Theorem. Let x be the number of maximum queens that share the same row/column in a minimum domination set. We have $x \leq 2\gamma - n + 2$.

First, consider the following theorem from this paper: "Cockayne, E.J., 1990. Chessboard domination problems. *Discrete Mathematics*, *86*(1-3), pp.13-20."

Theorem 3 (Spencer [14]). *For any* n, $\gamma(Q_n) \geqslant (n-1)/2$.

Proof. Consider a covering of the $n \times n$ board using $\gamma = \gamma(Q_n)$ queens. Suppose that the rows and columns are sequentially labelled $1, \ldots, n$ from top to bottom and left to right respectively. A row or column is said to be *occupied* if it contains a queen.

Let column a, (b) be the left most (right most) unoccupied column and let row $c\,(d)$ be the unoccupied row closest to the top (bottom). Further we set $\delta_1 = b - a$ and $\delta_2 = d - c$ and assume without lost of generality that $\delta_1 \geqslant \delta_2$.

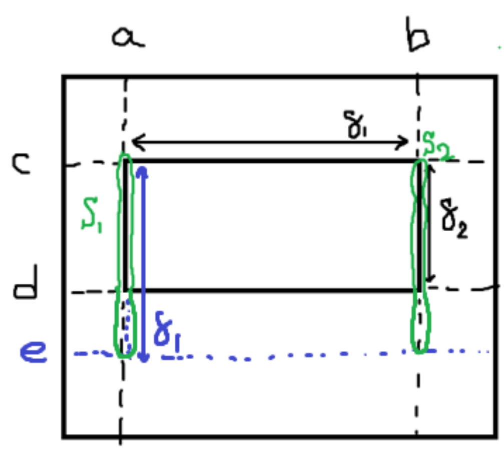
Consider the sets S_1 and S_2 of squares in columns a and b respectively, which lie between rows c and $c + \delta_1 - 1$ inclusive and let $S = S_1 \cup S_2$. Since $\delta_1 \geqslant \delta_2$, no diagonal intersects both S_1 and S_2. Hence every queen diagonally dominates at most two squares of S (i.e. at most one per diagonal). Further queens situated above row c or below row $c + \delta_1 - 1$ do not dominate squares of S by row or column.

By definition of c, there are at least $c - 1$ queens above row c. Each row below row d is occupied and $d = c + \delta_2 \leqslant c + \delta_1$. Therefore all the $n - c - \delta_1$ rows below row $c + \delta_1$ are occupied. Hence there are at least $n - c - \delta_1$ queens below row $c + \delta_1 - 1$.

It follows that at least $(c-1) + (n - c - \delta_1) = n - \delta_1 - 1$ queens dominate at most 2 squares of S. The remaining queens of which there are at most $\gamma - (n - \delta_1 - 1)$, may cover at most 4 squares of S. Since all the $2\delta_1$ squares of S must be dominated we have

$$2(n - \delta_1 - 1) + 4(\gamma - (n - \delta_1 - 1)) \geqslant 2\delta_1,$$

which gives $\gamma \geqslant (n-1)/2$ as required. □



Let $x \geq 2$ be the number of maximum queens that share the same row in a minimum domination set. Thus, there is at least one row with x queen.

Case 1. If there is such a row above or below the inner rectangle of the figure, we have:

$2(n-\delta_1-1+(x-1))+4\,(\gamma-(n-\delta_1-1+(x-1)))\geq 2\delta_1$ which gives $x\leq 2\gamma-n+2$

In other words, Case 1 represents the scenario in which such a row is one that each queen on it can at most attach 2 squares of S potentially.

Case 2. If there is such a row in the inner rectangle of the figure, we have:

$2(n-\delta_1-1)+4\,(\gamma-(n-\delta_1-1)-x)+\ 2\times 3+2(x-2)\geq 2\delta_1$ which gives $x\leq 2\gamma-n+2$

In other words, Case 2 represents the scenario in which such a row is one that each queen on it could at most attach 4 squares of S potentially.

Therefore, $x\leq 2\gamma-n+2$ in all cases. The board is symmetric, and so, the result is also valid for columns.

Corollary. If $\gamma=n/2$, $x\leq 2$.

Theorem. Consider two arbitrary rows named r_x, and r_y such that r_x != r_y.And denote the number of queens in those rows by x and y, respectively. We have $x+y\leq 2\gamma-n+3$.

Case 1. x, y both are placed in rows that can attack at most 2 sqaures of S. We have:

$2(n-\delta_1-1+(x-1)+(y-1))+4\,(\gamma-(n-\delta_1-1+(x-1)+(y-1)))\geq 2\delta_1$ which gives $x+y\leq 2\gamma-n+3$

Case 2. x, y both are placed in rows that can attack at most 4 sqaures of S. We have:

$2(n-\delta_1-1)+4\,(\gamma-(n-\delta_1-1)-x-y)+2\times 3\times 2+2(x-2)+2(y-2)\geq 2\delta_1$ which gives $x+y\leq 2\gamma-n+3$

Case 3. w/o lose of generality, x is placed in a row that can attack at most 2 sqaures of S. And y is placed in a row that can attack at most 4 sqaures of S. We have:

$2(n-\delta_1-1+(x-1))+4\,(\gamma-(n-\delta_1-1+(x-1))-y)+2\times 3+2(y-2)\geq 2\delta_1$ which gives $x+y\leq 2\gamma-n+3$

Corollary. If $\gamma=n/2$, in an arbitrary minimum domination set, at most one row/column can have more than one queen.