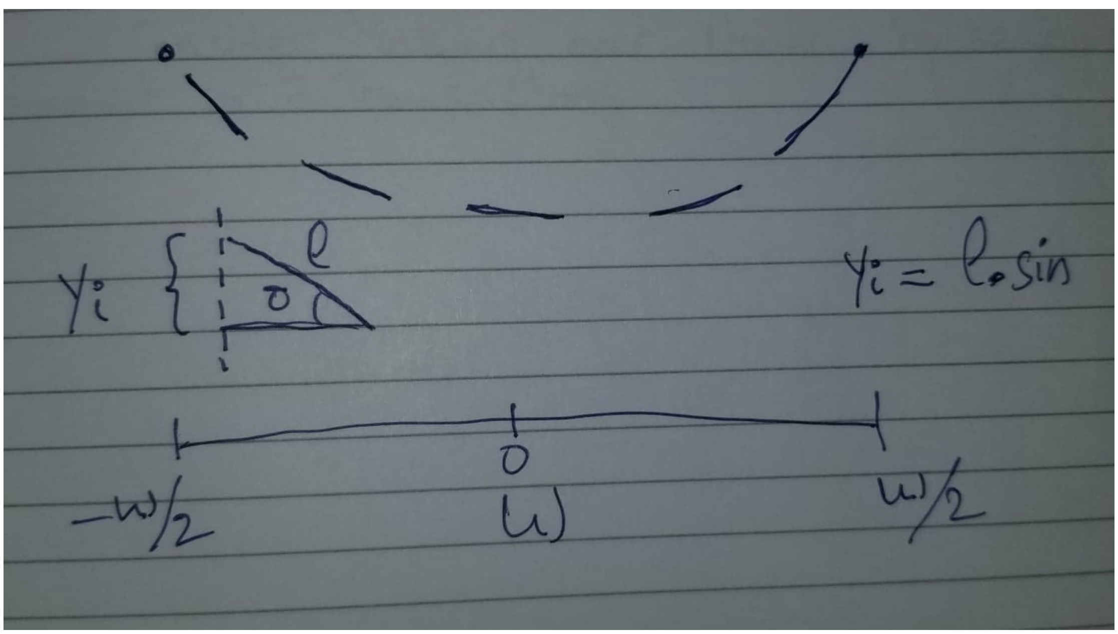
CS164 6.2 PCW

Equality Constraints with Lagrange Multipliers II

(1) A chain comprises N straight segments of length l, each having mass m. Assume that the ends of the chain are anchored at two points along the x axis, namely −w/2 and w/2, with Nl > w. The chain hangs downwards under gravity. We will number the chain segments from i = 1 . . . N, denoting the y coordinate of the left-hand edge of each link by yi. We will define a height yN+1 = 0 for convenience. Finally, we will denote the angle of each segment from the horizontal as θi

(a) Show that the joining constraint for the chain segments can be written as: y_(i+1) = y_i + l * sin(-theta) for all i = 1 . . . N

As illustrated in the drawing below, the height of each link is determined by the product of the length of the segment by the sinusoidal function of the angle between the segment and the horizontal.
For two consecutive links, the relation between them is additive. In other words, as we add up the heights of the link, we get the height of a link i with respect to the original height 0 at y_(n+1)



(b) If the centre of mass of each link lies halfway along its length, show that the centre of mass height for the i_th link is given by:
Y_i = (1/2) * (y_i + y_(i+1))

If we consider a single link a with y coordinates starting at y_i to y_(i+1). According to the given stated above, If the middle of the link is considered the center of gravity then since the links are considered straight segments we get that:
Y_i = (1/2) * (y_i + y_(i+1))

(c) The total potential energy of the chain is given by

$$P = m * g \sum_{i=1}^{N} Y_i$$

for constant g. Given that the chain should hang in a minimum energy configuration, write out (but do not solve) the optimization problem required to find the heights of the chain segments. How many variables and how many Lagrange multipliers are required?

Since the chain has to hang in a minimum energy configuration, then we need to solve for the derivative of the function P to be equal 0

$$P = \frac{1}{2} m * g \sum_{i=1}^{N} (y_i + y_{i+1})$$

$$y_{i+1} = y_i * l * \mathrm{Sin}[\theta_i]$$

$$P = \frac{1}{2} m * g \sum_{i=1}^{N} \left(y_i + y_i * l * \mathrm{Sin}[\theta_i]\right)$$

$$P = \frac{1}{2} m * g \sum_{i=1}^{N} \left(y_i \left(1 + l * \mathrm{Sin}[\theta_i]\right)\right)$$

The above equations mentions how the function of the energy configuration is dependent on only one changing variable which is the angle of each of the links.

However, we can just leave the function as it is

$$P = m * g \sum_{i=1}^{N} Y_i$$

And then consider these functions to be the constraints for the Lagrangian Multiplier:

$$y_{i+1} = y_i * l * \mathrm{Sin}[\theta_i]$$

$$Y_i = \frac{1}{2} (y_{i+1} + y_i)$$

In such a case, we would end up with two Lagrangian multipliers for each link of the chain, hence, we need 2*N multipliers.