CS164 PRECLASS WORK EXERCISES - QUADRATIC PROGRAMMING

In this exercise, we will look at the problem of portfolio optimization, which we briefly investigated in lesson 1.1. We will now use a more realistic model, where we consider risk levels and constraints on minimum rates of return. Suppose that we have a vector $x \in \mathbb{R}^N$, where the ith component x_i represents the fraction of our capital that we have invested in asset i. We treat the rate of return of each asset as a random variable, where the mean rate of return is represented by a vector $\mu \in \mathbb{R}^N$, and the covariance matrix for the rates of return over all assets is denoted $C \in \mathbb{R}^{N \times N}$. One way to allocate assets is to minimize risk, subject to our portfolio making some minimum rate or return.

(1) Explain why the quadratic form $\frac{1}{2}x^T C x$ provides a measure of overall portfolio risk. This is what we want to minimize

(2) Let r denote the minimum rate of return for the portfolio. Explain why this translates into a constraint

$$\mu^T x \geq r$$

(3) Explain why we additionally need the (element-wise) constraint $x \geq 0$ and the equality constraint

$$\sum_i x_i = 1$$

(4) A dataset of 225 different assets can be found here. The first line of the file tells us the number of assets (225). The next 225 lines list the mean rate of return and standard deviation for each of the 225 assets. The final 113×225 lines tell us the *correlation* between the rates of return of the different assets: the first and second

column are two assets i and j, and the third column is the correlation between asset i and asset j. Note that only the upper triangle of this matrix is specified, since correlations must be symmetric.

(a) Load the data into Python (pre-processing if necessary) and create a vector μ for the mean rate of return, a vector σ for the standard deviations, and a matrix K for the correlations.

(b) Compute the covariance matrix C by using the identity

$$C_{ij} = K_{ij}\sigma_i\sigma_j$$

(c) Using the cost and constraints described above, create and solve a quadratic program using CVXPY to find the optimal asset allocation, assuming a minimum return rate of 0.2%. Are there some assets in which we practically would not invest in this case?