# Unconstrained Optimization Algorithms:

## Gradient Descent & Line Searching

In [1]:
```
# Importing packages
import matplotlib.pyplot as plt
from numpy import linalg as lp
import numpy as np
import math

%matplotlib inline
```

Exmaple function from section 4.2 of Freund, R.M. (2004).

$$f\left(x\right)=\frac{1}{2}x^TQx-c^Tx+10$$

$$f\left(x\right)=\frac{1}{2}\times\begin{pmatrix}x_1\\x_2\end{pmatrix}^T\times\begin{pmatrix}20&5\\5&2\end{pmatrix}\times\begin{pmatrix}x_1\\x_2\end{pmatrix}-\begin{pmatrix}14\\6\end{pmatrix}^T\times\begin{pmatrix}x_1\\x_2\end{pmatrix}-10$$

$$f\left(x\right)=2\left(5+10x_1^2-3x_2+x_2^2+x_1\left(-7+5x_2\right)\right)$$

In [2]:
```
def function(x):
    return 2*(5+10*x[0]**2-3*x[1]+x[1]**2+x[0]*(-7+5*x[1]))
```

In [3]:
```
def gradient(x):
    value = np.full(len(x), 0)
    xd = x.copy()
    h = 10**-6
    for j in range(len(x)):
        xd[j] = x[j] + h
        value[j] = (function(xd) - function(x))/h
        xd = x.copy()
    return value
```

In [29]:
```
def step_size(x):
    alpha = 1
    beta = 0.8
    while function(x-alpha*gradient(x)) > \
    (function(x)-.5*alpha*lp.norm(gradient(x))**2):
        alpha *= beta
    return alpha
```

**Steepest Descent Algorithm:**

**Step 0.** Given $x^0$, set $k := 0$

**Step 1.** $d^k := -\nabla f(x^k)$. If $d^k = 0$, then stop.

**Step 2.** Solve $\min_\alpha f(x^k + \alpha d^k)$ for the stepsize $\alpha^k$, perhaps chosen by an exact or inexact linesearch.

**Step 3.** Set $x^{k+1} \leftarrow x^k + \alpha^k d^k, k \leftarrow k + 1$. Go to **Step 1.**

Note from Step 2 and the fact that $d^k = -\nabla f(x^k)$ is a descent direction, it follows that $f(x^{k+1}) < f(x^k)$.

In [27]:
```
"""
I would put the starting point at [0, 0]
but had to follow the example from Freund, R.M. (2004).
"""
x = [40, -100]
x1 = np.array([])
x2 = np.array([])

pre, k = 0, 0

# Gradient-Descent Algorithm
while abs(function(x) - pre) > 0:
    pre = function(x)
    x1 = np.append(x1, x[0])
    x2 = np.append(x2, x[1])

    alpha = step_size(x)
    x -= alpha*gradient(x)
    k += 1

print("Function Value: ", function(x), "\nAt x = ", x, "in", k, "iterations")
```

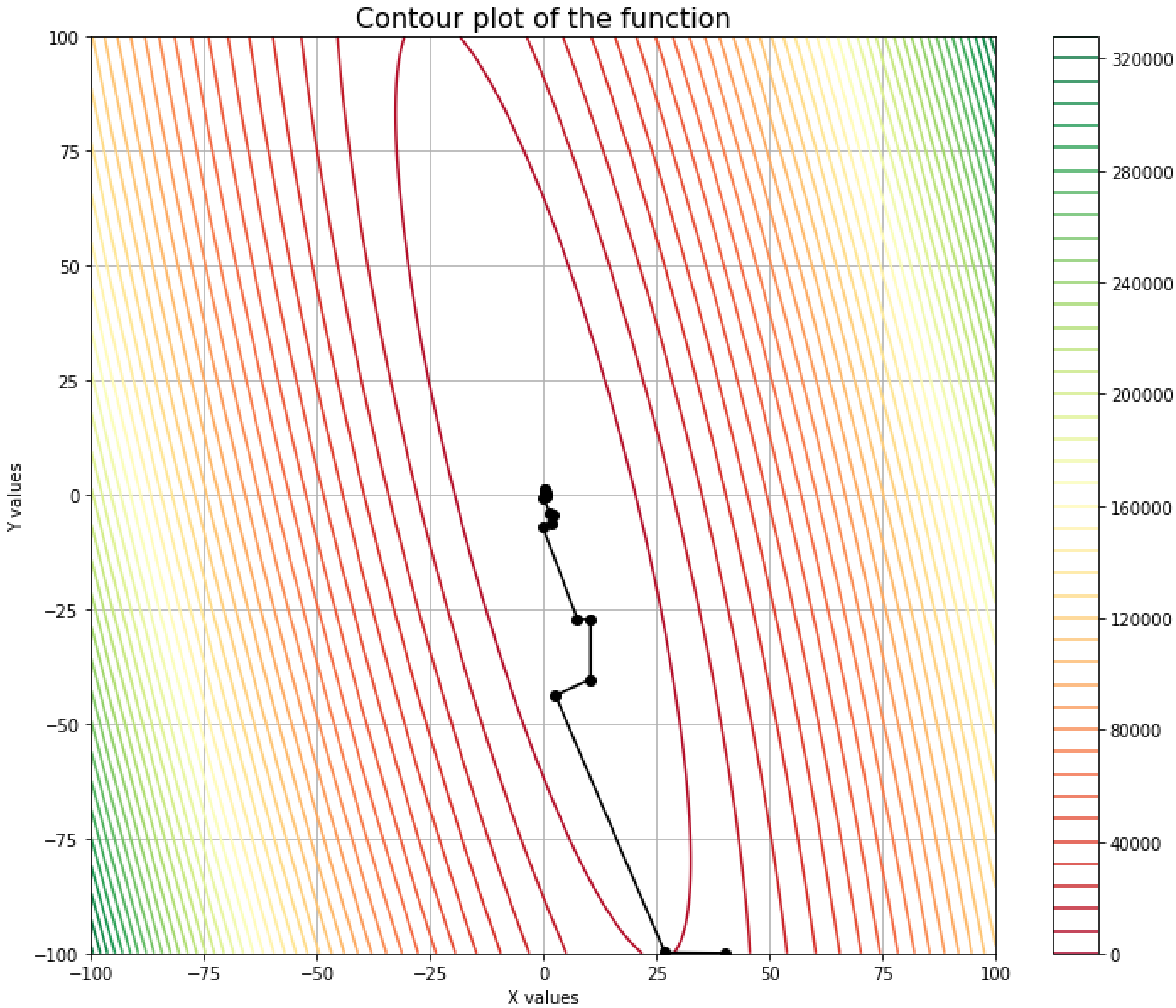
```
Function Value:  5.801090572384302
At x =  [0.10564042 0.99951905] in 20 iterations
```

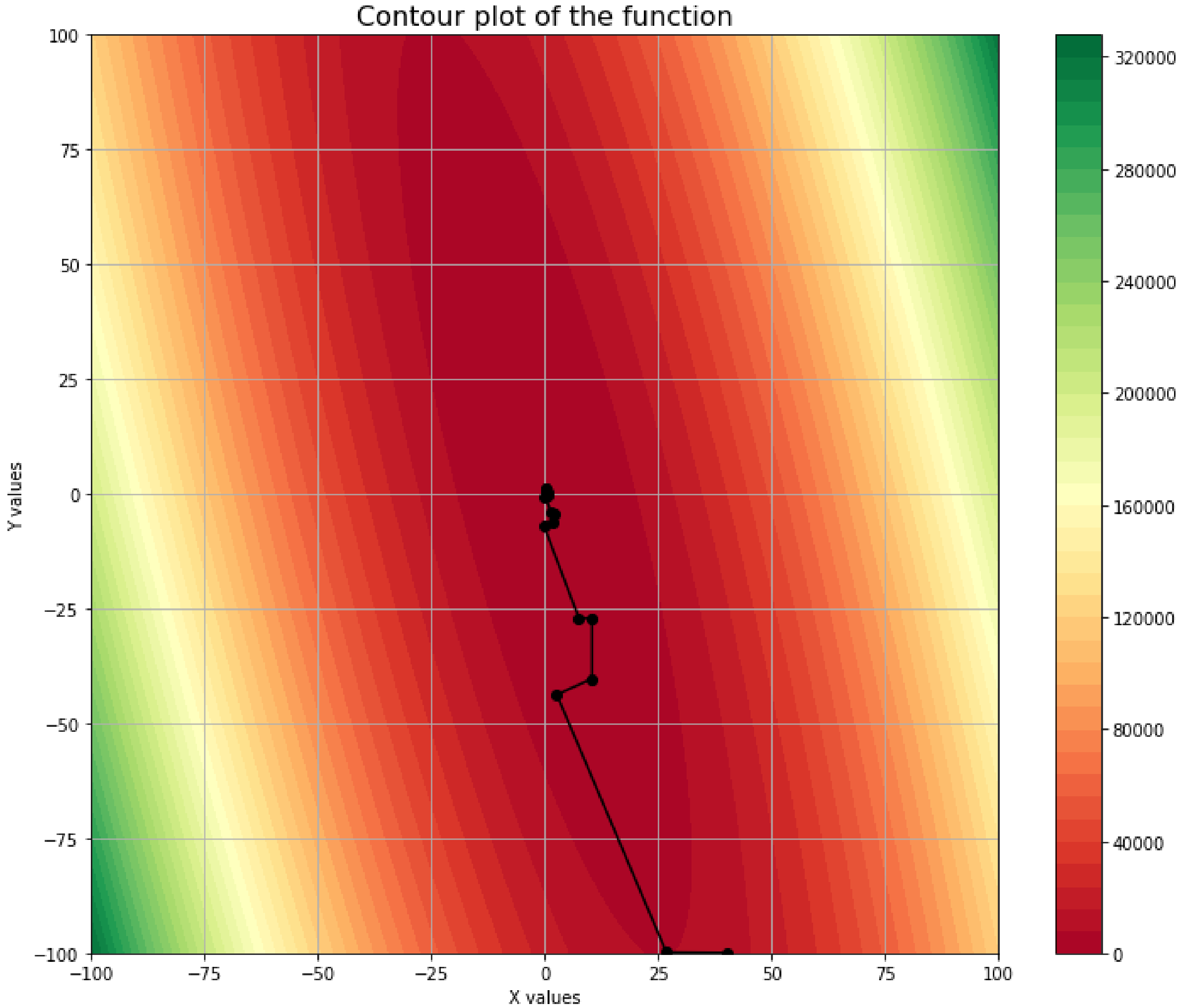
In [28]:

```
"""
Contour plot of a function
Ref: https://jakevdp.github.io/PythonDataScienceHandbook/04.04-density-and-con
tour-plots.html
"""
x = np.arange(-100, 100, 0.05)
y = np.arange(-100, 100, 0.05)

X, Y = np.meshgrid(x, y)
Z = 2*(5+10*X**2-3*Y+Y**2+X*(-7+5*Y))

for _ in [plt.contour, plt.contourf]:
    plt.figure(figsize=(12, 10))
    _(X, Y, Z, 50,cmap='RdYlGn')
    plt.plot(x1, x2, 'o-', color='black')
    plt.xlabel('X values')
    plt.ylabel('Y values')
    plt.title('Contour plot of the function', fontsize=16)
    plt.colorbar()
    plt.grid(True)
    plt.show()
```


Contour plot of the function
Y values
X values
100
75
50
25
0
−25
−50
−75
−100
−100
−75
−50
−25
0
25
50
75
100
320000
280000
240000
200000
160000
120000
80000
40000
0


Contour plot of the function
X values
Y values
100
75
50
25
0
−25
−50
−75
−100
−100
−75
−50
−25
0
25
50
75
100
320000
280000
240000
200000
160000
120000
80000
40000
0

## Checking the results with Mathematica

```
NMinimize[2 (5 + 10 x^2 - 3 y + y^2 + x (-7 + 5 y)), {x, y}]
```

```
{5.46667, {x → -0.0666667, y → 1.66667}}
```

```
Show[Plot3D[2 (5 + 10 x^2 - 3 y + y^2 + x (-7 + 5 y)),
   {x, -2, 1}, {y, -1, 3}, PlotStyle → Opacity[0.3]],
  Graphics3D[{Black, PointSize[0.02],
    Point[{-0.06667, 1.66667, 5.46667}]}]]
```