CS164 3.2 PCW

Sublevel Sets and Coercive

Pre-Class Work

(1) For each of the functions below, plot a diagram of their level sets and determine whether or not the functions are coercive. You may use a graphical package to plot the level sets.
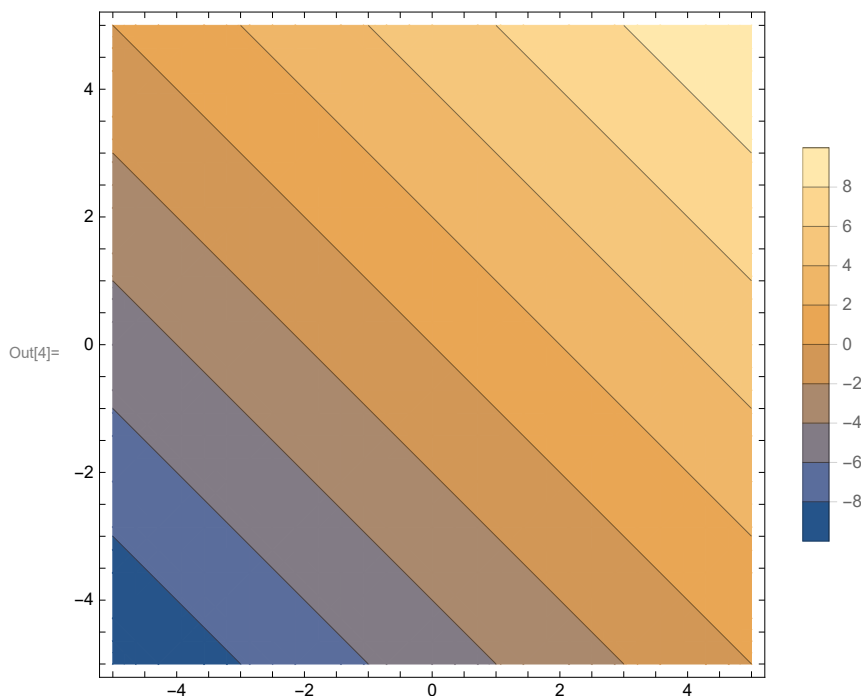
The function $f(x, y) = x + y$
is not coercive since the output is constant as the input $||X||$ tends towards infinity. As a general rule, all linear functions are not coercive since they can be written in the form:
$f(x, y) = ax + by + c$
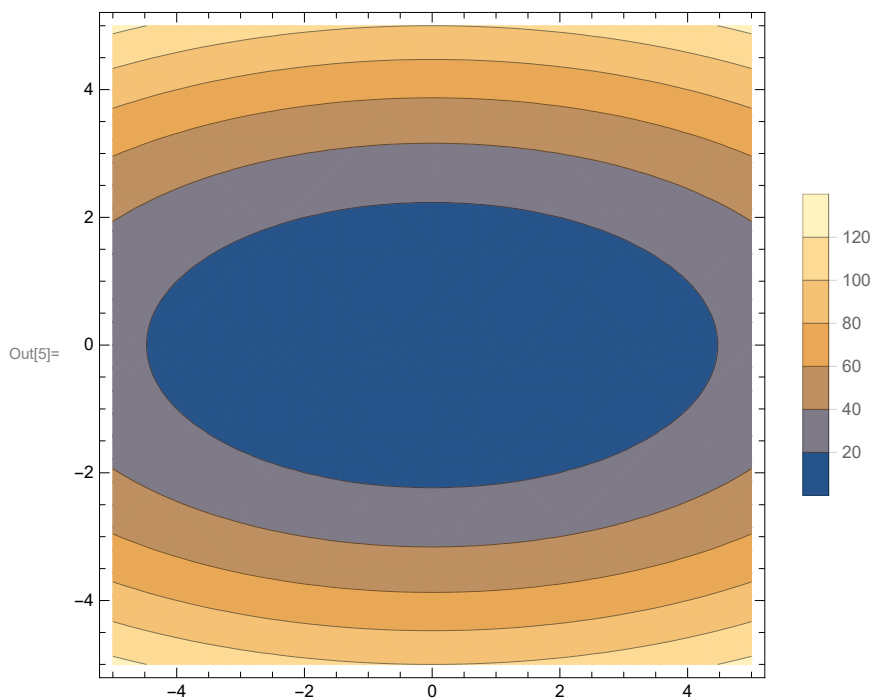As $||X||$ goes to infinity, the output of the function $f(x, y)$ converges towards c. Thus $f(x, y)$ is not coercive.

In[4]:= **ContourPlot[x + y, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]**

Out[4]=

The function $f(x, y) = x^2 + 4*y^2$
The function is coercive since when the norm $||X|| = Sqrt(x^2 + y^2)$
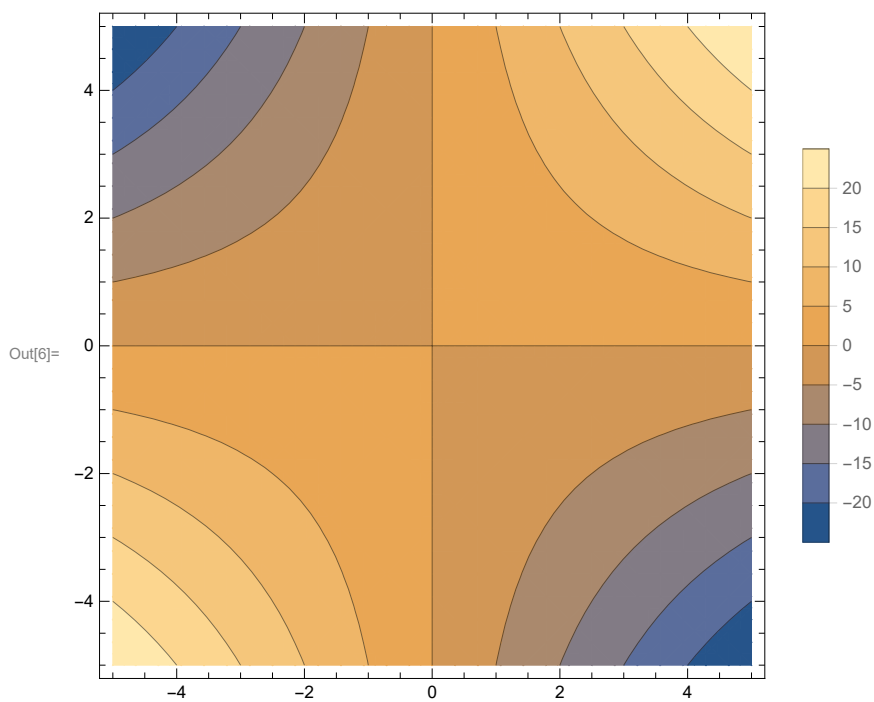tends towards infinity, the output of the function f goes to infinity.

In[5]:= **ContourPlot[x^2 + 4 * y^2, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]**

Out[5]=

The function $f(x, y) = x * y$

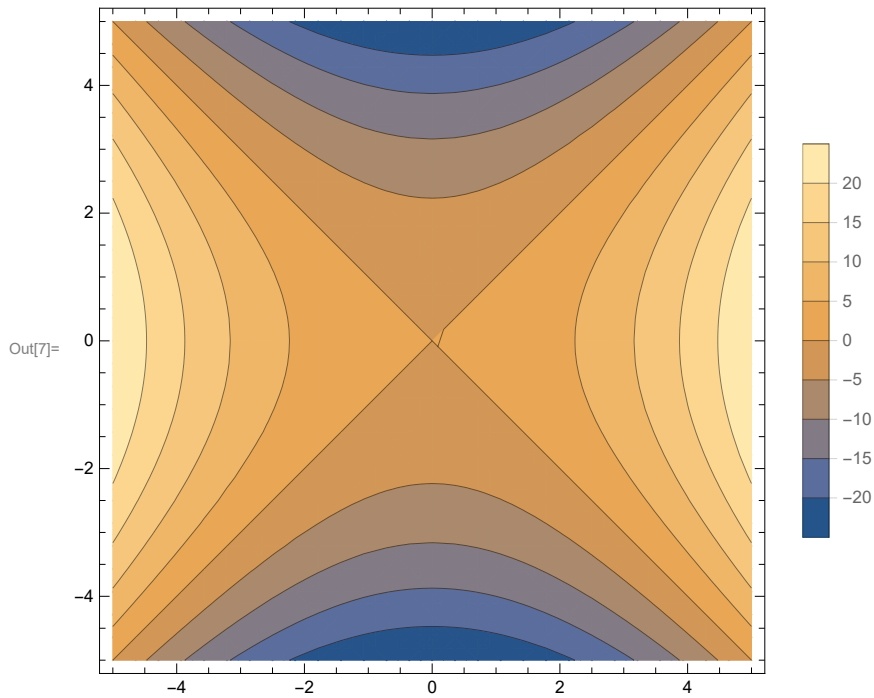The function is not coercive since it goes to infinity on some path on which the norm tends to infinity.

In[6]:= **ContourPlot[x * y, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]**

Out[6]=

The function $f(x, y) = x^2 - y^2$ is not coercive

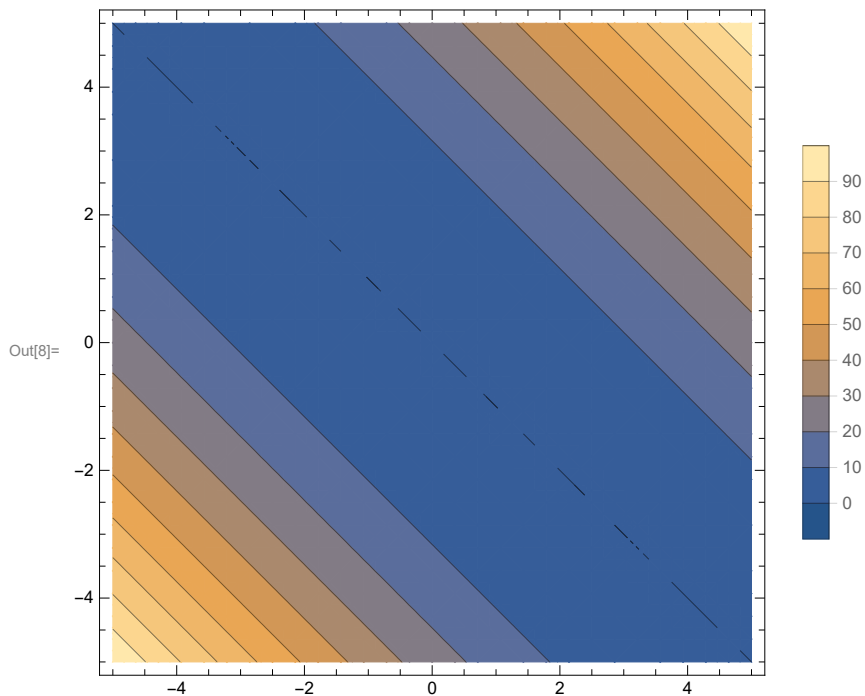Since the function is equal to 0 when x = y then the output of the function is constant along the line x = y

In[7]:= `ContourPlot[x^2 - y^2, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]`

Out[7]=

The function f(x, y) = x^2+2*x*y+y^2 = (x + y)^2

Coercive, as the norm tends to infinity, the output of the function also goes to infinity

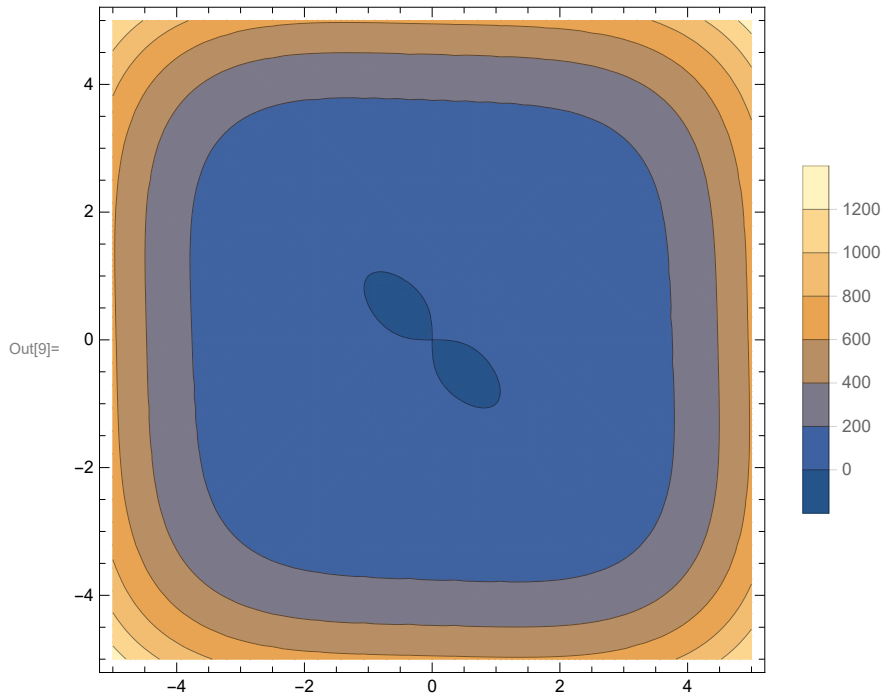In[8]:= `ContourPlot[x^2 + 2 * x * y + y^2, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]`

Out[8]=

The function f(x, y) = x^4+2*x*y+y^4

The function is coercive since it converges towards infinity on all path where the norm goes to infinity.

In[9]:= **ContourPlot[x^4 + 2 * x * y + y^4, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]**

Out[9]=

(2) Prove that the following function is coercive and describe its sublevel sets mathematically (no need to plot them).

The norm of this function is written as:

||X|| = Sqrt(x^2 + y^2 + z^2)

We can transform the function to be:

f(||X||) = Exp(||X||^2)

As the norm ||X|| goes to infinity, the function f(||X||) tends to infinity.

The sublevel sets are in the form of spheres as shown in the plot below.

In[19]:= `ContourPlot3D[Exp[x² + y² + z²], {x, -1/4, 1/4},`
`{y, -1/4, 1/4}, {z, -1/4, 1/4}, PlotLegends → Automatic]`

Out[19]=

■	1.05156
■	1.10312
■	1.15467