# Simulation and Modelling

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CS4056

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Notes



# Building Conceptual Models

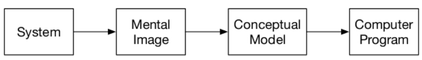


Notes



## WHAT IS A CONCEPTUAL MODEL?

- A conceptual model is a representation of a system using specialized concepts and terms.
- a mathematical model of a system can be thought of as a conceptual model that is constructed using specialized concepts such as constants, variables, and functions and specialized terms such as derivative and integral.
- before a conceptual model can be built, a mental image of the system under study must be developed in the mind of the modeler
- A mental image reflects how the modeler perceives the system and its operation.
- The mental image should include only those aspects of the system that are necessary for the simulation study.



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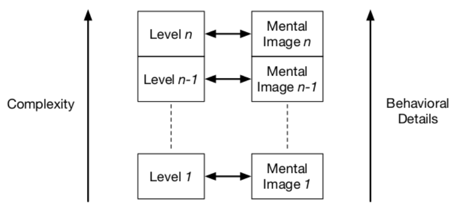
## Mental image

- Different mental images can be developed for the same system. They include different levels of details. Complexity increases as more details are added.






Notes

## Components of a System

- In order to understand and analyze a system, we need some terms
- General Terminology
  - **Entity** Object of interest in the system
  - **Attribute** Property of an entity
  - **Activity** A time period of specified length
  - **System state** Collection of variables required to describe the system at any time
  - **Event** An instantaneous occurrence that might change the state of the system
  - **Endogenous** Activities and Events occurring within the system
  - **Exogenous** Activities and Events in the environment (outside the system) that affect the system



Notes



## ELEMENTS OF A CONCEPTUAL MODEL

A conceptual model can be constructed using five elements:

- Entity:
  - An entity represents a physical (or logical) object in the system.
  - The server is a static entity because it does not move in the system and its purpose is to provide service only for other entities.
  - A person is a dynamic entity because it moves through the system.
  - A static entity maintains a state that can change during the lifetime of the system.
  - On the other hand, dynamic entities do not maintain any state.
- Attribute:
  - A dynamic entity typically has attributes which are used for storing data.
  - An entity is characterized using attributes, which are local variables defined inside the entity.
  - a person can have an attribute for storing the time of his arrival into the system (i.e., arrival time).

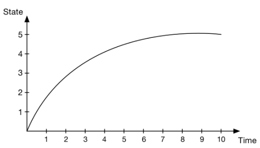


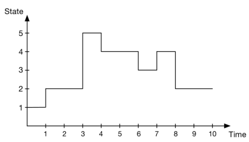
Notes



## ELEMENTS OF A CONCEPTUAL MODEL

- State Variable:
  - A state variable is used to track a property of a static entity over time.
  - for a memory module in a system, its state could be the number of data units it currently stores.
  - state of a cashier in the supermarket example. It is either free or busy.
  - A state variable is said to be continuous if it takes values that change continuously over time.
  - if the value of a state variable is from a discrete set, then it is referred to as a discrete state variable.



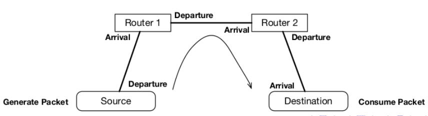



Notes



## ELEMENTS OF A CONCEPTUAL MODEL

- Event
  - An event represents the occurrence of something interesting inside the system.
  - It is a stimulus that causes the system to change its state.
  - In the supermarket example, the arrival of a new customer represents an event which will cause the state variable representing the number of people waiting in line to increase by one.
  - The departure of a customer will cause the cashier to become free.
  - Events can be used to delimit activities and move active entities
  - a packet is moved from a source to a destination using eight events.



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## ELEMENTS OF A CONCEPTUAL MODEL

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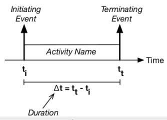
- Activity.
  - An activity is an action which is performed by the system for a finite (but random) duration of time.
  - The initiating event starts the activity.
  - The end of the activity is scheduled at the time of occurrence of the terminating event.
  - The difference in time between the two events represents the duration of the activity.
  - Durations of activities are modeled as random variables.
  - An important activity is the time a customer spends at the checkout.The duration of this activity depends on how many items the customer has.






Notes



## Components of a System

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| *System* | *Entities* | *Attributes* | *Activities* | *Events* | *State Variables* |
|---|---|---|---|---|---|
| Banking | Customers | Checking-account balance | Making deposits | Arrival; departure | Number of busy tellers; number of customers waiting |
| Rapid rail | Riders | Origin; destination | Traveling | Arrival at station; arrival at destination | Number of riders waiting at each station; number of riders in transit |
| Production | Machines | Speed; capacity; breakdown rate | Welding; stamping | Breakdown | Status of machines (busy, idle, or down) |
| Communications | Messages | Length; destination | Transmitting | Arrival at destination | Number waiting to be transmitted |
| Inventory | Warehouse | Capacity | Withdrawing | Demand | Levels of inventory; backlogged demands |



Notes



## SINGLE-SERVER QUEUEING SYSTEM

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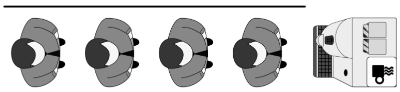

- Consider the situation depicted in Figure where there is one coffee machine and multiple users. Only one user can use the machine at a time. Thus, the others have to wait
- Mental Image
- conceptual model

| Element |
|---|
| Entity |
| State Variables |
| Events |
| Activities |



Notes



SINGLE-SERVER QUEUEING SYSTEM

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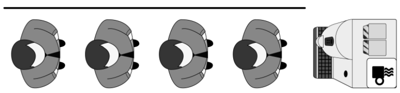

| Element | Details |
|---|---|
| Entity | Queue, Server, Person |
| State Variables | Q = Number of Persons in Queue<br>Q ∈ {0, 1, 2, ...}<br>S = Status of Server<br>S ∈ {Free, Busy} |
| Events | Arrival, Start_Service, End_Service (or Departure) |
| Activities | Generation, Waiting, Service, Delay |



Notes

## SINGLE-SERVER QUEUEING SYSTEM

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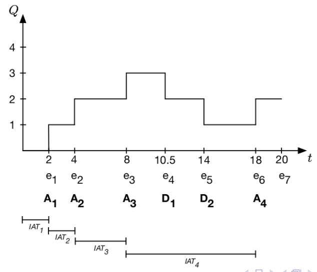
Sample paths of DESs have a special shape which can be represented by a piecewise constant function.
The time between two such arrivals is random and it is referred to as the Inter-Arrival Time (IAT).



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## SINGLE-SERVER QUEUEING SYSTEM

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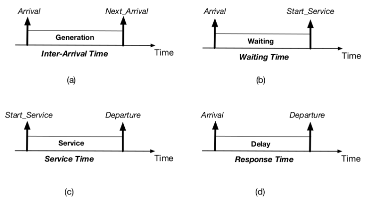
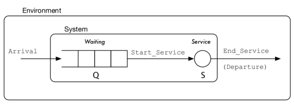
Four possible activities take place inside the single-server queueing system.






Notes



## Simulation using a Table

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- Introducing simulation by manually simulating on a table
- Can be done via pen-and-paper or by using a spreadsheet

| Repetition $i$ | Inputs $x_{i1}$ | $x_{i2}$ | … | $x_{ij}$ | … | $x_{ip}$ | Response $y_i$ |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| . | | | | | | | |
| . | | | | | | | |
| . | | | | | | | |
| $n$ | | | | | | | |

Static meta data

Dynamic data during simulation run

1. Determine the characteristics of each input to the simulation.
2. Construct a simulation table consisting of p inputs $x_{ij}, j = 1, 2, \ldots, p$, one response $y_i, i = 1, 2, \ldots, n$
3. For each repetition i, generate a value for each of the p inputs $x_{ij}$ and calculate the response $y_i$ .



Notes



## Simulation of Queueing Systems

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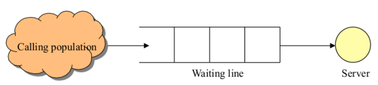
A queueing system is described by

- Calling population
- Arrival rate
- Service mechanism
- System capacity
- Queueing discipline






Notes

## Simulation of Queueing Systems

**Single server queue**

- Queueing system state
- Calling population is infinite⇒ Arrival rate does not change
- Units are served according FIFO
- Arrivals are defined by the distribution of the time between arrivals ⇒ inter-arrival time
- Service times are according to a distribution
- Arrival rate must be less than service rate ⇒ stable system
- Otherwise waiting line will grow unbounded ⇒ unstable system



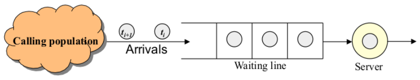
Notes



## Simulation of Queueing Systems

**Queueing system state**

- System
  - Server
  - Units (in queue or being served)
  - Clock
- State of the system
  - Number of units in the system
  - Status of server (idle, busy)
- Events
  - Arrival of a unit
  - Departure of a unit





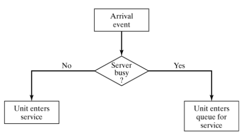

Notes



## Simulation of Queueing Systems

Arrival Event:

- If server idle unit gets service, otherwise unit enters queue.

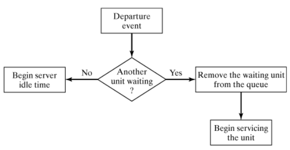



How do events occur?

- Events occur randomly
- Interarrival times $\in \{1, \ldots, 6\}$
- Service time $\in \{1, \ldots, 4\}$

Departure Event

- If queue is not empty begin servicing next unit, otherwise server will be idle.






Notes



## Simulation of Queueing Systems

The interarrival and service times are taken from distributions!

| Customer | Interarrival Time | Arrival Time on Clock | Service Time |
|---|---|---|---|
| 1 | - | 0 | 2 |
| 2 | 2 | 2 | 1 |
| 3 | 4 | 6 | 3 |
| 4 | 1 | 7 | 2 |
| 5 | 2 | 9 | 1 |
| 6 | 6 | 15 | 4 |

The simulation run is build by meshing clock, arrival, and service times!

| Customer Number | Arrival Time [Clock] | Time Service Begins [Clock] | Service Time [Duration] | Time Service Ends [Clock] |
|---|---|---|---|---|
| 1 | 0 | 0 | 2 | 2 |
| 2 | 2 | 2 | 1 | 3 |
| 3 | 6 | 6 | 3 | 9 |
| 4 | 7 | 9 | 2 | 11 |
| 5 | 9 | 11 | 1 | 12 |
| 6 | 15 | 15 | 4 | 19 |



Notes

## Simulating a Single-Server Queue

A small grocery store has one checkout counter. Customers arrive at the checkout counter at random times that range from 1 to 8 minutes apart. We assume that interarrival times are integer-valued with each of the 8 values having equal probability; this is a discrete uniform distribution. The service times vary from 1 to 6 minutes (also integer-valued), with the probabilities shown in Table below. Our objective is to analyze the system by simulating the arrival and service of first 10 customers and to compute a variety of typical measures of performance for queueing models.

| Service Times (Minutes) | Probability | Cumulative Probability |
|---|---|---|
| 1 | 0.10 | 0.10 |
| 2 | 0.20 | 0.30 |
| 3 | 0.30 | 0.60 |
| 4 | 0.25 | 0.85 |
| 5 | 0.10 | 0.95 |
| 6 | 0.05 | 1.00 |



Notes



## Simulating a Single-Server Queue

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|---|---|---|
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| 2 | 0.20 | 0.30 |
| 3 | 0.30 | 0.60 |
| 4 | 0.25 | 0.85 |
| 5 | 0.10 | 0.95 |
| 6 | 0.05 | 1.00 |

| Inter arrival Times (minutes) | |
|---|---|
| Minimum | 1 |
| Maximum | 8 |

Simulation Table

| Step | Activity | Clock | Activity | Clock | Output | Clock | Output | Output |
|---|---|---|---|---|---|---|---|---|
| Customer | Interarrival Time (Minutes) | Arrival Time | Service Time (Minutes) | Time Service Begins | Waiting Time in Queue (Minutes) | Time Service Ends | Time Customer Spends in System (Minutes) | Idle Time of Server (Minutes) |
| 1 | 3 | 3 | 3 | | | | | |
| 2 | 2 | 5 | 2 | | | | | |
| 3 | 8 | | 3 | | | | | |
| 4 | 6 | | 5 | | | | | |
| 5 | 2 | | 4 | | | | | |
| 6 | 1 | | 1 | | | | | |
| 7 | 7 | | 2 | | | | | |
| 8 | 6 | | 4 | | | | | |
| 9 | 7 | | 6 | | | | | |
| 10 | 6 | | 2 | | | | | |



Notes



## Simulating a Single-Server Queue

- The average waiting time for a customer is ______ minutes, computed as follows:

$$\text{Average waiting time(minutes)} = \frac{\text{total time customers wait in queue (minutes)}}{\text{total numbers of customers}}$$

- The probability that a customer has to wait in the queue is ______, computed as follows:

$$\text{Probability (wait)} = \frac{\text{numbers of customers who wait}}{\text{total number of customers}}$$

- The server is idle about ______% of the time, computed as follows:

$$\text{Probability of idle} = \frac{\text{total idle time of server (minutes)}}{\text{total run time of simulation (minutes)}}$$

  Therefore, the server is busy about ______% of the time.
- The average service time is ______ minutes, computed as follows:

$$\text{Average service time(minutes)} = \frac{\text{total service time (minutes)}}{\text{total number of customers}}$$

- Comparing average service time with the expected service time by finding the mean of the service-time distribution

$$E(S) = \sum_{s=0}^{\infty} s \times P(s)$$



Notes



## Simulating a Single-Server Queue

- The average time between arrivals is ______ minutes, computed as follows:

$$\text{Average time between arrivals} = \frac{\text{sum of all times between arrivals}}{\text{number of arrivals-1}}$$

- Compared to the expected time between arrivals by finding the mean of the discrete uniform distribution whose endpoints are $a = 1$ and $b = 8$. The mean is given by

$$E(A) = \frac{a+b}{2} =$$

- The average waiting time of those who wait is ______ minutes, computed as follows:

$$\text{Average waiting time of those who wait} = \frac{\text{total time customers wait in queue}}{\text{total number of customers who wait}}$$

- The average time a customer spends in the system is ______ minutes, computed in ways.

$$\text{Average time customer spends in the system} = \frac{\text{total time customers spend in the system}}{\text{total number of customers}}$$

$$\begin{array}{c} \text{Average time} \\ \text{customer spends} \\ \text{in the system} \\ \text{(minutes)} \end{array} = \begin{array}{c} \text{average time} \\ \text{customer spends} \\ \text{waiting in the} \\ \text{queue (minutes)} \end{array} + \begin{array}{c} \text{average time} \\ \text{customer spends} \\ \text{in service} \\ \text{(minutes)} \end{array}$$



Notes