CS164 5.2 PCW

Lagrange Multiplier

Problem One:

First, we find the partial derivatives:

In[∘]:=
```
function[x_, y_] := x^2 + 2 y^2 - 4 y
D[function[x, y], x]
D[function[x, y], y]
```

Out[∘]= $2x$

Out[∘]= $-4 + 4y$

Then we solve for the system of equations:

In[∘]:=
```
Solve[{2 x == L * 2 x}, {x, L}]
```

Out[∘]= $\{\{L \to 1\}, \{x \to 0\}\}$

In[∘]:=
```
Solve[{-4 + 4 y == L * 2 y}, {y}]
```

Out[∘]= $\left\{\left\{y \to -\frac{2}{-2 + L}\right\}\right\}$

In[∘]:= $\text{Solve}\left[\left(-\frac{2}{-2 + L}\right)\text{^}2 == 9, \{L\}\right]$

Out[∘]= $\left\{\left\{L \to \frac{4}{3}\right\}, \left\{L \to \frac{8}{3}\right\}\right\}$

The case where x = 0:

In[∘]:= $\text{function}\left[0, -\frac{2}{-2 + \frac{4}{3}}\right]$

Out[∘]= 6

In[∘]:= $\text{function}\left[0, -\frac{2}{-2 + \frac{8}{3}}\right]$

Out[∘]= 30

The case where L = 1:

The value of y would be (-2) if we plug in this value into the constraint function then the value of x would be ($\sqrt{5}$).

After inserting these values into the objective function, the output would be (5) which is considered to be the minimum under the given constraint.

We can then check the results on Mathematica built in function:

In[ ]:=
```
Minimize[x^2 + 2 y^2 - 4 y, x^2 + y^2 == 9, {x, y}]
```

Out[ ]= $\left\{5, \left\{x \to -\sqrt{5}, y \to 2\right\}\right\}$

In[ ]:=
```
Maximize[x^2 + 2 y^2 - 4 y, x^2 + y^2 == 9, {x, y}]
```

Out[ ]= $\{30, \{x \to 0, y \to -3\}\}$

Problem Two:

Cobb-Douglas function under constraints:

In[ ]:=
```
CobbDouglas[x_, y_] := 100 x^(3/4) * y^(1/4)
D[CobbDouglas[x, y], x]
D[CobbDouglas[x, y], y]
```

Out[ ]= $\frac{75\, y^{1/4}}{x^{1/4}}$

Out[ ]= $\frac{25\, x^{3/4}}{y^{3/4}}$

In[ ]:=
```
constraint[x_, y_] := 200 x + 250 y
D[constraint[x, y], x]
D[constraint[x, y], y]
```

Out[ ]= 200

Out[ ]= 250

In[ ]:=
```
Solve[D[CobbDouglas[x, y], x] == L * D[constraint[x, y], x], {x}]
```

Out[ ]= $\left\{\left\{x \to \frac{81\, y}{4096\, L^4}\right\}\right\}$

In[ ]:=
```
Solve[D[CobbDouglas[x, y], y] == L * D[constraint[x, y], y], {y}]
```

Out[ ]= $\left\{\left\{y \to \frac{x^{3/4}}{10 \times 10^{1/3}\, L \left(\frac{L}{x^{3/4}}\right)^{1/3}}\right\}\right\}$

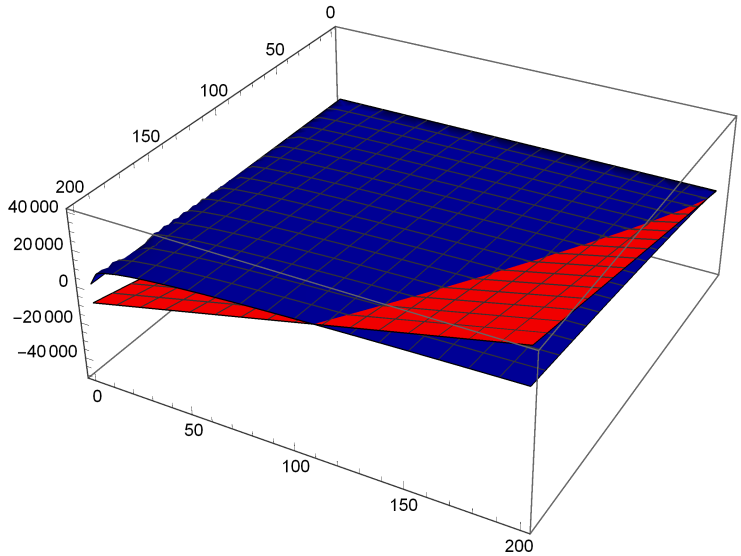
In[ ]:= $\text{Solve}\left[\text{constraint}\left[\frac{81\, y}{4096\, L^4}, y\right] == 50\,000, \{y\}\right]$

Out[ ]= $\left\{\left\{y \to \frac{1\,024\,000\, L^4}{81 + 5120\, L^4}\right\}\right\}$

In[ ]:= $\text{Solve}\left[\text{constraint}\left[x, \frac{x^{3/4}}{10 \times 10^{1/3}\, L \left(\frac{L}{x^{3/4}}\right)^{1/3}}\right] == 50\,000, \{x\} \in \text{Reals}\right]$

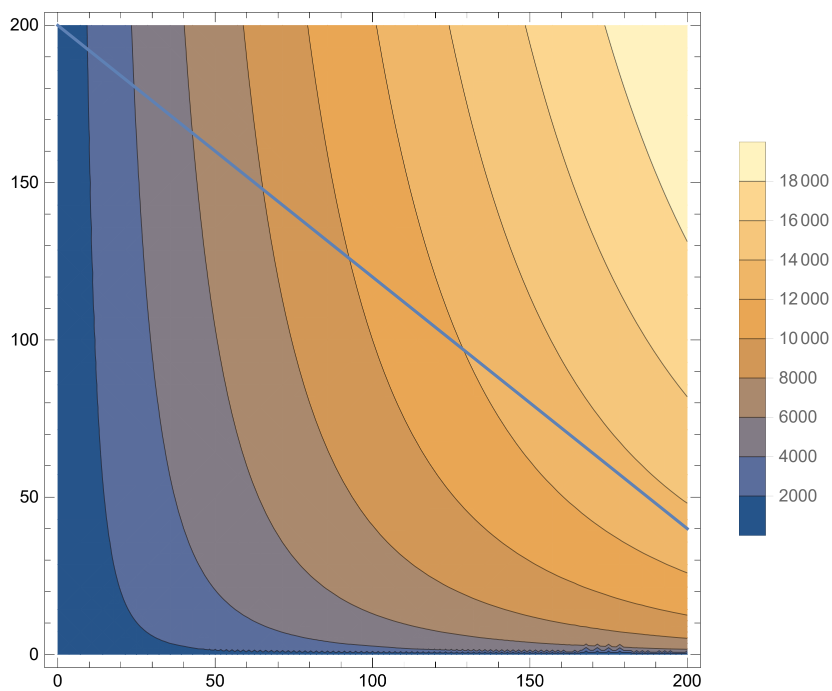
Out[ ]= $\text{Solve}\left[\frac{5 \times 5^{2/3}\, x^{3/4}}{2^{1/3}\, L \left(\frac{L}{x^{3/4}}\right)^{1/3}} + 200\, x == 50\,000, x \in \mathbb{R}\right]$

In[ ]:=
```
Show[Plot3D[CobbDouglas[x, y], {x, 0, 200}, {y, 0, 200}, PlotStyle → Blue],
 Plot3D[200 x + 250 y == 50000, {x, 0, 200}, {y, 0, 200}, PlotStyle → Red],
 PlotRange → Automatic]
```

Out[ ]=


The contour plot of the Cobb-Douglas function and the constraint:

In[ ]:=
```
Show[ContourPlot[CobbDouglas[x, y], {x, 0, 200}, {y, 0, 200}, PlotLegends → Automatic],
 ContourPlot[200 x + 250 y == 50000, {x, 0, 200}, {y, 0, 200}], PlotRange → Automatic]
```

Out[ ]=


In[ ]:=
```
Maximize[{CobbDouglas[x, y], 200 x + 250 y == 50000}, {x, y}]
```

Out[ ]= $\left\{-100\,\mathrm{Root}\left[-1318359375 + 4\,\#1^4\,\&,\,1\right],\ \left\{x \to \frac{375}{2},\ y \to 50\right\}\right\}$

```
In[•]:= Evaluate[CobbDouglas[375/2, 50]]
```

Out[•]= $1250\sqrt{2}\ 15^{3/4}$