Computation in Complex Systems

Unit 1 Exam

- **QUESTION 1: Polynomials & Exponentials**

Today, your computer can do T steps in a week. According to Moore's law, next year, your computer will be able to do 2T steps in a week. How does doubling T change the n that can be computed in a week?

a) If T = n^2, what does doubling T correspond to in terms of n? In other words, by what arithmetic factor does n change when T doubles?

Initially: $T = n^2 \rightarrow n = 1 \times \sqrt{T}$

Doubling T: $2 \times T = n^2 \rightarrow n = \sqrt{2 \times T} \rightarrow n = \sqrt{2} \times \sqrt{T}$

Computing $\sqrt{2}$ would yield roughly 1.41 which is a 0.41 improvement on the first equation. In other words, doubling T would improve n by $(\sqrt{2} - 1) \times \sqrt{T}$

b) If T = 2^n, what does doubling T correspond to in terms of n? In other words, by what arithmetic factor does n change when T doubles?

When T doubles, we can decrease the power of n to be $n - 1$. The time complexity would correspond to $T = 2^{n-1}$. In other words, doubling T leads to a linear increase in the power of n.

Important! Your answer should include an algebraic solution as well as the reasoning/logical steps - either in words or equations - by which you arrived at your answer. The latter will form a part of the exam grade.

- **QUESTION 2: Divide & Conquer**

For the Towers of Hanoi puzzle, there is a function ƒ(n) that computes the total number of moves needed to move n disks. For ƒ(0) = 0, but for n greater than 0, ƒ(n) = 2ƒ(n-1)+1. Additionally:

n	0	1	2	3	4	5
ƒ(n)	0	1	3	7	15	31

a) What is the function that allows the equation to be true?

$f(n) = 2 \times f(n-1) + 1$

$f(n-1) = 2 \times f(n-2) + 1$

$f(n-2) = 2 \times f(n-3) + 1$

$f(n-3) = 2 \times f(n-4) + 1$

$f(n) = 2 \times (2 \times (2 \times (2 \times f(n-4) + 1) + 1) + 1) + [1]$

Assuming that n=4, we notice that the left-hand side of the equation are a series of multiplications by 2 for n-1 times since f(0) = 0.

On the right-hand side, it's a summation of 1 multiplied by 2 except for the very last one (the one put in brackets). We can write that as:

$f(n) = 2^n - 1$

b) What is ƒ(n) for n = 64?

$f(64) = 2^{64} - 1 = e^{64 \times ln(2)} - 1 = 1.8446744\, e + 19$

Important! Your answer should include both an algebraic solution (the identity of function ƒ) as well as a numeric solution, solving for n=64.