Simulation and Modelling

Spring 2024
CS4056

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Notes

Introduction

Notes

Course Outline

- Hierarchy of knowledge about a system and Modeling Strategy.
- Simulating Probabilities
- Simulating Random Variables
- Simulating Stochastic Processes
- The Monte Carlo Simulation
- Simulating Queueing Processes
- Statistical Analysis of Simulated Data
- Markov Chains
- Random Variate Generation
- Random Number Generation

Notes

Grading Plan

1	Term-exams(M1,M2)	30%
2	Final Exam(F)	40%
3	Homework/Programming assignments(A1,A2,A3)	10%
4	Quiz	5%
5	Final Project(F)	15%

Notes

Software knowledge

- Excel
- Python
- Modelica
- Simulink
- Arena

Notes

Three Pillars of Science and engineering

- Observation (O)
  The outcome of an observational study is a set of facts. For example, what happens if a burning candle is covered with a glass cup.
- Experimentation (E)
  Experiment is a physical setup. It is performed to make measurements.Measurements are raw data.
- Computation (C)
  computation is a representation of the phenomenon or system under study in the form of a computer program.

Notes

- Consider the following situation faced by a pharmacist who is thinking of setting up a small pharmacy where he will fill prescriptions. He plans on opening up at 9 a.m. every weekday and expects that, on average, there will be about 32 prescriptions called in daily before 5 p.m. experience that the time that it will take him to fill a prescription, once he begins working on it, is a random quantity having a mean and standard deviation of 10 and 4 minutes, respectively. He plans on accepting no new prescriptions after 5 p.m., although he will remain in the shop past this time if necessary to fill all the prescriptions ordered that day. Given this scenario the pharmacist is probably, among other things, interested in the answers to the following questions:

1. What is the average time that he will depart his store at night?
2. What proportion of days will he still be working at 5:30 p.m.?
3. What is the average time it will take him to fill a prescription (taking into account that he cannot begin working on a newly arrived prescription until all earlier arriving ones have been filled)?
4. What proportion of prescriptions will be filled within 30 minutes?
5. If he changes his policy on accepting all prescriptions between 9 a.m.and 5 p.m., but rather only accepts new ones when there are fewer thanfive prescriptions still needing to be filled, how many prescriptions, on average, will be lost?
6. How would the conditions of limiting orders affect the answers to questions 1 through 4.

Notes

What is Simulation

- Simulation is the process of representing a system by a model.
- then executing this model to generate raw data.
- Involves the generation and observation of an artificial history of a system
- The raw data is not useful by itself.
- It must be statistically processed to produce insights about the performance of the system.
- Draw inferences about the characteristics of the real system

Notes

Simulation

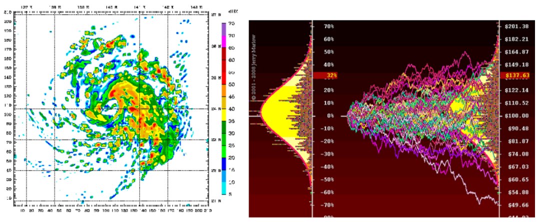


Figure: Typhoon and Financial Analysis simulation

Notes

Why Simulation

- It is often too costly or even impossible to do physical studies in reality with the actual system.
- May be disruptive, expensive, dangerous, or rare.
- The mathematical model (will be defined shortly) which can well represent the real problem, may be very difficult to solve.
- You can only solve it with high simplification.
- With simulation technique, we can easily make change and observe the effect, while keeping high fidelity.

Notes

Why Simulation

- Simulation can be used as both an analysis tool and a design tool.
    - An analysis tool: To answer "what if" questions about the existing real-world system.
      E.g., try alternative layout of a production line, try other staff shifts of a service center, test a financial system in some extreme situation, etc.
    - A design tool: To study systems in the design stage, before they are built.
      E.g., evaluate designs and operations for new transportation facilities, service organizations, manufacturing systems, etc.
- Simulation is also an important type of numerical methods.

Notes

Models

- A model is a representation of a system or problem.
- A set of assumptions and/or approximations about how the system works will often be imposed.
- It is only necessary to consider those aspects that affect the problem under investigation.
- However, the model should be sufficiently detailed to draw valid conclusions about the real system or problem.
- The trade-off: simplicity vs. accuracy.

Notes

Types of models

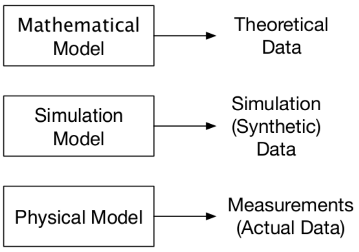


Figure: Types of models and the data generated from them.

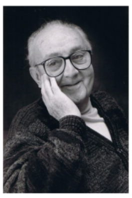
Notes

Physical model vs. Mathematical model

- Physical model: a scaled-down (or -up) version of the system.
- Mathematical model: uses symbolic notation and mathematical equations to represent the system.
- Instead of doing physical studies with the actual system in real world, we can study the model. It will be much easier, faster, cheaper, and safer!
- A simulation model is a particular type of mathematical model.

Notes

Models



“All models are wrong, but some are useful.”

— *George E. P. Box*

George E. P. Box (1919.10 – 2013.03) was a British statistician, who worked in the areas of quality control, time-series analysis, design of experiments, and Bayesian inference. He has been called “one of the great statistical minds of the 20th century”.

Notes

Models

- When a mathematical model is simple enough, we can solve it
  - analytically, with mathematical tools like algebra, calculus,probability theory.(e.g.,projectile motion)
  - numerically, with computational procedures (e.g., solving a quintic equation).
- But not all mathematical models can be “solved”.
- In simulation, the mathematical models (more specifically,simulation models) are run rather than solved:
  - Artificial history of the system is generated from the model assumptions.
  - Observations of system status are collected for analysis;
  - System performance measures are estimated.
- Essentially, running simulation is still one type of numerical methods.
  - Real-world simulation models can be large, and such runs are usually conducted with the aid of a computer.

Notes

Classification of Simulation models

- Simulation models may be classified as being static or dynamic.
- Static: Time does not play a natural role.
  - Example 1 – Finance: evaluate portfolio return and risk.
  - Example 2 – Project Management: evaluate projects payoff in different scenarios.
  - Often used in the complex numerical calculation in financial engineering, computational physics, etc.
- Dynamic: Time does play a natural role.
  - Example 1 – Logistics Management: evaluate the efficiency of a terminal.
  - Example 2 – Service Management: evaluate waiting time of customers under different staff shifts.
  - Often used to simulate the logistics/transportation/service systems, whose status naturally changes over time.

Notes

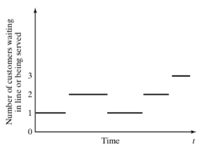
Classification of Simulation models

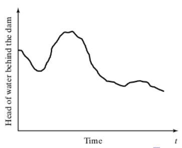
- Simulation models may be classified as being deterministic or stochastic.
- Deterministic: Everything is known with certainty.
- e.g., patients arrive at a hospital precisely on schedule, the service time is precisely fixed, the transfer among different units is pre-determined.
- Stochastic: Uncertainty exists.
- e.g., arrival times and service times of patients have random variations, the transfer is random.
- Used much more often (uncertainty is more or less involved in a real-world system).

Notes

Classification of Simulation models

- Simulation models may be classified discrete or continuous.
- Discrete: System states change only at discrete time points.
- E.g., the number of customers in the bank, changes only when a customer arrives or leaves after service
- Continuous: System states change continuously over time.
- the head of water behind a dam changes continuously during a period of time.





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- In summary, simulation models may be classified as being
  - static or dynamic,
  - deterministic or stochastic, and
  - discrete or continuous.
- For most operational decision-making problems, the suitable simulation models are dynamic, stochastic and discrete.
- The simulation is called Discrete-Event System Simulation
  - The word "event" indicates that the simulation is advanced by the occurrence of events.
  - The word "discrete" means that events occur at discrete points of time.

Notes

Phases of a simulation study

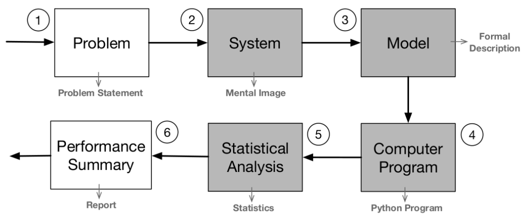

Figure: Basic Paradigm of A Simulation Study

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Phases of a simulation study

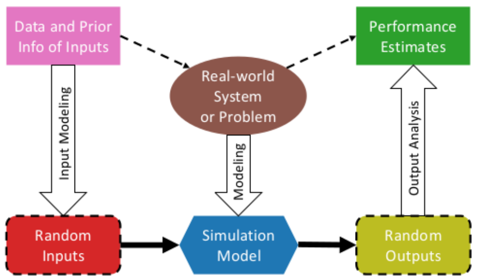
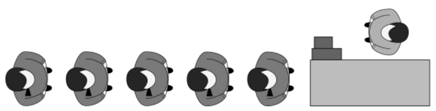


Figure: Basic Paradigm of A Simulation Study

Notes

Queueing

Consider the situation in Figure below where five people have to wait in a queue at the checkout counter in a supermarket. This situation arises because there is only one cashier and more than one person wants to have access to him. This phenomenon is referred to as queueing. Let us see how observation, experimentation, and computation can be used to study this phenomenon.



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No.	Phase	Description
1	Problem	Customers experience delays longer than 5 minutes. The checkout process has to be speeded up. Potential solutions include changing the cashier and installing a new software system. The raw data to be collected include the delay experienced by each customer i (D_i) which is defined as the difference between his departure time (D_i) and arrival time (A_i).
2	System	Customers, waiting line, and cashier.
3	Model	A customer arrives at the system. If the cashier is free, he will be served immediately. Otherwise, he has to wait. Service time of each customer is random.
4	Computer Program	Model is expressed in Python code.
5	Statistical Analysis	Response time of the system (i.e., the average delay). $T_{avg} = \frac{\sum_i^N D_i}{N}$, where N is the number of participating customers.
6	Performance Summary	Response time for each possible solution. Pick the one that gives the best response time as the optimal solution.

Notes

Advantages of Simulation

- There is no need to build the physical system under study and then observe it. Thus, knowledge about the behavior of the system can be acquired with a minimum cost.
- Critical scenarios can be investigated through simulation with less cost and no risk.
- Using a simulation model, the effect of changing values of system variables can be studied with no interruption to the physical system.
- Simulation is more flexible and convenient than mathematical analysis.
- In simulation, there is no need for simplifying assumptions
- Simulation allows us to compress and expand the behavior of the system under study. For example, several years' worth of system evolution can be studied in a few minutes of computer time.

Notes

Limitation of Simulation

- The outcome of a simulation study is an estimate subject to a statistical error. For example, different simulation runs typically produce different numbers although the same simulation model is used.
- Simulation can become costly and time consuming. For example, very powerful computers and skillful people are required.
- Simulation models are not easy to develop. Existing methodologies are not universal. This is why development of simulation models is still an art, not a science.
- Existing programming languages are not designed to support simulation. Thus, a lot of programming is involved.

Notes

Practice problem

- The following data yield the arrival times and service times that each customer will require, for the first 13 customers at a single server system. Upon arrival, a customer either enters service if the server is free or joins the waiting line. When the server completes work on a customer, the next one in line (i.e., the one who has been waiting the longest) enters service.
  Arrival Times:12 31 63 95 99 154 198 221 304 346 411 455 537
  Service Times:40 32 55 48 18 50 47 18 28 54 40 72 12
  - a Determine the departure times of these 13 customers.
  - b Repeat (a) when there are two servers and a customer can be served by either one.
  - c Repeat (a) under the new assumption that when the server completes a service, the next customer to enter service is the one who has been waiting the least time.

Notes

Practice problem

- Consider a service station where customers arrive and are served in their order of arrival. Let A_n , S_n , and D_n denote, respectively, the arrival time, the service time, and the departure time of customer n. Suppose there is a single server and that the system is initially empty of customers.
  - With $D_0 = 0$, argue that for $n > 0$

$$D_n - S_n = Maximum\{A_n, D_{n-1}\}$$

  - Determine the corresponding recursion formula when there are two servers.
  - Determine the corresponding recursion formula when there are k servers.
  - Write a computer program to determine the departure times as a function of the arrival and service times and use it to check your answers in parts (a) and (b) of previous Exercise

Notes