

a) Longest Common Subsequence

A subsequence is a sequence that appears in the same relative order, but not necessarily contiguous. In LCS, we have to find the Longest Common Subsequence that is in the same relative order.

In the give question, let the name of Group Member 1 is **Danish** and Group member 2 is **Salman**, so:

X: {B, D, C, A, B, A} and Y: {A, B, C, B, D, A, B}

j		0	1	2	3	4	5	6
i	y_j	B	D	C	A	B	A	
0	x_i	0	0	0	0	0	0	0
1	A	0	↑	↑	↑	↖1	←1	↖1
2	B	0	↖1	←1	←1	↑1	↖2	←2
3	C	0	↑1	↑1	↖2	←2	↑2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3	←3
5	D	0	↑1	↖2	↑2	↑2	↑3	↑3
6	A	0	↑1	↑2	↑2	↖3	↑3	↖4
7	B	0	↖1	↑2	↑2	↑3	↖4	↑4

LCS-LENGTH(*X*, *Y*)

```

1  m = X.length
2  n = Y.length
3  let b[1..m, 1..n] and c[0..m, 0..n] be new tables
4  for i = 1 to m
5      c[i, 0] = 0
6  for j = 0 to n
7      c[0, j] = 0
8  for i = 1 to m
9      for j = 1 to n
10         if  $x_i == y_j$ 
11             c[i, j] = c[i - 1, j - 1] + 1
12             b[i, j] = "↖"
13         elseif c[i - 1, j] ≥ c[i, j - 1]
14             c[i, j] = c[i - 1, j]
15             b[i, j] = "↑"
16         else c[i, j] = c[i, j - 1]
17             b[i, j] = "←"
18  return c and b
```

PRINT-LCS(*b*, *X*, *i*, *j*)

```

1  if i == 0 or j == 0
2      return
3  if b[i, j] == "↖"
4      PRINT-LCS(b, X, i - 1, j - 1)
5      print  $x_i$ 
6  elseif b[i, j] == "↑"
7      PRINT-LCS(b, X, i - 1, j)
8  else PRINT-LCS(b, X, i, j - 1)
```

(b) Shortest-Common-Supersequence

Example Problem

S1 = "abdul"

S2 = "muhammad"

Now we first need to find the LCS of both strings by dynamic programming approach.

The image shows a handwritten dynamic programming table for finding the Longest Common Subsequence (LCS) between the strings S1 = "abdul" and S2 = "muhammad". The table is a grid with rows representing S2 and columns representing S1. The top row of the grid is labeled with the characters of S1: a, b, d, u, l. The leftmost column is labeled with the characters of S2: m, u, h, a, m, m, a, d. The grid cells contain numerical values representing the length of the LCS for the substrings up to that point. The values are as follows:

		a	b	d	u	l
m	0	0	0	0	0	0
u	1	0	0	0	0	0
h	2	0	0	0	1	1
a	3	0	0	0	1	1
m	4	0	1	1	1	1
m	5	0	1	1	1	1
a	6	0	1	1	1	1
d	7	0	1	1	1	1
	8	0	1	1	2	2

Arrows are drawn from the cell (row 8, column 4) to (row 7, column 3), from (row 7, column 4) to (row 6, column 3), and from (row 6, column 4) to (row 5, column 3). Below the table, it is written: "So LCS is 'ad'".

Now for the shortest common supersequence we need to find the following;

1) Length of shortest common supersequence

Length of SCS = (length of S1 + length of S2) - Length of LCS

$$= (5 + 8) - 2$$

$$= 11$$

So in our example the length of SCS will be **11**

Find SCS :

Note:

1. Add LCS characters only once .
2. Assume first common character belongs to LCS character.
3. Add non LCS characters in order of either S1 then S2 or vice versa.

S1= a b d u l
 ↗ ↗ ↗ ↗ ↗

S2 = m u h a m m a d
 ↑ ↗ ↗ ↗ ↗ ↗ ↗ ↗ ↗

LCS = a d
 ↑ ↗

SCS = muhabmmadul

So “muhabmmadul” is the SCS in our example of length
11

Algorithm

```
int p1=0,p2=0;
for(char c: lcs)
    while(str1[p1]!=c) //Add all non-LCS chars from str1
        ans += str1[p1++];
    while(str2[p2]!=c) //Add all non-LCS chars from str2
        ans += str2[p2++];
    ans += c; //Add LCS-char and increment both ptrs
    ++p1
    ++p2
Return ans
```

c) Longest Increasing subsequence

Algo Assignment #3

Date: _____

Roll No of Member #1 = 18K-0264
Roll No of Member #2 = 18K-0350

Problem# longest increasing subsequence.

Q) 4 10 2 0 20

Algorithm used:-

$$L(i) = 1 + \max(L(j)) \text{ where } 0 \leq j < i \text{ and } \text{array}[j] < \text{array}[i];$$

$$L(i) = 1, \text{ if no } j \text{ exists}$$

Solution:- making new array of same size with all values 1

iteration #1(i) $i=1, j=0 \Rightarrow \text{array}[j] < \text{array}[i], \text{ add } 1, j++$
 $i=i, \text{ new iteration}$

1	1	1	1	1
---	---	---	---	---

iteration #2(i) $i=2, j=0 \Rightarrow \text{array}[j] > \text{array}[i], j++$

iteration #2(ii) $i=2, j=1 \Rightarrow \text{array}[j] > \text{array}[i], j++$
 $i=i, \text{ new iteration}$

1	2	1	1	1
---	---	---	---	---

iteration #3(i) $i=3, j=0 \Rightarrow \text{array}[j] > \text{array}[i], j++$

iteration #3(ii) $i=3, j=1 \Rightarrow \text{array}[j] > \text{array}[i], j++$

iteration #3(iii) $i=3, j=2 \Rightarrow \text{array}[j] > \text{array}[i], j++$
 $i=i, \text{ new iteration}$

iteration #4(i) $i=4, j=0 \Rightarrow \text{array}[j] < \text{array}[i], \text{ add } 1, j++$

1	2	1	1	2
---	---	---	---	---

iteration #4(ii) $i=4, j=1 \Rightarrow \text{array}[j] < \text{array}[i], \text{ add } 1, j++$
to i th position
Pick from
 j th position

1	2	1	1	3
---	---	---	---	---

iteration #4(iii) $i=4, j=2 \Rightarrow \text{array}[j] < \text{array}[i]$
but we didn't add because already max value present
 $j++$

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iteration #4(iv) $i=4, j=3 \Rightarrow \text{array}[j] < \text{array}[i]$
we didn't add because max value already present,
 $j++$
new iteration

So longest common subsequence is = 3

d) Levenshtein-Distance

The minimum edit distance between two strings is the minimum number of editing operations:

- Insertion
- Deletion
- Substitution

Needed to transform one into the other.

Examples include Spell correction, computational biology etc.

Normally, each operation has cost of 1. However, in Levenshtein, the substitution cost can be taken as 2.

For two strings X of length n Y of length m. We define $D(i,j)$ the edit distance between $X[1..i]$ and $Y[1..j]$ i.e., the first i characters of X and the first j characters of Y. The edit distance between X and Y is thus $D(n,m)$.

Algorithm

```
//Initialization
D(i,0) = i
D(0,j) = j
//Recurrence Relation:
For each i = 1...n
    For each j = 1...m
        D(i,j)= min(
            D(i-1, j) + 1,
            D(i, j-1) + 1,
            D(i-1, j-1) + 2,   if X(i) ≠ Y(j)
            D(i-1, j-1) + 0   if X(i) = Y(j) )
//Termination:
D(n,m) is distance
```

The “RELEVANT” word has values for every letter next to them in ascending order from 1 to 8 since it is 8 letters. Besides, that same condition is valid for the other word “ELEPHANT” shown in the figure as well. Therefore, there is an extra cell that is assigned as 0 because there is no letter in a column or row naturally. At the beginning of the two words comparison starts and goes on at the end of the matrix the last element on the diagonal. In this manner, comparison can be approached cell by cell and the responding row and column letter to that cell. In the beginning, the trivial cell which has 0 value on the leftmost and the top one is selected, and there is no letter for his cell. When we pass to the next cell one by one, we need to imply some execution to make the right action according to the algorithm.

		E	L	E	P	H	A	N	T
	0	1	2	3	4	5	6	7	8
R	1	1	2	3	4	5	6	7	8
E	2	1	2	2	3	4	5	6	7
L	3	2	1	2	3	4	5	6	7
E	4	3	2	1	2	3	4	5	6
V	5	4	3	2	2	3	4	5	6
A	6	5	4	3	3	3	4	5	
N	7	6	5	4	4	4	4	3	4
T	8	7	6	5	5	5	5	4	3

e) Matrix Chain Multiplication

Input: $p[] = \{40, 20, 30, 10, 30\}$

Output: 26000

There are 4 matrices of dimensions 40×20 , 20×30 , 30×10 and 10×30 .

Let the input 4 matrices be A, B, C and D. The minimum number of multiplications are obtained by putting parenthesis in following way
 $(A(BC))D \rightarrow 20 \times 30 \times 10 + 40 \times 20 \times 10 + 40 \times 10 \times 30$

Input: $p[] = \{10, 20, 30, 40, 30\}$

Output: 30000

There are 4 matrices of dimensions 10×20 , 20×30 , 30×40 and 40×30 .

Let the input 4 matrices be A, B, C and D. The minimum number of multiplications are obtained by putting parenthesis in following way
 $((AB)C)D \rightarrow 10 \times 20 \times 30 + 10 \times 30 \times 40 + 10 \times 40 \times 30$

Input: $p[] = \{10, 20, 30\}$

Output: 6000

There are only two matrices of dimensions 10×20 and 20×30 . So there is only one way to multiply the matrices, cost of which is $10 \times 20 \times 30$

```
#include <bits/stdc++.h>
using namespace std;

// Matrix Ai has dimension p[i-1] x p[i]
// for i = 1..n
int MatrixChainOrder(int p[], int i, int j)
{
    if (i == j)
        return 0;
    int k;
    int min = INT_MAX;
    int count;

    // place parenthesis at different places
    // between first and last matrix, recursively
    // calculate count of multiplications for
    // each parenthesis placement and return the
    // minimum count
    for (k = i; k < j; k++)
    {
        count = MatrixChainOrder(p, i, k)
                + MatrixChainOrder(p, k + 1, j)
                + p[i - 1] * p[k] * p[j];

        if (count < min)
            min = count;
    }
}
```

```
        // Return minimum count
        return min;
    }

    // Driver Code
    int main()
    {
        int arr[] = { 1, 2, 3, 4, 3 };
        int n = sizeof(arr) / sizeof(arr[0]);

        cout << "Minimum number of multiplications is "
              << MatrixChainOrder(arr, 1, n - 1);
    }
```


f) Knapsack Problem

Given a set 'S' of 'n' items,

such that each item i has a positive value v_i and positive weight w_i

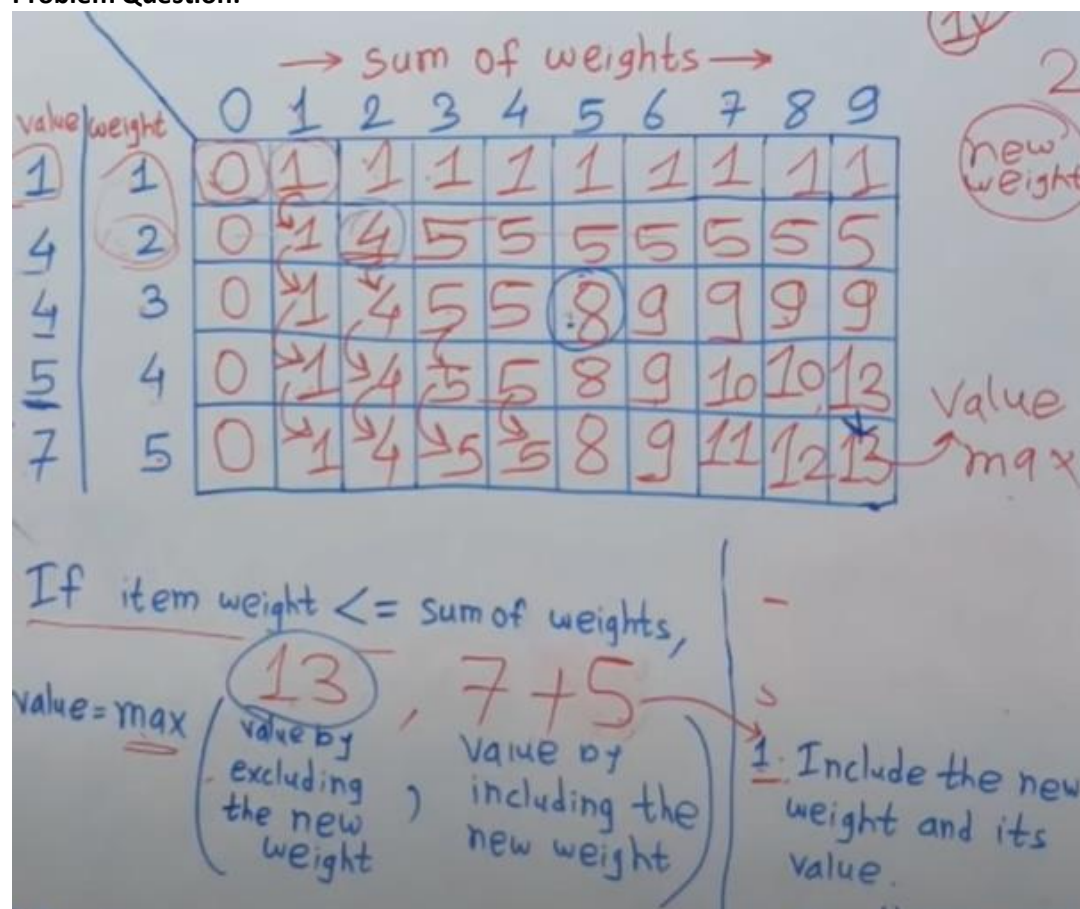
The goal is to find maximum-benefit subset that does not exceed the given weight W .

Algorithm

```
Knapsack(j,w)
  for i ← 0 to n
    M[i,0] ← 0
  for w ← 0 to W
    M[0,w] ← 0

  for j ← 1 to n
    for w ← 0 to W
      if  $w_j > w$ 
        M[j,w] = M[j - 1, w]
      else M[j,w] ← MAX( $v_j + M[j - 1, w - w_j]$ ,
        M[j - 1, w])
  return M[n,W]
```

Problem Question:



The maximum benefit/value = 13

The selected items' indexes are: $s_i = [0,1,1,1,0]$

g) Partition Problem

Given a set of positive integers, check if it can be divided into two subsets with equal sum.

First. The sum of all elements in the set is calculated. If sum is odd, we can't divide the array into two sets. If sum is even, check if a subset with $\text{sum}/2$ exists or not.

Let the first three alphabets converted to numbers:

Group member 1: Abid 1,2,9

Group member 2: Adam 1,4,1

Set= 1,2,9,1,4,1

Sum=18 (Since the sum is even, so partition might be possible)

$\text{Sum}/2=9$

We will make table of size 9, and if the last element results in True, then the partition is possible, otherwise not.

This somewhat resembles to the 0/1 knapsack problem.

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
1	T	T	F	F	F	F	F	F	F	F
2	T	T	T	F	F	F	F	F	F	F
9	T	T	T	F	F	F	F	F	F	T
1	T	T	T	T	F	F	F	F	F	T
4	T	T	T	T	T	T	T	T	T	T
1	T	T	T	T	T	T	T	T	T	T

The last box of the table is true, that shows that the partition is possible.

The result showed that set can be partitioned into $S1 = \{9\}$, and $S2 = \{1,2,1,4,1\}$, with each subset having a sum of 9. However, note that the size of sets are not equal.

h) Rod Cutting Problem

Rod cutting problem is a type of allocation problem. Allocation problem involves the distribution of resources among the competing alternatives in order to minimize the total costs or maximize total return (profit).

In the rod cutting problem we have given a rod of length n , and an array that contains the prices of all the pieces smaller than n , determine the maximum profit you could obtain from cutting up the rod and selling its pieces.

Length[] = {1,2,3,4,5,6,7,8}

price[] = {1,5,8,9,10,16,18,20}

Rod Length: 8

Rod Length (i)	Cut 0	Cut 1	Cut 2	Cut 3	Cut 4	Cut 5	Cut 6	Cut 7	Cut 8
----------------	-------	-------	-------	-------	-------	-------	-------	-------	-------

0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---

1	0	1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---	---	---

2	0	1	5	6	10	11	15	16	20
---	---	---	---	---	----	----	----	----	----

3	0	1	5	8	10	13	15	18	20
---	---	---	---	---	----	----	----	----	----

4	0	1	5	8	10	13	15	18	20
---	---	---	---	---	----	----	----	----	----

5	0	1	5	8	10	13	15	18	20
---	---	---	---	---	----	----	----	----	----

6	0	1	5	8	10	13	16	18	21
---	---	---	---	---	----	----	----	----	----

7	0	1	5	8	10	13	16	18	21
---	---	---	---	---	----	----	----	----	----

8	0	1	5	8	10	13	16	18	22
---	---	---	---	---	----	----	----	----	----

Maximum Profit = 22

Selected Pieces = 8 pieces of Rod Length 2, to get maximum profit for rod length 22.

[illegible]

j) Word Break

Given an input string and a dictionary of words, find out if the input string can be segmented into a space-separated sequence of dictionary words.

We will try to search from the left of the string to find a valid word when a valid word is found, we will search for words in the next part of that string.

For Word Break Problem, $S = \{i, like, ice, cream, icecream, mobile, apple\}$

INPUT:

Ilikeapple

OUTPUT:

i like apple

[illegible]