Learn & Explore

MATHEMATICS

2

ELEMENTS
LEARNING SYSTEM

Publisher Printing Year etc

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بسم الله الرحمن الرحيم

(In the Name of Allah, the Most Compassionate, the Most Merciful)

Contents

Preface

The second in the Elements Learning series of Mathematics books is the Grade 2 book that, in addition to covering the contents to be taught, also contains the "how" of teaching. In the words of George Polya, "Mathematics is not a spectator sport". To understand math is to do math. We are following best practices based on the CPA (Concrete, Pictorial, Abstract) methodology, proposed by Jerome Bruner, combined with some innovative methods from Richard Skemp, Zoltán Pál Dienes, Caleb Gattegno, and others. Each topic is integrated with a variety of tools and contexts to make ***learning through discovery.***

The book starts with a Refresher which recaps all the preliminary concepts that the student is expected to know before moving forward with the new grade. It is followed by twelve subsequent units placed in a conceptually logical and sequential manner. Each unit is composed of different topics and activities, broadly divided into demo and interactive teaching, small group activities, and individual tasks. Small Group activities provide opportunities for learning from each other, in addition to the benefits of communicating, collaborating, and thinking together. Each unit culminates with an exercise to be performed individually and independently by each learner. In order to let learning percolate in pupils' minds as a natural process, "Review Exercises" have been placed after every three units. Each unit acts as a benchmark after which the teacher, following AFL (assessment for learning) approach, must assess the students before moving on.

The book also equips our educators with ***How-to-Teach*** videos via QR codes embedded throughout the different topics. For extra knowledge and practice where needed to keep learners in the challenge zone, the teacher may refer to the "References" provided at the end of each chapter.

Benchmarks

Grade 1-2-3

Numbers, Data & Measurements

[Number] Students will be able to demonstrate knowledge of place value (up to 4 digit numbers); represent whole numbers with words, diagrams, number lines, or symbols; order and compare numbers.

They will add and subtract numbers up to 4-digit numbers; multiply (up to 3-digit numbers with 1-digit) and divide (3-digit by 1-digit number). Solve problems involving odd and even numbers, addition, subtraction, multiplication and division of numbers (involving missing numbers, money, quantities and measures), round numbers to nearest tens, hundreds and thousands and make estimates.

[Measurements] Students will be able to measure, compare and order mass (kilograms/grams/milligrams),

They would also solve problems involving weight/mass, and time (including addition and subtraction).

Read, write and compare time (hours and minutes);

[Data & Stat] Read and interpret data from pictographs, bar graphs, tally charts, block graphs and Carroll diagrams.

Organize and represent data using pictographs, bar graphs, tally charts, block graphs and Carroll diagrams to answer questions.

Describe the probability of an event.

Fractions

Recognize fractions as parts of wholes or collections; represent fractions using words, numbers, equivalent fractions in simplest form; compare and order simple fractions; add and subtract simple like and unlike fractions, including those set in problem situations.

Demonstrate knowledge of decimal place value to the tenth.

Geometry

[Geometry] Students will be able to use properties to describe and compare three dimensional shapes (cube, cuboid, cone, cylinder, sphere, prism and pyramids) and relate those with two dimensional shapes; differentiate and classify polygons.

Identify parallel and perpendicular lines; reflective symmetry, right angles and angles smaller and larger than a right angle; positions, directions and movements, centre, radius, and diameter of a circle.

[Measurements] Students will be able to measure, compare and order

lengths (Kilometers/meters/ centimeters),

capacity (liters/ milliliters);

They would also solve problems involving length and capacity (including addition and subtraction). Measure area and perimeter using square grids.

Algebra

Students will be able to analyze and complete geometrical and number patterns; find the missing number or operation in a number sentence.

Legend

This describes specific learning objectives of this topic.

This describes activity that the teacher has to demonstrate before the children.

This icon describes a teacher led activity in which the whole class participates.

Activity that children do in a group of four to six.

Activity given to children for individual homework.

Single player game.

Double player game.

Four player game.

How to play.

This icon describes an activity to be repeated in different contexts.

Indicative duration for the activity.

REFRESHER

Let’s have some fun with the things we have learned in previous grades!

Capacity and Using its Vocabulary

A game for four players

You will need:

- Game board: Page 1- 6(Gameboards for Grade 2 Maths)
- Game cards: Page 7- 12(Gameboards for Grade 2 Maths)
- Bag for the game cards

Game 1

- The water' cards are mixed up in the bag. In turn, each player takes a card from the bag and places it on their 'glass' on the game board, starting at the bottom.
- Players must fill the glass. They cannot place a card that will go over the 'full' line. If the card taken is too big, the player puts it back in the bag and the other player has their turn.
- The first player to 'fill' the glass to the 'full' line is the winner.

Game 2

- If a player takes a card that fills their glass to more than half full, the player must put that card back in the bag and the other player has their turn.
- When a player's glass is exactly half full, that player can continue playing to fill their glass. It must be exactly full. If the card taken is too big, the player puts it back in the bag and the other player has their turn.
- First player to fill the glass to the full line is the winner.

Game 3

- Players must first fill their glass to the 'half full' mark. If the card taken is too big, the player can either swap it for one already on the glass, or put it back in the bag. The other player then has their turn.
- When a player's glass is exactly half full, that player can continue playing to 'fill' their glass with the same rules.
- The first player to 'fill' the glass to the full line is the winner.

These games can be played by four players with each pair having a game board with two glasses. The pair have to fill both glasses. They discuss the options before placing the card on a glass. They can choose to fill one glass first and then the other, or place pieces on either glass each time.

Time and Its Units

A game for two players

You will need:

- Game board: Page I - II (End of the Book)
- A counter (or alternative) per player
- A 1-6 dice
- A set Game cards
- Paper and pencil

Shuffle the cards and place them face down on the board

Game 1: Race to the Moon

- Players 'START' on the earth and 'FINISH' on the moon.2.
- Players take turns to roll the dice and move that number of spaces:

  If a player lands on a star they take a card and keep it.

  If a player lands on an instruction space they must follow the instruction.

- When a player reaches the 'FINISH (the moon) they calculate the times shown on their cards.
- The player with the shortest amount of time is the winner, as they travelled to the moon the fastest.

Game 2: Moon to Earth and Back

- Play like Game 1, but once at the moon the player must travel backwards along the board to earth, as the new finish.
- Players must collect time cards as you travel back
- The winner is the player with the shortest amount of time once they get back to earth, as they took the shortest time to travel from the earth to the moon and back again.

Number Bonds up to 20

A game for two players

You will need:

- Game board: Page III-IV (End of the book)
- A 1-6 dice
- (1 & 3) 20 counters (or alternative); different colour per player
- (2 & 4) 11 to 19 counters; depending on target number

Game 1: 20 pairs

- Place the 20 counters in the central oval. Each player uses one pair of Ten frames as their base board. Players take turns to roll a dice and take that number of counters from the oval. They then place one counter in each square on their base board.
- When there are no more counters in the oval, the players have to stop and count how many counters they each have.
- Players should recognise that between them they have a number pair to 20. They write down the resulting number pair, for example if Player 1 has 14 counters and Player 2 has 6: 14 + 6 = 20. The player whose turn it is next, announces what number of counters they need in order to make 20, for example Player 1 would say 'I need 6 counters. The player then rolls the dice.
- If the number rolled is the number required to make 20, the player automatically wins the game. If not, they either take counters or miss a turn: if the number rolled is less than the number of counters they need, they take that many counters from their opponent; if they roll a number that is greater than the number they need, they miss a turn.
- Play passes to the next player who starts by writing the new number pair and announces how many counters they need to make 20. Play continues in this way.
- The winner is the first player to make 20.

Game 2: Playing with 11 to 19

- Players decide together what the target number will be. They then colour in squares on the second Ten frame from the bottom up, on their side of the Game board; the coloured squares represent spaces that cannot be used.
- In doing so. the players create a Game board with the target number of squares on either side of the board.
- For example, if the players choose 18 as their target number, they each colour in the bottom two squares of their second Ten frame so that there are only 18 white squares on each side of the board.
- Put a number of counters that matches the target number in the oval. In our example, this would be 18 counters. Players take turns to roll a dice and take that number of counters from the oval. They then place one counter in each square on their base board.
- When there are no more counters in the oval, the players have to stop and count how many counters they each have
- Players should recognise that between them they have a number pair to their target number (in this case 18). They write down the resulting number pair, for example if Player 1 has 8 counters and Player 2 has 10: 8+10= 18
- The player whose turn it is next, announces what number of counters they need in order to make the target number. The player then rolls the dice.
- If the number rolled is the number required to make the target number, the player automatically wins the game. If not, they either take counters or miss a turn: if the number rolled is less than the number of counters they need, they take that many counters from their opponent; if they roll a number that is greater than the number they need, they miss a turn.
- Play passes to the next player who starts by writing the new number pair and announces how many counters they need to make the target number. Play continues in this way.
- The winner is the first player to reach the target number.

Game 3: 20 Take away

- Each player starts with 20 counters on their two ten frames. They take it in turns to roll a dice and remove that number of counters from their frame, placing them on the oval. Players can only remove the number of counters that is indicated by the number on the dice. For example, if they throw a 5, but have only four counters, they miss a turn.
- As a player removes counters from their ten frames, they record the subtraction they have carried out. So if a player starts by rolling a 3 and removes three counters, they can record this as 20-317. If their next roll of the dice is a 5, they remove five more counters. Their ten frames now show 20-8 = 12 as they have taken away eight counters altogether and they have 12 counters left on their ten frames.
- The winner is the first player to have no counters left on their Ten frames.

Game 4: 11 to 19 Take away

- Players choose a target number and set up the Game board as per Playing with 20 (2), colouring in squares that are not to be used, and putting the appropriate number of counters on the oval.
- As players remove counters from their ten frames, they record the subtraction they have carried out. So if the target number is 18 and a player starts by rolling a 3 and removes three counters, they can record this as 18-3= 15. If their next roll of the dice is a 5, they remove five more counters. Their ten frames now show 18-8=10 as they have taken away eight counters altogether and as they have ten counters left on their ten frame.
- The winner is the first player to have no counters left on their Ten frames.

Taking off!

Lets's dive into the new Grade Maths!

MISTAKES + ALLOW - x =

THINKING TO HAPPEN % <

Unit: 1

The Fundamentals of Numerals and Symbols

Learning Objectives:

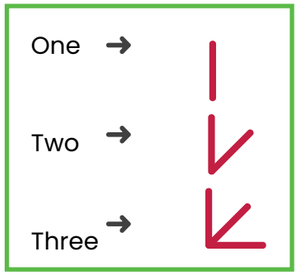
- Read and write Roman numbers up to 12.
- Recognize the place value in 3-digit numbers.
- Compare and order numbers up to 999.
- Count numbers up to and across 999 (3-digit numbers) forwards and backwards, beginning from zero or one, or from any given number.
- Count and write in 10s and 100s.
- Count backward in tens from any given number.
- Recognize the position of objects and write it using ordinal numbers up to 20.

Vocabulary:

Symbol | Figure | Numeral | Place Value | Greater than | Less than

Writing Numbers in Figures (Symbols & Numerals)

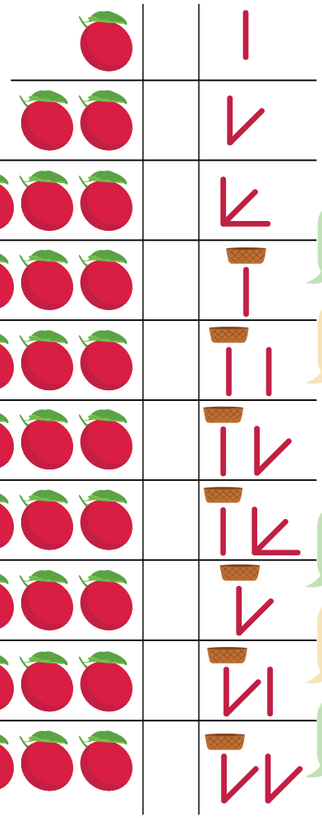
An “Amazon” tribe had only symbols for one, two & three



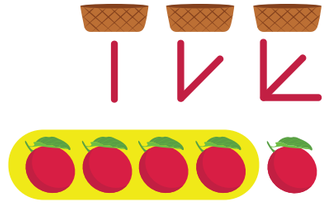
How would they write up to ten?

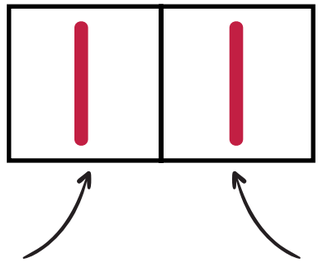
Counting

PASSION FRUIT



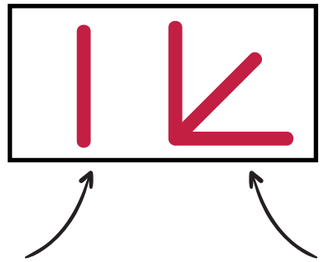
Why not make baskets and have a place value for baskets rather than making more symbols?





This place is for basket.

First place is fixed for units



One basket of fruits

Three fruits (units)

So  is Seven

What does  indicate?

HINT:

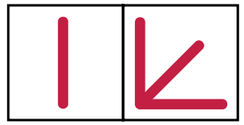
How many units? How many baskets?

When there are NO UNITS, Only Baskets

How will Amazon tribe write twelve?

Let's make groups of four i.e. three baskets(groups), and NO UNITS

Why groups of four? Why not of three?



Baskets

(three baskets)

if we write it  ----------

Units

(first place is for units)

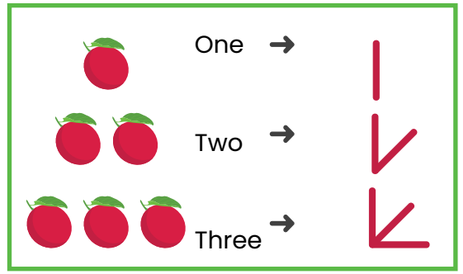
For units we have left a space

This may cause an error or confusion

So we write ZERO for that space

THREE BASKETS NO UNITS

Finally the Amazon Tribe would write (numerals) for numbers, like the following:



(Numbers)

No. of fruits (Numerals)	
One	
Two	
Three	
Four	
Five	
Six	
Seven	
Eight	
Nine	
Ten	
Eleven	
Twelve	

and so on

Number Making Activities

A 'penta' tribe has only four symbols.

Can you make(write) numerals for them.
Their symbols are:

One →	
Two →	
Three →	
Four →	

(Numbers)

No. of fruits (Numerals)		
Nil		
One		
Two		
Three		
Four		
Five		
Six		
Seven		
Eight		
Nine		
Ten		
Eleven		
Twelve		

Number-writing activities

How can "binary" tribe write their numbers? They have only two symbols.

Nothing => 0 (zero)

One => 1

Write their numerals. (use Numbers-Numeral sheet on next page)

.....

You are "urban" tribe and have ten symbols.

Write your numerals on Numbers-Numeral sheet on next page

Symbols

Nothing => 0

One => 1, two => 2, three => 3,

Four => 4, five => 5 , six => 6,

seven => 7, eight => 8, nine => 9,

.....

Coin your own numeral (activities)

Name you tribe:_____

How many symbols you choose to have?

Write your numerals

(use Numbers-Numeral sheet on next page)

Zero => 0

One => _____

Two => _____

____ => _____

Numbers › Numerals

What are your symbols?

Numbers (Numerals)		
Nil		
One	▲	
Two	▲ ▲	
Three	▲ ▲ ▲	
Four	▲ ▲ ▲ ▲	
Five	▲ ▲ ▲ ▲ ▲	
Six	▲ ▲ ▲ ▲ ▲ ▲	
Seven	▲ ▲ ▲ ▲ ▲ ▲ ▲	
Eight	▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲	
Nine	▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲	
Ten	▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲	
Eleven	▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲	
Twelve	▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲ ▲	

What strategy did you apply?

Real life Examples of Bundling

MEDICINE

Carton of 1000s

Packet of 100s

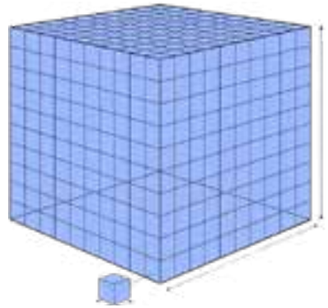
Strip of 10s

Tablets

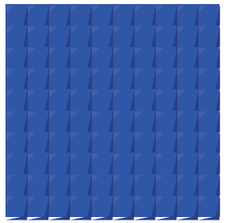
units

MONEY

NUMBER BLOCKS



Cube (1000s)

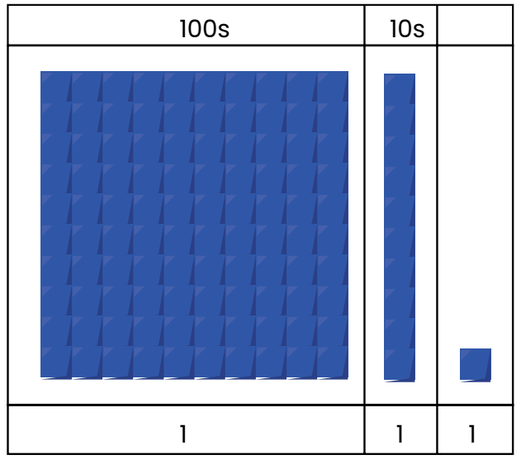


Fat (100s)

Long (10s)

Unit

Why we call it place value system?



PLACE VALUE
PLACE of digit tells is value.

First digit place is for UNITS.

Second-digit place is for TENS.

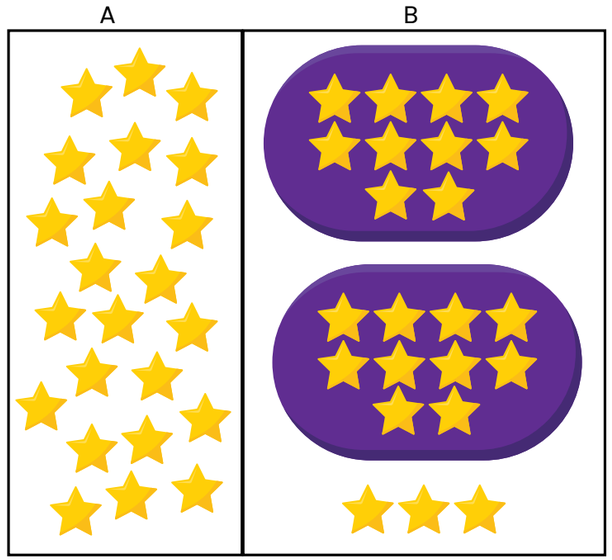
Third-digit place if for HUNDREDS.

One hundreds *One tens* *one unit*

Solve **Question 1** from **Exercise 1**

Count the following sets

Which set A or B was easier to count and why?



Comparing 3-Digit Numbers

A game for two players

You will need:

- Game cards: Page V (End of the book)
- Scissors

- Cut out the number cards.
- Place them face down on the table and mix them up.
- Both players pick up a card and turn it over at the same time.
- They both look at the numbers.
- The player whose number is greater says:
- _____is greater than_____
- The player whose number is less says:
- _____is less than_____.
- The player who speaks first claims both number cards.
- The winner is the player who collects the most cards..

Solve **Question 2** from **Exercise 1**

Skip Counting: 2s, 5s, and 10s to 100

A game for two players

You will need:

- Game board: Page VI (End of the book)
- A counter for each player
- A 1-6 dice
- A 100 square

- Place your counter on the 'Start' square.
- Roll the dice and move your counter that number of spaces.
- When you land on an instruction square, count out loud as told.
- If you count correctly, move ahead one space. Otherwise, stay where you are.
- The game ends when someone reaches the 'Finish' square. The first one to do so is the winner.

Ordering & Comparing Numbers Using Place Value

A game for two players

You will need:

- Game board
- A counter
- A 'Player card' for each player
- A 1-6 dice
- Two or three sets of Place value cards
- (2) A set of 'Less than/greater than' cards

Game 1:

- Players place their counter on 'Start'. They take turns to roll the dice and move their counter that number of spaces. Players collect the Place value card matching the numbers they land on. Players cannot collect more than one copy of each card, so if they land on a space of a card they already have, they do not collect a card.
- When they reach 'Finish', they use their place value cards to make the numbers requested by the Player cards. Players score a point if their number is the closest or highest.
- The winner is the player with the most points.

Game 2:

As Game 1 until players reach 'Finish'. They then use their cards to create number sentences using the Less than/greater than cards. The winner (or winners) is the player who uses all their cards in correct number sentences.

Solve **Question 3** from **Exercise 1**

Ordinal Numbers

A game for the whole class

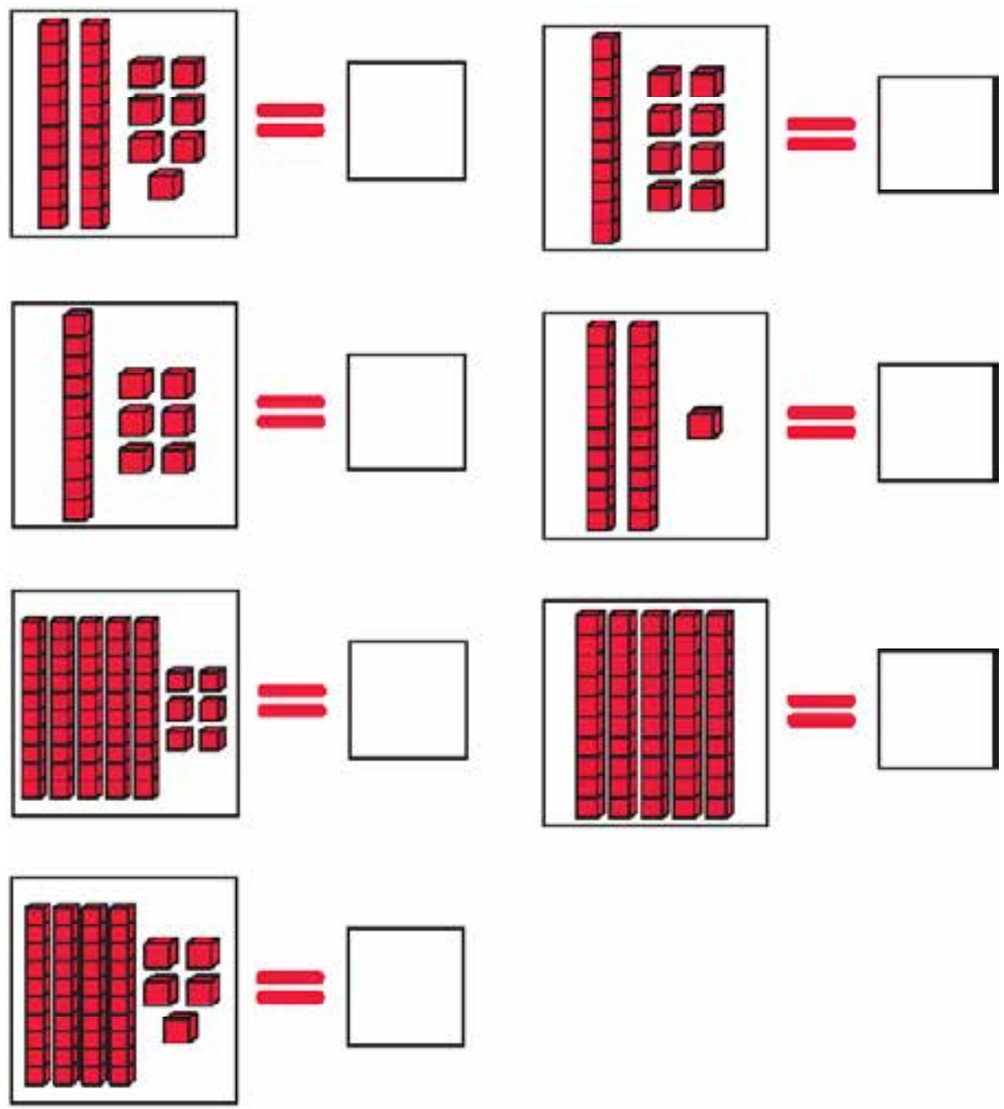
You will need:

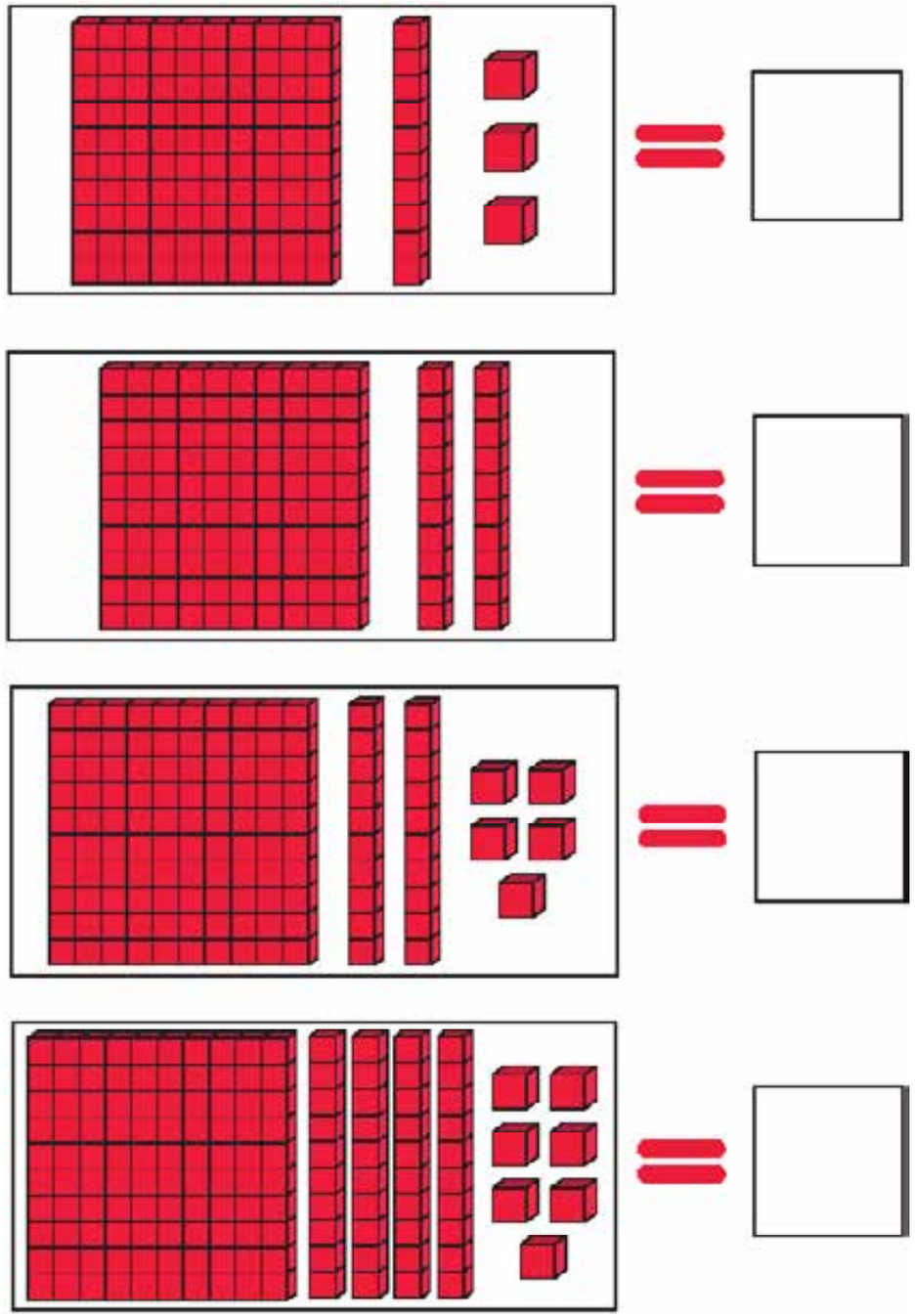
- Ball or a parcel that can be thrown

- Ask all students to sit in a line.
- Start at one point and that student is first in line.
- Say different numbers such as the 5th one and the student with the ball in their hand, will throw it to the 5th students.
- Repeat for many numbers up to 20.

Exercise 1

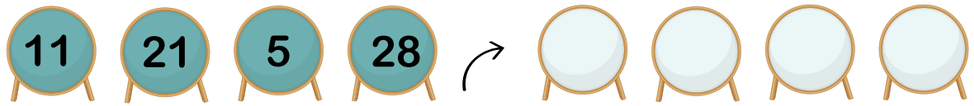
Question 1: Write the following base-10 block figures into numerals.

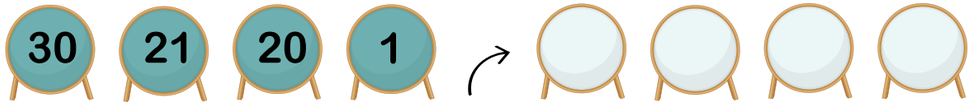


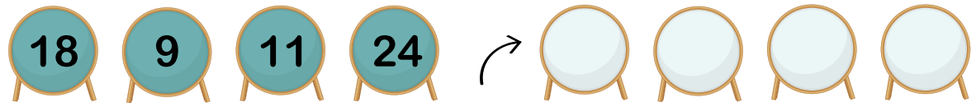


Exercise 1

Question 2: Write the numbers from smallest to biggest.

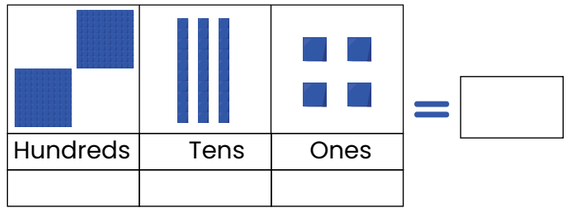


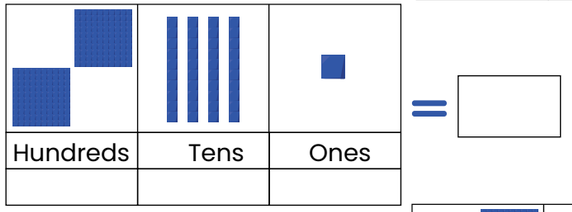


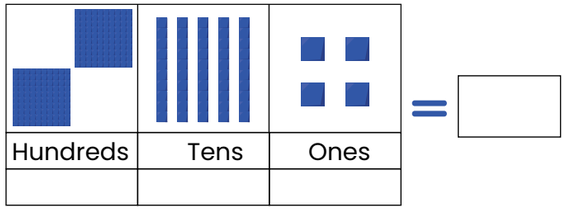


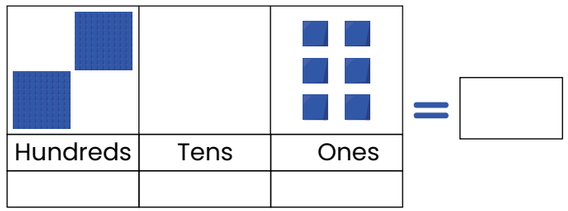
Exercise 1

Question 3: Write the following base-10 block figures into numerals and fill in the place value blanks.









References:

https://youtu.be/lzRGl0b61ug

https://youtu.be/Awth8srDk0g

https://youtu.be/GjHZxtf0tRs

Unit: 2

Money Maths and Estimation

Learning Objectives:

- Identify international currency and denominations.
- Adding numbers up to 25 on a numberline.
- Mentally adding numbers up to 20.
- Adding and subtracting in ones and tens.
- Patterns in a 100 grid.

Vocabulary:

Currency | Notes | Addition | Mental Math | Denomination