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# ASSIGNMENT 03

## Q1)

Sol) $u = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}, v = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

$\langle u, v \rangle = tr(u^T v)$

$u^T = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, V = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

$u^T v = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

$u^T v = \begin{bmatrix} -2 & -1 \\ 14 & -3 \end{bmatrix}$

$tr(u^T v) = -2 + (-3)$

$tr(u^T v) = -5$

$\boxed{\langle u, v \rangle = -5}$

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# Q2)(a)

Sol) $\|u+v\|^2 = \|u\| + 2\langle u,v\rangle + \|v\|^2$

$L.H.S = \|u+v\|^2$ $\qquad \because \|u\|^2 = \langle u,u\rangle$

$= \langle u+v, u+v\rangle$

$= \langle u,u\rangle + \langle u,v\rangle + \langle v,u\rangle$
$+ \langle v,v\rangle$

$= \|u\|^2 + \langle u,v\rangle + \langle v,u\rangle$
$+ \|v\|^2$

A/c to definition 1:

$\therefore \langle u,v\rangle = \langle v,u\rangle$

$L.H.S = \|u\|^2 + 2\langle u,v\rangle + \|v\|^2$

$L.H.S = R.H.S$

proved

# Q2)(b)

Sol) $\langle u+v, u-v\rangle = \|u\|^2 - \|v\|^2$

$L.H.S = \langle u+v, u-v\rangle$ $\qquad \because \|u\|^2 = \langle u,u\rangle$

$= \langle u,u\rangle - \langle u,v\rangle + \langle v,u\rangle$
$- \langle v,v\rangle$

$= \|u\|^2 - \langle u,v\rangle + \langle v,u\rangle$
$- \|v\|^2$

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A/c to dy 1

$\therefore \langle u, v\rangle = \langle v, u\rangle$

L.H.S $= \|u\|^2 - \langle u, \cancel{v}\rangle + \langle \cancel{v}, u\rangle - \|v\|^2$

L.H.S $= \|u\|^2 - \|v\|^2$

L.H.S = R.H.S

proved

Q3)

Sol) $\|u\| = 4$ , $v = (2, -1, -2)$

$\|u+v\| = ?$

A/c to pythagorus theorem:

$\|u+v\|^2 = \|u\|^2 + \|v\|^2$

$\|u+v\| = \sqrt{\|u\|^2 + \|v\|^2}$

$\|v\| = \sqrt{(2)^2 + (-1)^2 + (-2)^2}$

$\|v\| = 3$

$\|u\| = 4$

$\|u+v\| = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\boxed{\|u+v\| = 5}$ Ans

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Q4)

Sol) $<p,q> = \int_{-1}^{1} p(x)q(x)\,dx$

$P_2 = \{1, x, x^2\}$

$S = \{p_1, p_2, p_3\}$

$S = \{1, x, x^2\}$

By Gram Schmidth Process:

Step 1:

$V_1 = p_1 = 1$

Step 2:

$$V_2 = p_2 - \frac{<p_2 - V_1>}{\|V_1\|^2} V_1$$

~~Vsee~~

$$\|V_1\| = \sqrt{<V_1, V_1>} = \sqrt{\int_{-1}^{1} (1)^2 dx}$$

$$= \sqrt{1\int_{-1}^{1} dx} = \sqrt{[x]_{-1}^{1}} = \sqrt{1-(-1)}$$

$\|V_1\| = \sqrt{2}$

$$\boxed{\|V_1\|^2 = 2}$$

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$$\langle p_2, v_1 \rangle = \int_{-1}^{1} (x) \times (1)\, dx$$

$$= \int_{-1}^{1} x\, dx$$

$$= \left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$= \left[\frac{1}{2} - \left(\frac{1}{2}\right)\right]$$

$$\boxed{\langle p_2, v_1 \rangle = 0}$$

$$v_2 = x - \frac{0}{2}(1)$$

$$\boxed{v_2 = x}$$

Step 3:

$$v_3 = p_3 - \frac{\langle p_3, v_1 \rangle}{\|v_1\|^2} v_1 - \frac{\langle p_3, v_2 \rangle}{\|v_2\|^2} v_2$$

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$\|V_2\| = \sqrt{\langle V_2, V_2 \rangle}$

$\|V_2\|^2 = \int_{-1}^{1} (x)(x)\,dx = \int_{-1}^{1} x^2\,dx$

$= \left[\frac{x^3}{3}\right]_{-1}^{1} = \left[\frac{1}{3} - \left(-\frac{1}{3}\right)\right]$

$\boxed{\|V_2\|^2 = \frac{2}{3}}$

$\langle p_3, V_1 \rangle = \int_{-1}^{1} (x^2)(1)\,dx = \int_{-1}^{1} x^2\,dx$

$= \left[\frac{x^3}{3}\right]_{-1}^{1} = \left[\left(\frac{1}{3}\right) - \left(-\frac{1}{3}\right)\right]$

$\langle p_3, V_1 \rangle = \frac{2}{3}$

$\langle p_3, V_2 \rangle = \int_{-1}^{1} (x^2)(x)\,dx = \int_{-1}^{1} x^3\,dx$

$\langle p_3, V_2 \rangle = \left[\frac{x^4}{4}\right]_{-1}^{1} = \left[\frac{1}{4} - \left(\frac{1}{4}\right)\right] = 0$

$V_3 = x^2 - \left[\frac{2}{3}/2 \times (1)\right] - \left[\frac{0}{(2/3)} \times (x)\right]$

$V_3 = x^2 - \left[\frac{2}{3} \times \frac{1}{2}\right] - 0$

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$$\boxed{V_3 = x^2 - \frac{1}{3}}$$

Normalizing, (For Orthonormal)

$$\cancel{q_1 = \frac{V_1}{\|V_1\|^2} = \frac{x}{2}}$$

$$q_1 = \frac{V_1}{\|V_1\|} = \boxed{1/\sqrt{2}}$$

$$q_2 = \frac{V_2}{\|V_2\|} = \frac{x}{\sqrt{\frac{2}{3}}} = x\sqrt{\frac{3}{2}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{q_2 = \frac{3x}{\sqrt{6}}}$$

$$q_3 = \frac{V_3}{\|V_3\|} =$$

$$\cancel{\|V_3\| = \sqrt{(x^2)^2 + \left(-\frac{1}{3}\right)^2}}$$

$$\|V_3\| = \sqrt{\langle V_3, V_3 \rangle}$$

$$\|V_3\| = \sqrt{\int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)\left(x^2 - \frac{1}{3}\right) dx}$$

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$$= \sqrt{\int_{-1}^{1} x^4 - \frac{2}{3}x^2 + \frac{1}{9}\, dx}$$

$$= \sqrt{\int_{-1}^{1} x^4 dx - \frac{2}{3}\int_{-1}^{1} x^2 dx + \frac{1}{9}\int_{-1}^{1} dx}$$

$$= \sqrt{\left[\frac{x^5}{5}\right]_{-1}^{1} - \frac{2}{3}\left[\frac{x^3}{3}\right]_{-1}^{1} + \frac{1}{9}\left[x\right]_{-1}^{1}}$$

$$= \sqrt{\left[\frac{x^5}{5} - \frac{2x^3}{9} + \frac{x}{9}\right]_{-1}^{1}}$$

$$= \sqrt{\left[\frac{1}{5} - \frac{2}{9} + \frac{1}{9}\right] - \left[\frac{-1}{5} + \frac{2}{9} - \frac{1}{9}\right]}$$

$$\|v_3\| = \sqrt{\frac{4}{45} - \left(\frac{-4}{45}\right)} = \sqrt{\frac{8}{45}} = \frac{2\sqrt{10}}{15}$$

$$q_3 = x^2 - \frac{1}{3} \Big/ \frac{2\sqrt{10}}{15}$$

$$q_3 = \left(x^2 - \frac{1}{3}\right) \times \frac{15}{2\sqrt{10}}$$

$$q_3 = \left(15x^2 - \frac{\cancel{15}^{5}}{\cancel{3}}\right) \times \frac{1}{2\sqrt{10}}$$

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$$\boxed{v_3 = \frac{15x^2-5}{2\sqrt{10}}}$$

Orthonormal Basis $= \left\{ \frac{1}{\sqrt{2}}, \frac{3x}{\sqrt{6}}, \frac{15x^2-5}{2\sqrt{10}} \right\}$

Ans

Q5)

Sol) $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_n|$

$\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

$\|x\|_\infty = \max\{|x_1|, |x_2|, \dots, |x_n|\}$

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\|x\|_1 = |2| + |3| = 5$

$\|x\|_2 = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$

$\|x\|_\infty = \max(|2|, |3|) = \max(2,3) = 3$

(b) $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$

$\|x\|_1 = |0| + |-2| =$

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$\|x\|_1 = 0 + 2 = 2$

$\|x\|_2 = \sqrt{(0)^2 + (-2)^2} = \sqrt{4} = 2$

$\|x\|_\infty = \max(|0|, |-2|) = \underset{\max}{(0, 2)} = 2$

(c) $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$

$\|x\|_1 = |-4| + |-1|$

$\|x\|_1 = 4 + 1 = 5$

$\|x\|_2 = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$

$\|x\|_\infty = \max(|-4|, |-1|)$

$\|x\|_\infty = \max(4, 1) = 4$

Q6)

Sol) $\langle u, v \rangle = u_1 v_1 + 2u_2 v_2 + 3u_3 v_3$

$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

<u>Orthogonal Basis:</u>

<u>Step 1:</u>

$v_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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$$V_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} v_1$$

$\langle u_2, v_1 \rangle = (1)(1) + 2(1)(1) + 3(0)(1)$

$\langle u_2, v_1 \rangle = 1 + 2 + 0 = 3$

$\cancel{\|v_1\|^2 = \sqrt{(1)^2 + 2(1)^2 + 3(1)^2}}$

$\cancel{= \sqrt{1+2+3} = \sqrt{6}}$

$\|V_1\|^2 = (1)^2 + 2(1)^2 + 3(1)^2$

$\|V_1\|^2 = 6$

$\|V_1\| = \sqrt{6}$

$$V_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{\cancel{3}}{\cancel{6}\,2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$$

<u>Step 3:</u>

$$V_3 = u_3 - \frac{\langle u_3, V_1 \rangle}{\|V_1\|^2} V_1 - \frac{\langle u_3, V_2 \rangle}{\|V_2\|^2} V_2$$

$\langle u_3, V_1 \rangle = (1)(1) + 2(0)(1) + 3(0)(1)$

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$\langle u_3, v_1 \rangle = 1 + 0 + 0 = 1$

$\langle u_3, v_2 \rangle = (1)\left(\frac{1}{2}\right) + 2(0)\left(\frac{1}{2}\right) + 3(0)\left(\frac{-1}{2}\right)$

$\langle u_3, v_2 \rangle = \frac{1}{2} + 0 + 0 = \frac{1}{2}$

$\|v_2\|^2 = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{-1}{2}\right)^2$

$\|v_2\|^2 = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} = \frac{3}{2} \Rightarrow \|v_2\| = \sqrt{3/2}$

$$v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{\frac{1}{2}}{3/2}\begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1/6 \\ 1/6 \\ 1/6 \end{bmatrix} - \begin{bmatrix} 1/6 \\ 1/6 \\ -1/6 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 2/3 \\ -1/3 \\ 0 \end{bmatrix}$$

Orthogonal Basis $= \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \end{bmatrix}, \begin{bmatrix} 2/3 \\ -1/3 \\ 0 \end{bmatrix} \right\}$

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Normalizing: (For Orthonormal)

$$q_1 = \frac{v_1}{\|v_1\|} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Big/ \sqrt{6}$$

$$q_1 = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

$$q_2 = \frac{v_2}{\|v_2\|} = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \end{bmatrix} \Big/ \sqrt{3/2}$$

$$q_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \end{bmatrix} \times \sqrt{\frac{2}{3}}$$

$$q_2 = \begin{bmatrix} (\sqrt{2/3})/2 \\ (\sqrt{2/3})/2 \\ (-\sqrt{2/3})/2 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}$$

$$q_3 = \frac{v_3}{\|v_3\|} = \begin{bmatrix} 2/3 \\ -1/3 \\ 0 \end{bmatrix} \Big/ \sqrt{6}/3 = \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 0 \end{bmatrix}$$

$$\|v_3\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + (0)^2} = \frac{\sqrt{6}}{3}$$

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Orthonormal Basis $= \left\{ \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{6}} \\ 1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 0 \end{bmatrix} \right\}$

Ans

Q7)

Sol) $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 0 & 1 & -2 \end{bmatrix}$

let $u_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

Applying Gram Schmidt Process:

Step 1:

$V_1 = u_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

Step 2:

$V_2 = u_2 - \dfrac{\langle u_2, V_1 \rangle}{\|V_1\|^2} V_1$

$\langle u_2, V_1 \rangle = (-1)(2) + (2)(1) + (0)(1)$

$\langle u_2, V_1 \rangle = 0$

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$$\|V_1\|^2 = (2)^2 + (1)^2 + (0)^2 = 5$$

$$\|V_1\| = \sqrt{5}$$

$$V_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} - \frac{0}{5} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

$$V_3 = u_3 - \frac{\langle u_3, V_1 \rangle}{\|V_1\|^2} V_1 - \frac{\langle u_3, V_2 \rangle}{\|V_2\|^2} V_2$$

$$\langle u_3, V_1 \rangle = (2)(1) + (1)(-2) + (0)(\cdot)$$
$$\langle u_3, V_1 \rangle = 0$$
$$\langle u_3, V_2 \rangle = (-1)(1) + (2)(-2) + (1)(-2)$$
$$\langle u_3; V_2 \rangle = -1 - 4 - 2 = -7$$

$$\|V_2\|^2 = (-1)^2 + (2)^2 + (1)^2$$
$$\|V_2\|^2 = 1 + 4 + 1 = 6$$
$$\|V_2\| = \sqrt{6}$$

$$V_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} - \frac{0}{5} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} - \frac{(-7)}{6} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

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$$V_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} - 0 + \frac{7}{6}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\cancel{V_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} + \begin{bmatrix} -7/\sqrt{6} \\ 14/\sqrt{6} \\ 7/\sqrt{6} \end{bmatrix}}$$

$$V_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} + \begin{bmatrix} -7/6 \\ 14/6 \\ 7/6 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} -1/6 \\ 1/3 \\ -5/6 \end{bmatrix}$$

Normalizing,

$$q_1 = \frac{V_1}{\|V_1\|} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \Big/ \sqrt{5} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$$

$$q_2 = \frac{V_2}{\|V_2\|} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \Big/ \sqrt{6} = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

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$$q_3 = \frac{V_3}{\|V_3\|}$$

$$\cancel{\|V_3\| = \sqrt{(-\tfrac{1}{6})^2 + (-2)\ldots}}$$

$$\|V_3\| = \sqrt{\left(-\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{-5}{6}\right)^2} = \frac{\sqrt{30}}{6}$$

$$q_3 = \begin{bmatrix} -1/6 \\ 1/3 \\ -5/6 \end{bmatrix} \bigg/ \frac{\sqrt{30}}{6} = \begin{bmatrix} -1/\sqrt{30} \\ 2/\sqrt{30} \\ -5/\sqrt{30} \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{6} & -1/\sqrt{30} \\ 1/\sqrt{5} & 2/\sqrt{6} & 2/\sqrt{30} \\ 0 & 1/\sqrt{6} & -5/\sqrt{30} \end{bmatrix}$$

$$\therefore R = \begin{bmatrix} \langle u_1, q_1 \rangle & \langle u_2, q_1 \rangle & \langle u_3, q_1 \rangle \\ 0 & \langle u_2, q_2 \rangle & \langle u_3, q_2 \rangle \\ 0 & 0 & \langle u_3, q_3 \rangle \end{bmatrix}$$

$$\langle u_1, q_1 \rangle = (2)\left(\frac{2}{\sqrt{5}}\right) + (1)\left(\frac{1}{\sqrt{5}}\right) + (0)(0)$$

$$\langle u_1, q_1 \rangle = \sqrt{5}$$

$$\langle u_2, q_1 \rangle = (-1)\left(\frac{2}{\sqrt{5}}\right) + (2)\left(\frac{1}{\sqrt{5}}\right) + (1)(0)$$

$$\langle u_2, q_1 \rangle = 0$$

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$$\langle u_3, q_1 \rangle = (1)\left(\frac{2}{\sqrt{5}}\right) + (-2)\left(\frac{1}{\sqrt{5}}\right) + (-2)(0)$$

$$\langle u_3, q_1 \rangle = 0$$

$$\langle u_2, q_2 \rangle = (-1)\left(-\frac{1}{\sqrt{6}}\right) + (2)\left(\frac{2}{\sqrt{6}}\right) + (1)\left(\frac{1}{\sqrt{6}}\right)$$

$$\langle u_2, q_2 \rangle = \sqrt{6}$$

$$\langle u_3, q_2 \rangle = (1)\left(-\frac{1}{\sqrt{6}}\right) + (-2)\left(\frac{2}{\sqrt{6}}\right) + (-2)\left(\frac{1}{\sqrt{6}}\right)$$

$$\langle u_3, q_2 \rangle = \frac{-7\sqrt{6}}{6}$$

$$\langle u_3, q_3 \rangle = (1)\left(\frac{-1}{\sqrt{30}}\right) + (-2)\left(\frac{2}{\sqrt{30}}\right) + (-2)\left(\frac{-5}{\sqrt{30}}\right)$$

$$\langle u_3, q_3 \rangle = \frac{\sqrt{30}}{6}$$

$$R = \begin{bmatrix} \sqrt{5} & 0 & 0 \\ 0 & \sqrt{6} & -7\sqrt{6}/6 \\ 0 & 0 & \sqrt{30}/6 \end{bmatrix}$$

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$\therefore A = QR$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{6} & -1/\sqrt{30} \\ 1/\sqrt{5} & 2/\sqrt{6} & 2/\sqrt{30} \\ 0 & 1/\sqrt{6} & -5/\sqrt{30} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 0 & 0 \\ 0 & \sqrt{6} & -\frac{7\sqrt{6}}{6} \\ 0 & 0 & \frac{\sqrt{30}}{6} \end{bmatrix}$$

Q8)

Sol) $A = \begin{bmatrix} -7 & 24 & 0 & 0 \\ 24 & 7 & 0 & 0 \\ 0 & 0 & -7 & 24 \\ 0 & 0 & 24 & 7 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} -7-\lambda & 24 & 0 & 0 \\ 24 & 7-\lambda & 0 & 0 \\ 0 & 0 & -7-\lambda & 24 \\ 0 & 0 & 24 & 7-\lambda \end{bmatrix}$

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} -7-\lambda & 24 & 0 & 0 \\ 24 & 7-\lambda & 0 & 0 \\ 0 & 0 & -7-\lambda & 24 \\ 0 & 0 & 24 & 7-\lambda \end{vmatrix} = 0$$

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Expand by $R_1$ :

$$(-7-\lambda)\begin{vmatrix} 7-\lambda & 0 & 0 \\ 0 & -7-\lambda & 24 \\ 0 & 24 & 7-\lambda \end{vmatrix} - 24\begin{vmatrix} 24 & 0 & 0 \\ 0 & -7-\lambda & 24 \\ 0 & 24 & 7-\lambda \end{vmatrix}$$

$$+0-0=0$$

$$\Rightarrow \begin{vmatrix} 7-\lambda & 0 & 0 \\ 0 & -7-\lambda & 24 \\ 0 & 24 & 7-\lambda \end{vmatrix} = (7-\lambda)\begin{vmatrix} -7-\lambda & 24 \\ 24 & 7-\lambda \end{vmatrix} -0 +0$$

$$=(7-\lambda)\{(-7-\lambda)(7-\lambda)-(24)(24)\}$$

$$=(7-\lambda)\{(-49+\cancel{7\lambda}-\cancel{7\lambda}+\lambda^2-576)\}$$

$$=(7-\lambda)(\lambda^2-625)$$

$$\Rightarrow \begin{vmatrix} 24 & 0 & 0 \\ 0 & -7-\lambda & 24 \\ 0 & 24 & 7-\lambda \end{vmatrix} = 24\begin{vmatrix} -7-\lambda & 24 \\ 24 & 7-\lambda \end{vmatrix} -0+0$$

$$=24\{(-7-\lambda)(7-\lambda)-(24)(24)\}$$

$$=24(-49+\cancel{7\lambda}-\cancel{7\lambda}+\lambda^2-576)$$

$$=24(\lambda^2-625)$$

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Now,

$(-7-\lambda)(7-\lambda)(\lambda^2-625)-(24)(24)(\lambda^2-625)=0$

$(\lambda^2-49)(\lambda^2-625)-(576\lambda^2+360000)=0$

$\lambda^4-625\lambda^2-49\lambda^2+30625-576\lambda^2+36000=0$

$\Rightarrow \lambda^4-1250\lambda^2+390625=0$

$\therefore \lambda=25,25,-25,-25$

$\underline{\lambda=25}$:

$$A-\lambda I=\begin{bmatrix} -7-\lambda & 24 & 0 & 0 \\ 24 & 7-\lambda & 0 & 0 \\ 0 & 0 & -7-\lambda & 24 \\ 0 & 0 & 24 & 7-\lambda \end{bmatrix}$$

$$A-\lambda I=\begin{bmatrix} -32 & 24 & 0 & 0 \\ 24 & -18 & 0 & 0 \\ 0 & 0 & -32 & 24 \\ 0 & 0 & 24 & -18 \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 24 & -18 & 0 & 0 \\ 0 & 0 & -32 & 24 \\ 0 & 0 & 24 & -18 \end{bmatrix} R_1 \to \frac{1}{32}R_1$$

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$$\sim \begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -32 & 24 \\ 0 & 0 & 24 & -18 \end{bmatrix} R_2 - 24R_1$$

$$\sim \begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 24 & -18 \end{bmatrix} \left(-\frac{1}{32}\right) R_3$$

$$\sim \begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_4 - 24R_3$$

$$\sim \begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_{23}$$

$$x_1 - \frac{3}{4} x_2 = 0$$

$$x_3 - \frac{3}{4} x_4 = 0$$

Let $x_2 = t$, $x_4 = s$

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$\Rightarrow x_1 - \frac{3}{4}t = 0$

$\boxed{x_1 = \frac{3}{4}t}$

$\Rightarrow x_3 - \frac{3}{4}s = 0$

$\boxed{x_3 = \frac{3}{4}s}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t\begin{bmatrix} 3/4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ 3/4 \\ 1 \end{bmatrix}$$

OR

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t\begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix}$$

$\underline{\lambda = -25:}$

$$A - \lambda I = \begin{bmatrix} 18 & 24 & 0 & 0 \\ 24 & 32 & 0 & 0 \\ 0 & 0 & 18 & 24 \\ 0 & 0 & 24 & 32 \end{bmatrix}$$

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$$\sim\begin{bmatrix} 1 & 4/3 & 0 & 0 \\ 24 & 32 & 0 & 0 \\ 0 & 0 & 18 & 24 \\ 0 & 0 & 24 & 32 \end{bmatrix} \left(\frac{1}{18}\right)R_1$$

$$\sim\begin{bmatrix} 1 & 4/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 24 \\ 0 & 0 & 24 & 32 \end{bmatrix} R_2 - 24R_1$$

$$\sim\begin{bmatrix} 1 & 4/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1\cancel{8} & \cancel{2}4/3 \\ 0 & 0 & 24 & 32 \end{bmatrix} \left(\frac{1}{18}\right)R_3$$

$$\sim\begin{bmatrix} 1 & 4/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_4 - 24R_3$$

$$\sim\begin{bmatrix} 1 & 4/3 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_{23}$$

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$$x_1 + \frac{4}{3}x_2 = 0$$

$$x_3 + \frac{4}{3}x_4 = 0$$

Let $x_2 = t$, $x_4 = s$

$$\Rightarrow x_1 + \frac{4}{3}t = 0$$

$$\boxed{x_1 = -\frac{4t}{3}}$$

$$\Rightarrow x_3 + \frac{4}{3}s = 0$$

$$\boxed{x_3 = -\frac{4}{3}s}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t\begin{bmatrix} -4/3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ -4/3 \\ 1 \end{bmatrix}$$

OR

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t\begin{bmatrix} -4 \\ 3 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ -4 \\ 3 \end{bmatrix}$$

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Basis = orthogonal $\left\{ \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -4 \\ 3 \end{bmatrix} \right\}$

Let $v_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} -4 \\ 3 \\ 0 \\ 0 \end{bmatrix},$

$v_4 = \begin{bmatrix} 0 \\ 0 \\ -4 \\ 3 \end{bmatrix}$

For Orthonoral, Normalizing:

$$q_1 = \frac{v_1}{\|v_1\|}$$

$$\|v_1\| = \sqrt{(3)^2+(4)^2+0+0} = \sqrt{9+16} = \sqrt{25} = 5$$

$$q_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} \Big/ 5 = \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \\ 0 \end{bmatrix}$$

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$$q_2 = \frac{V_2}{\|V_2\|}$$

$$\|V_2\| = \sqrt{0+0+(3)^2+(4)^2} = 5$$

$$q_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix} \Big/ 5 = \begin{bmatrix} 0 \\ 0 \\ 3/5 \\ 4/5 \end{bmatrix}$$

$$q_3 = \frac{V_3}{\|V_3\|}$$

$$\|V_3\| = \sqrt{(-4)^2+(3)^2+0+0} = 5$$

$$q_3 = \begin{bmatrix} -4 \\ 3 \\ 0 \\ 0 \end{bmatrix} \Big/ 5 = \begin{bmatrix} -4/5 \\ 3/5 \\ 0 \\ 0 \end{bmatrix}$$

$$q_4 = \frac{V_4}{\|V_4\|}$$

$$\|V_4\| = \sqrt{0+0+(-4)^2+(3)^2} = 5$$

$$q_4 = \begin{bmatrix} 0 \\ 0 \\ -4 \\ 3 \end{bmatrix} \Big/ 5 = \begin{bmatrix} 0 \\ 0 \\ -4/5 \\ 3/5 \end{bmatrix}$$

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$$P = \begin{bmatrix} 3/5 & 0 & -4/5 & 0 \\ 4/5 & 0 & 3/5 & 0 \\ 0 & 3/5 & 0 & -4/5 \\ 0 & 4/5 & 0 & 3/5 \end{bmatrix}$$

$\therefore$ For $P^T AP$ :

$$\begin{bmatrix} 3/5 & 4/5 & 0 & 0 \\ 0 & 0 & 3/5 & 4/5 \\ -4/5 & 3/5 & 0 & 0 \\ 0 & 0 & -4/5 & 3/5 \end{bmatrix}
\begin{bmatrix} -7 & 24 & 0 & 0 \\ 24 & 7 & 0 & 0 \\ 0 & 0 & -7 & 24 \\ 0 & 0 & 24 & 7 \end{bmatrix}
\begin{bmatrix} 3/5 & 0 & -4/5 & 0 \\ 4/5 & 0 & 3/5 & 0 \\ 0 & 3/5 & 0 & -4/5 \\ 0 & 4/5 & 0 & 3/5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & -25 & 0 \\ 0 & 0 & 0 & -25 \end{bmatrix}$$

Ans

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Q9)

Sol) $A = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 6-\lambda & -2 \\ -2 & 3-\lambda \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\Rightarrow \begin{vmatrix} 6-\lambda & -2 \\ -2 & 3-\lambda \end{vmatrix} = 0$

$(6-\lambda)(3-\lambda) - (4) = 0$

$18 - 3\lambda - 6\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 9\lambda + 14 = 0$

$\lambda^2 - 7\lambda - 2\lambda + 14 = 0$

$\lambda(\lambda - 7) - 2(\lambda - 7) = 0$

$(\lambda - 7)(\lambda - 2) = 0$

$\Rightarrow \lambda = 7, \lambda = 2$

$\lambda = 7$ :-

$A - \lambda I = \begin{bmatrix} 6-\lambda & -2 \\ -2 & 3-\lambda \end{bmatrix}$

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$$A-\lambda I=\begin{bmatrix}-1 & -2\\ -2 & -4\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 2\\ -2 & 4\end{bmatrix} (-1)R_1$$

$$\sim\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix} R_2+2R_1$$

$x_1+2x_2=0$

let $x_2=t$

$\Rightarrow x_1+2t=0$

$x_1=-2t$

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix}=t\begin{bmatrix}-2\\ 1\end{bmatrix}$$

$\lambda=2$:

$$A-\lambda I=\begin{bmatrix}6-\lambda & -2\\ -2 & 3-\lambda\end{bmatrix}$$

$$A-\lambda I=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & -1/2\\ -2 & 1\end{bmatrix} \frac{1}{4}R_1$$

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$$\sim \begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix} R_2 + 2R_1$$

$x_1 - \frac{1}{2}x_2 = 0$

let $x_2 = t$

$x_1 - \frac{1}{2}t = 0$

$x_1 = \frac{1}{2}t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

OR

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For Orthogonal Basis:

$u_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Step1:

$V_1 = u_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

Step2:

$$V_2 = u_2 - \frac{\langle u_2, V_1 \rangle}{\|V_1\|^2} V_1$$

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$\langle u_2, v_1 \rangle = (1)(-2) + (2)(1)$
$\langle u_2, v_1 \rangle = 0$

$\|v_1\|^2 = (-2)^2 + (1)^2 = 5$

$v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \frac{0}{5} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Normalizing, (for Orthonormal)

$q_1 = \frac{v_1}{\|v_1\|} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} / \sqrt{5} = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$q_2 = \frac{v_2}{\|v_2\|}$

$\|v_2\| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$

$q_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} / \sqrt{5} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

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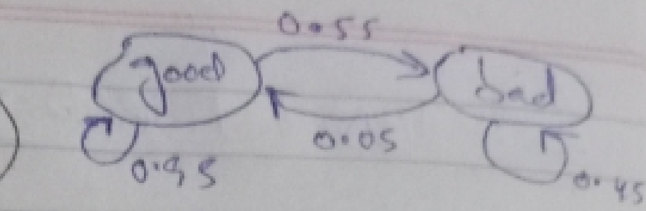
## Spectral Decomposition:

$$A = \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T$$

$$\cancel{A = (2)\begin{bmatrix} .2 \\ 1 \end{bmatrix}\begin{bmatrix} .2 & 1 \end{bmatrix} +}$$

$$A = 2\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} + 7\begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}\begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

21k-3881

Q10) (a) 


Sol) $P = \begin{array}{cc} & \begin{array}{cc} \text{good} & \text{bad} \end{array} \\ \begin{array}{c} \text{good} \\ \text{bad} \end{array} & \begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.48 \end{bmatrix} \end{array}$

(b) $\begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix} \begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0.93 \\ 0.07 \end{bmatrix} \Rightarrow \boxed{\text{Probability} = 0.93}$

c) $\begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix} \begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix} \begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.858 \\ 0.142 \end{bmatrix}$

$\boxed{\text{Probability} = 0.142}$

d) $\begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix}$, $X_0 = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$

$X_{k+1} = P X_k$

$X_1 = \begin{bmatrix} 0.95 & 0.55 \\ 0.05 & 0.45 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$

$X_1 = \begin{bmatrix} 0.63 \\ 0.37 \end{bmatrix} \Rightarrow \boxed{\text{Probability} = 0.63}$