Date:

Ex 7.6

Q.66. $\int \frac{dx}{2+\sin x}$

$dx = \frac{2}{1+u^2}du$

$\sin x = \frac{2u}{1+u^2}$

$u = \tan\frac{x}{2}$

u^2+u+1

$\left(u+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1$

$\int \frac{2\,du}{1+u^2} \div 2+\frac{2u}{1+u^2}$

$\int \frac{2\,du}{\cancel{1+u^2}} \times \frac{\cancel{1+u^2}}{2+2u^2+2u}$

$\int \frac{2}{2+2u^2+2u}du \Rightarrow \int \frac{1}{1+u+u^2}du$

$\int \frac{1}{\left(u+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}du \Rightarrow \int \frac{1}{\left(u+\frac{1}{2}\right)^2+\frac{3}{4}}$

$\int \frac{1}{\left(u+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} \Rightarrow \int \frac{1}{t^2+a^2}dt$

$$\boxed{\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C}$$

Date:

$$\int \frac{1}{t^2+a^2}\,dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)+C$$

$$\frac{2u+1}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{u+1/2}{\sqrt{3}/2}\right)+C$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2u+1}{\sqrt{3}}\right)+C$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\left(\tan\frac{x}{2}\right)+1}{\sqrt{3}}\right)+C$$

$$\left(u+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$u^2+\frac{2u}{2}+\frac{1}{4}+\frac{3}{4}$$

$$u^2+u+1$$

Date:

Q.67 $\int \frac{d\theta}{1-\cos\theta}$

$d\theta = \frac{2}{1+u^2}du$

$\cos\theta = \frac{1-u^2}{1+u^2}$

$u = \tan\frac{\theta}{2}$

$\int \frac{2du}{1+u^2} \div 1 - \left(\frac{1-u^2}{1+u^2}\right)$

$\int \frac{2du}{\cancel{1+u^2}} \div \frac{1+u^2-1+u^2}{\cancel{1+u^2}} \Rightarrow \int \frac{\cancel{2}du}{\cancel{2}u^2}$

$\int \frac{1}{u^2}du$

$\frac{u^{-2+1}}{-2+1}$

$-\frac{1}{u} \Rightarrow \frac{-1}{\tan\frac{x}{2}}$

$= -\cot\frac{\theta}{2} + C$

Date:

Q68 $\displaystyle\int \frac{1}{4\sin x - 3\cos x}\,dx$

$dx = \dfrac{2}{1+u^2}\,du$

$\sin x = \dfrac{2u}{1+u^2}$

$\cos x = \dfrac{1-u^2}{1+u^2}$

$$\int \frac{2\,du}{1+u^2} \div \left(\frac{8u}{1+u^2} - \frac{3+3u^2}{1+u^2}\right)$$

$$\int \frac{2}{1+u^2}\,du \div \frac{3u^2+8u-3}{1+u^2}$$

$$\int \frac{2}{3u^2+8u-3}\,du$$

$$\int \frac{2}{3\left(u^2+\frac{8u}{3}-1\right)}\,du \Rightarrow \frac{2}{3}\int \frac{1}{\left(u+\frac{8}{6}\right)^2-\left(\frac{8}{6}\right)^2-1}\,du$$

$$\frac{2}{3}\int \frac{1}{\left(u+\frac{4}{3}\right)^2-\frac{25}{9}} \Rightarrow \frac{2}{3}\int \frac{1}{\left(u+\frac{4}{3}\right)^2-\left(\frac{5}{3}\right)^2}$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

$$= \frac{2}{3}\left[\frac{1}{2\times\frac{5}{3}}\ln\left|\frac{u+\frac{4}{3}-\frac{5}{3}}{u+\frac{4}{3}+\frac{5}{3}}\right|\right.$$

Date:

$$= \frac{2}{10} \ln \left| \frac{u + \frac{4}{3} - \frac{5}{3}}{u + \frac{4}{3} + \frac{5}{3}} \right| + C \qquad u = \tan\frac{x}{2}$$

$$= \frac{1}{5} \ln \left| \frac{u - \frac{1}{3}}{u + \frac{1}{3}} \right|$$

$$= \frac{1}{5} \ln \left| \frac{\tan\frac{x}{2} - \frac{1}{3}}{\tan\frac{x}{2} + \frac{1}{3}} \right|$$

$$\int \frac{1}{4\sin x - 3\cos x}\, dx = \frac{1}{5} \ln \left| \frac{\tan\frac{x}{2} - \frac{1}{3}}{\tan\frac{x}{2} + \frac{1}{3}} \right| + C$$

Date:

Q69. $\int \frac{dx}{\sin x + \tan x}$

$u = \tan\frac{x}{2}$

$\sin x = \frac{2u}{1+u^2}$

$\tan x = \frac{2u}{1-u^2}$

$dx = \frac{2}{1+u^2}$

$$\int \frac{2du}{1+u^2} \div \left\{ \frac{2u}{1+u^2} + \frac{2u}{1-u^2} \right\}$$

$$\int \frac{2}{1+u^2} du \div \frac{2u(1-u^2)+2u(1+u^2)}{1^2-u^2}$$

$$\int \frac{2\,du}{\cancel{1+u^2}} \times \frac{(\cancel{1+u^2})(1-u^2)}{2u-\cancel{2u^3}+2u+\cancel{2u^3}}$$

$$\int \frac{2(1-u^2)}{4u} du \Rightarrow \int \frac{2}{4u} - \frac{2u^2}{4u} du$$

$$\int \frac{1}{2u} du + \int \frac{-2u^2}{4u} du$$

$$\cancel{\frac{1}{2} \frac{u^{-1+1}}{}}, \quad \frac{1}{2} \ln u - \frac{1}{2} \frac{u^{1+1}}{(1+1)}$$

$$\frac{1}{2} \ln u - \frac{1}{4} u^2$$

$$\frac{1}{2} \ln\left(\tan\frac{x}{2}\right) - \frac{1}{4}\left(\tan\frac{x}{2}\right)^2$$

Date:

Q.70 $\int \frac{\sin x}{\sin x + \tan x} dx$

$\sin x = \frac{2u}{1+u^2}$

$\tan x = \frac{2u}{1-u^2}$

$dx = \frac{2}{1+u^2}$

$$\int \frac{\frac{1-u^2}{1+u^2} \times \frac{2}{1+u^2}}{\frac{1-u^2}{1+u^2}+1} du$$

$$\int \frac{2(1-u^2)}{(1+u^2)^2} du \div \frac{1-u^2+1+u^2}{1+u^2} du$$

$$\int \frac{2(1-u^2)}{\cancel{1+u^2}} \times \frac{\cancel{1+u^2}}{2} du$$

$$\int \frac{1}{1+u^2} - \frac{u^2}{1+u^2} du$$

$$\tan^{-1}(u) - (u + \tan^{-1}(u))$$

$$\tan^{-1}\left(\tan\frac{x}{2}\right) - \tan\frac{x}{2}\ \cancel{x} + \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$x - \tan\frac{x}{2} + C$$

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Ex 7.8

Q.4 $\displaystyle\int_{-1}^{+\infty} \frac{x}{1+x^2}\,dx$

$\displaystyle\lim_{a\to\infty} \frac{1}{2}\int_{-1}^{a} \frac{x \times 2}{1+x^2}\,dx$

$\displaystyle\lim_{a-\infty} \frac{1}{2} \ln(1+x^2)\Big|_{-1}^{a}$

$\displaystyle\lim_{a\to\infty} \frac{1}{2}\left\{\ln(1+a^2) + \ln(2)\right\}$

app lim

$= \frac{1}{2}\left\{\ln(1+\infty^2) + \ln(2)\right\}$

$= +\infty$ (divergent)

Date:

Q.6 $\int_0^\infty x e^{-x^2} dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int_0^\infty \cancel{x} e^{-u} \times \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int_0^\infty e^{-u}$

$\lim_{a \to \infty} \frac{1}{2}\int_0^a e^{-u}$

$\lim_{a \to \infty} \frac{1}{2}\{-e^{-a} - e^0\}$

app lim

$\frac{1}{2}(-e^{-\infty}+1)$

$\frac{1}{2}(-\frac{1}{e^\infty}+1)$

$\frac{1}{2}(0+1)$

$\frac{1}{2}$

Date:

Q.8 $\int_{2}^{+\infty} \frac{1}{x\sqrt{\ln x}} dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\lim a \to +\infty \int_{2}^{a} \frac{1}{x\sqrt{u}} \times x\,du$

$\lim_{a \to +\infty} \int_{2}^{a} \frac{1}{\sqrt{u}}$

$\lim_{a \to +\infty} \left. \frac{(u)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right|_{2}^{a}$

$\lim_{a \to +\infty} \left. 2u^{\frac{1}{2}} \right|_{2}^{a}$

$\lim_{a \to +\infty} 2\left\{a^{\frac{1}{2}} - (2)^{\frac{1}{2}}\right\}$

$\lim_{a \to +\infty} 2\left\{a^{\frac{1}{2}} - 2^{\frac{1}{2}}\right\}$

app lim

$= \infty - 4^{\frac{1}{2}}$

$=$ (divergent)

Date:

Q.10 $\int_{-\infty}^{3} \frac{dx}{x^2+9}$ $\tan^{-1}\frac{\infty}{3}$

$\lim_{a\to-\infty} \int_{a}^{3} \frac{dx}{x^2+9}$

$\lim_{a\to-\infty} \frac{1}{3}\tan^{-1}\frac{x}{3}\Big|_{a}^{3}$

$\lim_{a\to-\infty} \frac{1}{3}\left\{\tan^{-1}1 - \tan^{-1}\frac{a}{3}\right\}$

$\lim_{a\to-\infty} \frac{1}{3}\left\{\frac{\pi}{4} - \tan^{-1}\frac{a}{3}\right\}$

app lim

$\frac{1}{3}\left\{\frac{\pi}{4} + \frac{\pi}{2}\right\}$

$\frac{1}{\cancel{3}} \times \frac{\cancel{3}\pi}{4}$

$\frac{\pi}{4}$

Date:

Q.12 $\int_{-\infty}^{0} \frac{e^x dx}{3-2e^x}$

$u = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\lim_{a \to -\infty} \int_{a}^{0} \frac{e^x dx}{3-2e^x}$

$\lim_{a \to -\infty} \int_{a}^{0} \frac{\cancel{e^x} \cancel{dx}}{3-2u} \times \frac{du}{\cancel{e^x}}$

$\lim_{a \to -\infty} \frac{-1}{2} \int_{a}^{0} \frac{du}{3-2u} \times -2$

$\lim_{a \to -\infty} \frac{-1}{2} \ln(3-2u) \Big|_{a}^{0} \Rightarrow \frac{-1}{2} \left\{ \ln(3-2a) - \ln 3 \right\}$

app lim

$= -\frac{1}{2} \left\{ \underbrace{\ln(-\infty)}_{\to \text{zero}} - \ln 3 \right\}$

$= \frac{1}{2} \ln 3$

$\frac{-1}{2} \ln(3-2u) \Big|_{-a}^{0}$

$\frac{-1}{2} \left\{ \ln(3-2a) - \cancel{\ln 3} \right\}$

$\frac{-1}{2} \left\{ \ln(3) - \cancel{\ln 3} \right\}$

Date:

Q.14 $\int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx$

$$\lim_{a\to-\infty}\int_a^0 \frac{x}{\sqrt{x^2+2}}\,dx + \lim_{b\to+\infty}\int_0^b \frac{x}{\sqrt{x^2+2}}\,dx$$

$$\lim_{a\to-\infty}\int_a^0 \frac{\cancel{x}}{\sqrt{u}} \times \frac{du}{2\cancel{x}}$$

$u = x^2+2$

$\frac{du}{dx} = 2x + \cancel{2}$

$= 2x$

$$\lim_{a\to-\infty} \frac{1}{2}\int \frac{1}{u^{1/2}}\,du$$

$$\lim_{a\to-\infty} \frac{1}{2}\,\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_a^0 + \lim_{b\to+\infty} \frac{1}{2}\,\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_0^b$$

$$\lim_{a\to-\infty} \frac{\cancel{2}\sqrt{u}}{\cancel{2}}\Bigg|_0^a + \lim_{b\to+\infty} \cancel{\sqrt{2}}\sqrt{\cancel{a}}\Bigg|_0^b$$

$$\lim_{a\to-\infty} \sqrt{a} + \lim_{b\to+\infty} \sqrt{b}$$

app limits

$\infty + \infty \Rightarrow$ divergent.

Victory

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Q.16 $\displaystyle\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt$

$$\int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}}\,dt + \int_{0}^{\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$\lim_{a\to-\infty}\int_{a}^{0} \frac{e^{-t}}{1+e^{-2t}}\,dt + \lim_{b\to\infty}\int_{0}^{b} \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$\cancel{\lim_{a\to-\infty}\int_{a}^{0} \frac{e^{-t}}{u} \times \frac{du}{-2e^{-2t}}}$$

$\cancel{u = e^{-2t}+1}$

$\cancel{\frac{du}{dt} = -2e^{-2t}}$

$$\lim_{a\to-\infty}\int_{a}^{0} \frac{\cancel{e^{-t}}}{1+u^2} \times \frac{du}{-\cancel{e^{-t}}}$$

$u = e^{-t}$

$\frac{du}{dt} = -e^{-t}$

$$\lim_{a\to-\infty}\int_{a}^{0} -\frac{1}{1+u^2}\,du$$

$$\lim_{a\to-\infty} -\tan^{-1}(u)\Big|_{a}^{0}$$

$$\lim_{a\to-\infty} -\tan^{-1}(a)$$

app lim

$= +\tan^{-1}(\infty)$

$= \pi/2$

$$\lim_{b\to\infty} -\tan^{-1}(u)\Big|_{0}^{b}$$

$$\lim_{b\to\infty} -\tan^{-1}(b)$$

app lim

$= \tan^{-1}(b)$

$= \pi/4$

$$\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Date:

Q.20 $\int_0^4 \frac{dx}{\sqrt{4-x}}$

$\lim_{a\to 4} \int_0^a \frac{1}{\sqrt{4-x}}\,dx$

$\lim_{a\to 4} \frac{1}{2}(4-x)^{\frac{1}{2}-1} \times -1 \Big|_0^a$

$\lim_{a\to 4} \frac{-1}{2\sqrt{4-x}} \Big|_0^a$

$\lim_{a\to 4} \frac{-1}{2\sqrt{4-a}} + \frac{1}{2\sqrt{4}}$

$\lim_{a\to 4} \frac{-1}{2\sqrt{4-a}} + \frac{1}{4}$

app lim

$= \frac{-1}{2\sqrt{4-4}} + \frac{1}{4}$

$= 4.$

Date:

Q.22 $\int_{-3}^{1} \frac{x\,dx}{\sqrt{9-x^2}}$

$$\int_{-3}^{0} \frac{x\,dx}{\sqrt{9-x^2}} + \int_{0}^{1} \frac{x\,dx}{\sqrt{9-x^2}}$$

$$\lim_{a\to -3} \int_{a}^{0} \frac{x\,dx}{\sqrt{9-x^2}} + \int_{0}^{1} \frac{x}{\sqrt{9-x^2}}\,dx$$

$$\lim_{a\to -3} \int_{a}^{0} \frac{\cancel{x}}{\sqrt{9-u^2}} \times \frac{du}{-2\cancel{x}}$$

$u = -x^2 + 9.$

$\frac{du}{dx} - 2x$

$$\lim_{a\to -3} \cancel{\sin^{-1}\frac{x}{a}\Big|_{a}^{0}} \quad \int_{0}^{a} (u)^{-\frac{1}{2}}\,du$$

$$\lim_{a\to -3} 2u^{-\frac{1}{2}+1}\Big|_{0}^{9}$$

app lim

$$\lim_{a\to -3} 2\sqrt{9-x^2}\Big|_{0}^{9}$$

$= -2\sqrt{9}$

$= -6$

$$\lim_{a\to -3^+} 2\sqrt{9-a^2} - 2\sqrt{9}$$

Date:

Q.24 $\int_0^{\pi/4} \frac{\sec^2 x}{1-\tan x}\, dx$ $u = 1-\tan x$

$\frac{du}{dx} = -\sec^2 x$

$\lim_{a\to\pi/4} \int_0^a \frac{\sec^2 x}{1-\tan x}$

$\lim_{a\to\pi/4} \int_0^a \frac{\cancel{\sec^2 x}}{u} \times \frac{-du}{\cancel{\sec^2 x}}$

$\lim_{a\to\pi/4} \int_0^a \frac{1}{u}$

$\lim_{a\to\pi/4} \ln u \Rightarrow \lim_{a\to\pi/4} \cancel{\lim} \ln(1-\tan x)\Big|_0^a$

$\lim_{a\to\pi/4} \cancel{\ln}(1-\tan a)$

app lim

$= +\infty$ (divergent)

Date:

Q.26 $\int_{-2}^{2} \frac{dx}{x^2}$

$= \lim_{a \to 0} \int_{-2}^{a} \frac{dx}{x^2} + \lim_{a \to 0} \int_{a}^{2} \frac{dx}{x^2}$

$= \lim_{a \to 0} \left. \frac{-1}{x} \right|_{-2}^{a} + \lim_{a \to 0} \left. \frac{-1}{x} \right|_{a}^{2}$

$= \left(\infty - \frac{1}{2} \right) + \left(-\frac{1}{2} + \infty \right)$

$= \infty$ (divergent)

Date:

Q.28 $\int_0^1 \frac{dx}{(x-1)^{2/3}}$

$u = x-1$
$du = dx$

$\int_0^1 \frac{1}{(u)^{2/3}} du$

$= \int u^{-2/3} du$

$= 3u^{1/3}$
$= 3\sqrt[3]{x-1} \Big|_0^1$

$= |0| - |-3|$
$= -3$

Q.30 $\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}} \Rightarrow \lim_{a\to\infty} \int_1^a \frac{dx}{x\sqrt{x^2-1}}$

$u = \sqrt{x^2-1}$
$du = \frac{x}{\sqrt{x^2-1}}$

$= \int \frac{1}{x^2} du \Rightarrow \int \frac{1}{u^2+1} du$

$= \tan^{-1}(u)$
$= \tan^{-1}(\sqrt{x^2-1})$

$\lim_{a\to\infty} \tan^{-1}(\sqrt{x^2-1}) \Big|_1^a$

$\lim_{a\to\infty} \tan^{-1}(\sqrt{a^2-1})$

app lim

$= \frac{\pi}{2}$

Date:

Q.32 $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x}\,(x+1)}$

$u = \sqrt{x}$

$x = u^2$

$dx = 2u\,du$

$$\int \frac{1}{\sqrt{x}(x+1)}\,dx \Rightarrow \int \frac{2}{u^2+1}\,du$$

$= 2\tan^{-1}(u)$

$= 2\tan^{-1}(\sqrt{x})$

$$\lim_{a\to\infty} 2\tan^{-1}(\sqrt{x})\Big|_0^a$$

$$\lim_{a\to\infty} 2\tan^{-1}(\sqrt{a})$$

app lim

$= 2 \times \frac{\pi}{2}$

$= \pi$

Date:

Q.38 $\displaystyle\int_{12}^{+\infty} \frac{dx}{\sqrt{x}(x+4)}$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\lim\limits_{a\to\infty} \displaystyle\int_{12}^{a} \frac{dx}{\sqrt{x}(x+4)}$

$\lim\limits_{a\to\infty} \displaystyle\int_{12}^{a} \frac{2\cancel{\sqrt{x}}\,du}{\cancel{\sqrt{x}}(u^2+4)}$

$\lim\limits_{a\to\infty} 2\displaystyle\int_{12}^{a} \frac{1}{u^2+4}\,du$

18, 20, 30, 36

$\lim\limits_{a\to\infty} \cancel{2}\left(\frac{1}{\cancel{2}}\tan^{-1}\left(\frac{4}{2}\right)\right)\Big|_{12}^{a}$

$\lim\limits_{a\to\infty} \tan^{-1}\frac{\sqrt{x}}{2}\Big|_{12}^{a}$

$\lim\limits_{a\to\infty} \tan^{-1}\frac{\sqrt{a}}{2} - \tan^{-1}\frac{\sqrt{12}}{2}$

app lim

$= \tan^{-1}\infty - \tan^{-1}\frac{\pi}{3}$

$= \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Date:

Q.40 $\displaystyle\int_0^{+\infty} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$

$u = e^{-x}$

$\frac{du}{dx} = -e^{-x}$

$$\lim_{a\to\infty}\int_0^a \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$$

$$\lim_{a\to\infty}\int_0^a \frac{\cancel{e^{-x}}}{\sqrt{1-u^2}}\,\frac{du}{\cancel{e^{-x}}}$$

$$\lim_{a\to\infty}\int_0^a \frac{1}{\sqrt{1-u^2}}\,du$$

$$\lim_{a\to\infty} \sin^{-1}(u)\Big|_0^a$$

$$\lim_{a\to\infty} \sin^{-1}(e^{-x})\Big|_0^a$$

$$\lim_{a\to\infty} \sin^{-1}(e^{-a}) - \sin^{-1}(1)$$

app lim

$$= \frac{\pi}{2}$$