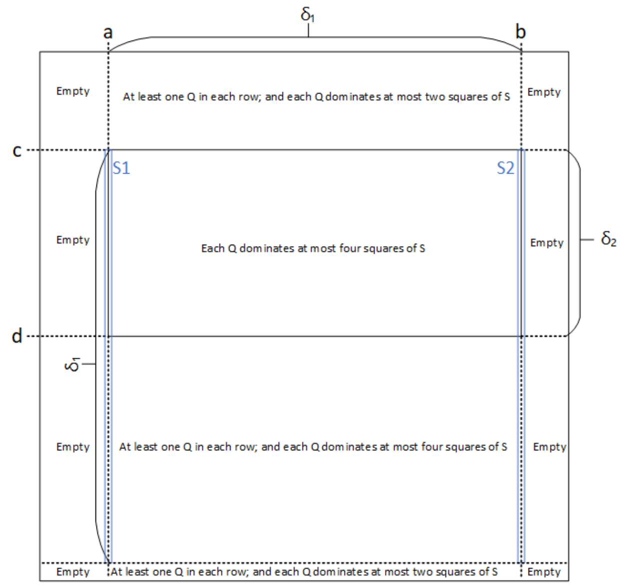
A significant portion of the theorem we aim to prove relies on the insights provided in the paper titled "Chessboard Domination Problems" by Cockayne, E.J., published in Discrete Mathematics in 1990 (Volume 86, Issues 1-3, Pages 13-20).

The following figure illustrates the settings that will be employed consistently in our proofs.



Lemma 1. *Given an n×n chessboard, let γ be the minimum number of queens required to cover all squares of the chessboard. Let x be the number of queens that are placed on a row/column of some domination set of queens of size γ. We have the following inequality:* $x \leq 2\gamma - n + 2$.

Number the rows and columns of the chessboard in sequence 1...n from top to bottom and from left to right, respectively. A row or column is considered empty if it does not contain any queens; otherwise, it is non-empty.

Consider some domination set of size γ. Let a be the leftmost empty column, b be the rightmost empty column, c be the empty row closest to the top, and d be the empty row closest to the bottom. Set $\delta_1 = b - a$ and $\delta_2 = d - c$. Without loss of generality, assume $\delta_1 \geq \delta_2$.

Let S_1 be the set of squares in column a and between rows c and c + - 1 inclusive. Let S_2 be the set of squares in column b and between rows c and $c + \delta_1 - 1$ inclusive. Also, let $S = S_1 \cup S_2$.

It is easy to see that any queen placed above row c or below row $c + \delta_1 - 1$ may dominate at most two squares of S since it does not dominate squares of S by row or column.

Since $\delta_1 \geq \delta_2$, no diagonal intersects both S_1 and S_2. Therefore, every queen diagonally dominates at most two squares of S (i.e., at most one per diagonal). This implies that every queen that does not lie above row c and below row $c + \delta_1 - 1$ may dominate at most four squares of S (i.e., two squares diagonally and two by row).

There are $c - 1$ rows above row c, and there are $n - c + \delta_1$ rows below row $c + \delta_1$ that, by definition, each contains at least one queen. Therefore, at least $(c - 1) + (n - c + \delta_1) = (n - \delta_1 - 1)$ queens that each of them dominates at most two squares of S.

The remaining queens, of which there are at most $\gamma - (n - \delta_1 - 1)$, may cover at most four squares of S. Since all $2\delta_1$ of S must be dominated, we have: $2(n - \delta_1 - 1) + 4(\gamma - (n - \delta_1 - 1)) \geq 2\delta_1$, which gives $\gamma \geq (n - 1)/2$. Obviously, if $x = 0$ or $x = 1$, the $x \leq 2\gamma - n + 2$ inequality is maintained. If, in the assumed domination set, there is no row with more than one queen, the work is done. Thus, we consider a case in which the assumed minimum domination set has at least one row that contains more than one queen. Choose one of such rows arbitrarily and let $x \geq 2$ be the number of queens it contains.

Case 1: If the chosen row is above row c or below row $c + \delta_1 - 1$, that row may dominate at most $2x$ squares of S. We already know that each row above c or below $c + \delta_1 - 1$ contains at least one queen. Thus, there are at least $(n - \delta_1 - 1 + (x - 1))$ queens that each of them may dominate at most 2 squares of S. It remains at most $(\gamma - (n - \delta_1 - 1 + (x - 1)))$ queens that each of them may dominate at most 4 squares of S. All in one, we have $2(n - \delta_1 - 1 + (x - 1)) + 4(\gamma - (n - \delta_1 - 1 + (x - 1))) \geq 2\delta_1$, which gives $x \leq 2\gamma - n + 2$.

Case 2: If the chosen row is not above row c nor below row $c + \delta_1 - 1$, since $x \geq 2$, that row may dominate at most $2\times3 + (x - 2)$ squares of S. All in one, we have $2(n - \delta_1 - 1) + 4(\gamma - (n - \delta_1 - 1) - x) + 2\times3 + 2(x - 2) \geq 2\delta_1$, which gives $x \leq 2\gamma - n + 2$.

The board is symmetric, and so the inequality also holds for columns. □

Lemma 2. *Given an $n\times n$ chessboard, let γ be the minimum number of queens required to cover all squares of the chessboard. Let x and y be the number of queens placed on two distinct rows/columns of some domination set of queens of size γ. We have the following inequality: $x + y \leq 2\gamma - n + 3$.*

Consider the same setting as Lemma 1, i.e., the same numbering and the same naming convention for a, b, c, d, δ_1, δ_2, S_1, S_2, and S. Consider some domination set of size γ. Choose two distinct rows arbitrarily and denote the number of queens on one row by x and another by y.

Case 1: Both of the rows might be chosen above row c or below row $c + \delta_1 - 1$. Thus, we have $2(n - \delta_1 - 1 + (x - 1) + (y - 1)) + 4(\gamma - (n - \delta_1 - 1 + (x - 1) + (y - 1))) \geq 2\delta_1$, which gives $x + y \leq 2\gamma - n + 3$.

Case 2: The chosen rows are not above row c nor below row $c + \delta_1 - 1$. Thus, we have $2(n - \delta_1 - 1) + 4(\gamma - (n - \delta_1 - 1) - x - y) + 2\times3\times2 + 2(x - 2) + 2(y - 2) \geq 2\delta_1$, which gives $x + y \leq 2\gamma - n + 3$.

Case 3: One of the chosen rows is above row c or below row $c + \delta_1 - 1$, and the other is not. Thus, we have $2(n - \delta_1 - 1 + (x - 1)) + 4(\gamma - (n - \delta_1 - 1 + (x - 1)) - y) + 2\times3 + 2(y - 2) \geq 2\delta_1$, which gives $x + y \leq 2\gamma - n + 3$.

The board is symmetric, so the inequality also holds for columns. □

Theorem 1. *Given an $n \times n$ chessboard, let γ be the minimum number of queens required to cover all squares of the chessboard. If $\gamma = n/2$, in every minimum queen domination set of that chessboard, no row/column contains more than two queens, and there is at most one row that contains two queens.*

If $\gamma = n/2$, according to Lemma 1, each row contains at most two queens. Also, according to Lemma 2, every two rows contain at most 3 queens in total. As a result, we have $x + y \leq 3$ such that $x, y \leq 2$, which implies that at most, one of x and y could be two. The same argument is valid for columns.

□