & e \\
0 & 0 & 0 & 0 & 0 &
\end{array}\right]$$

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Rank(A) = 5

Rank(A_b) = 5

Rank(A) = Rank(A_b)

∴ System is Consistent

It will span.

∴ Hence it is proved that all the vectors form basis for P_4.

Q9)

$$A_b = \begin{bmatrix} 1 & -4 & 5 & 1 \\ 2 & -8 & 9 & 0 \\ 1 & -4 & 3 & -3 \\ -1 & 4 & -2 & 5 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 1 & -4 & 5 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & 3 & 6 \end{bmatrix} \begin{matrix} R_2 - 2R_1, \\ R_3 - R_1, \\ R_4 + R_1 \end{matrix}$$

$$A_b = \begin{bmatrix} 1 & -4 & 5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & 3 & 6 \end{bmatrix} (-1)R_2$$

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$$A_3 = \begin{bmatrix} 1 & -4 & 5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_3+2R_2, \\ R_4-3R_3 \end{matrix}$$

$$\text{Rank}(A) = 2$$

$$A^T = \begin{bmatrix} 1 & 2 & 1 & -1 \\ -4 & -8 & -4 & 4 \\ 5 & 9 & 3 & -2 \\ 1 & 0 & -3 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 3 \\ 0 & -2 & -4 & 6 \end{bmatrix} \begin{matrix} R_2+4R_1, \\ R_3-5R_1, \\ R_4-R_1 \end{matrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & -2 & -4 & 6 \end{bmatrix} (-1)R_3$$

$$A^T = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_4+2R_3$$

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$\text{Rank}(A^T) = 2$

$\text{Rank}(A) = \text{Rank}(A^T)$

Hence Proved