CS164 PRECLASS WORK EXERCISES - LAGRANGE MULTIPLIERS II

Complete the exercises below and be prepared to share your results during the class.

(1) A chain comprises N straight segments of length l, each having mass m. Assume that the ends of the chain are anchored at two points along the x axis, namely $-w/2$ and $w/2$, with $Nl > w$. The chain hangs downwards under gravity. We will number the chain segments from $i = 1 \dots N$, denoting the y coordinate of the left-hand edge of each link by y_i. We will define a height $y_{N+1} = 0$ for convenience. Finally, we will denote the angle of each segment from the horizontal as θ_i.

(a) Show that the joining constraint for the chain segments can be written as

$$y_{i+1} = y_i + l \sin \theta_i,$$

for all $i = 1 \dots N$.

(b) If the centre of mass of each link lies halfway along its length, show that the centre of mass height for the ith link is given by

$$\bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$$

(c) The total potential energy of the chain is given by

$$P = mg \sum_{i=1}^{N} \bar{y}_i,$$

for constant g. Given that the chain should hang in a minimum energy configuration, write out (but do not solve) the optimization problem required to find the heights of the chain segments. How many variables and how many Lagrange multipliers are required?