CS164 PRECLASS WORK EXERCISES - LAGRANGE MULTIPLIERS I

Complete the exercises below and be prepared to share your results during the class.

(1) Use Lagrange Multipliers to find the global *maximum* and *minimum* values of $f(x, y) = x^2 + 2y^2 - 4y$, subject to the constraint $x^2 + y^2 = 9$. Note that no second derivative test is needed here - simply look at the function values at each critical point.

(2) A manufacturer's production is modeled by the Cobb-Douglas function

$$f(x, y) = 100x^{3/4}y^{1/4}$$

where x represents the units of labor, y represents the units of capital and $f(x, y)$ represents the production level. Each labor unit costs $200 and each capital unit costs $250. The total expenses for labor and capital cannot exceed $50,000. Given the form of the cost function, we can be sure that the maximum will occur when the labor and capital expenses are exactly at their maximum total value.

(a) Plot level sets of the production function and total cost constraint to graphically estimate the maximum production level.

(b) Find the maximum production level exactly using the method of Lagrange Multipliers.

(c) Use the Lagrange Multipliers to estimate the increase in maximum production if we allowed the total expense threshold to increase to $51000.