CS164 PRECLASS WORK EXERCISES - UNCONSTRAINED MULTIVARIABLE OPTIMIZATION

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Complete the exercises below and be prepared to share your results during the class.

(1) Find and classify the critical points of the following functions
   (a) $f(x, y) = -x^2 + y^2$
   (b) $f(x, y) = x^6 + y^3 + 6x - 12y + 7$
   (c) $f(x, y, z) = (2x^2 + 3y^2 + z^2)e^{-(x^2+y^2+z^2)}$

   For (a)-(b), plot the functions if you are unsure.

(2) A box is made of cardboard with double thick sides, a triple thick bottom, single thick front and back and no top. It's total volume is 3 units. What box dimensions will use the least amount of cardboard? Demonstrate that the dimensions you have found actually minimize the cardboard used.

(3) Given a set of N datapoints of the form (x_i, y_i), where $i = 1, 2, \ldots, N$, use multivariable calculus to find the values of $a \in \mathbb{R}$ and $b \in \mathbb{R}$ such that the following fit error is minimized:

$$e(a, b) = \sum_{i=1}^{N} (ax_i + b - y_i)^2.$$