Kelly Criterion Experiment

November 29, 2019

#

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A simulation inspired by the work of Victor Haghani and Richard Dewey in their paper: **Rational Decision-Making Under Uncertainty: Observed Betting Patterns on a Biased Coin** retrieved from: https://arxiv.org/pdf/1701.01427.pdf

0.0.1 Description of the experiment:

Starting by 25$ in their balance, the participants were told that a simulated coin is likely to land heads 60% and fall tails 40% of the time. Furthermore, the rule is that you place any amount you fancy on either heads or tail. A win douples the stake of money you bet, a loss substracts the amount they bet from their balance.

The game ends if: - 1 The participant bust with less than 2 dollars in their balance. - 2 The participant reaches a maximum payout of 250$ (10 folds the starting balance). - 3 The amount of time they spend playing the game hits 30 minutes.

0.0.2 Optimal strategy

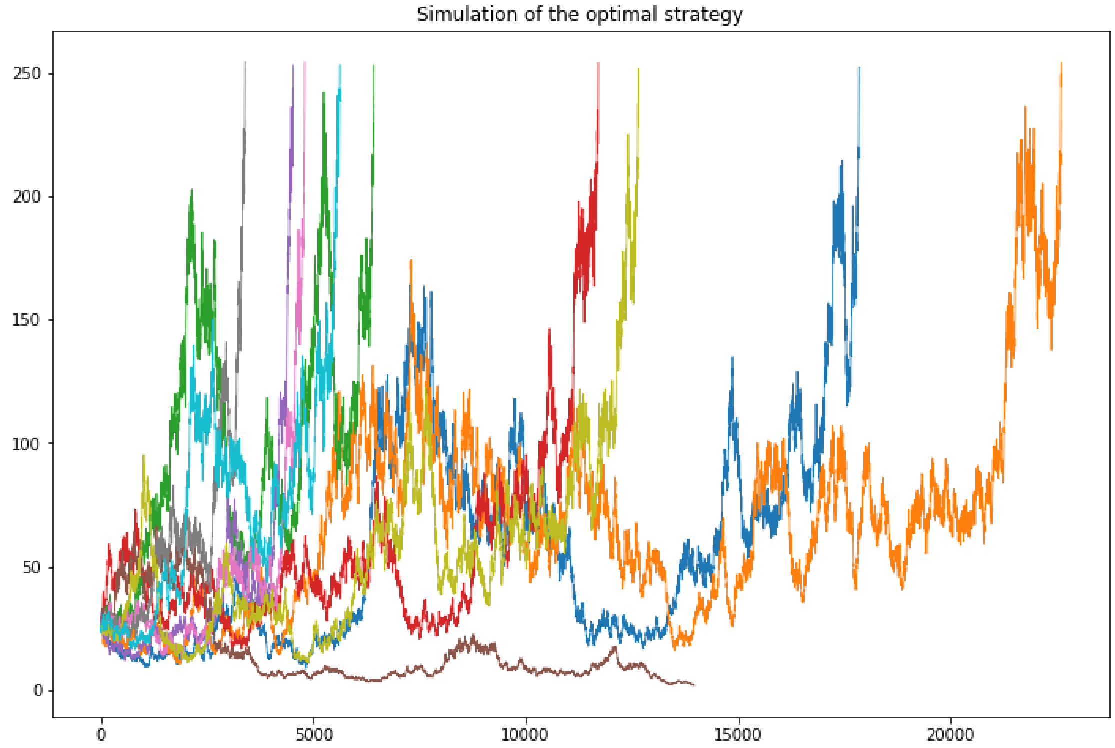
According to Kelly criterion, the subject is supposed to bet $[2 \cdot (0.6 - 1)] = 20\%$ of their balance on heads on each flip.

```
In [75]: import random as rn
         from math import *
         import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt
         %matplotlib inline

         # Function to impliment a biased coin
         def flip(p):
             return "H" if rn.random() < p else "T"

         summ = []
         for _ in range(10**3):
             summ.append(flip(0.6))

         print('Count of landing heads: ', summ.count("H"),
               '\nCount of landing tails: ', summ.count("T"))
```

```
Count of landing heads:  581
Count of landing tails:  419
```

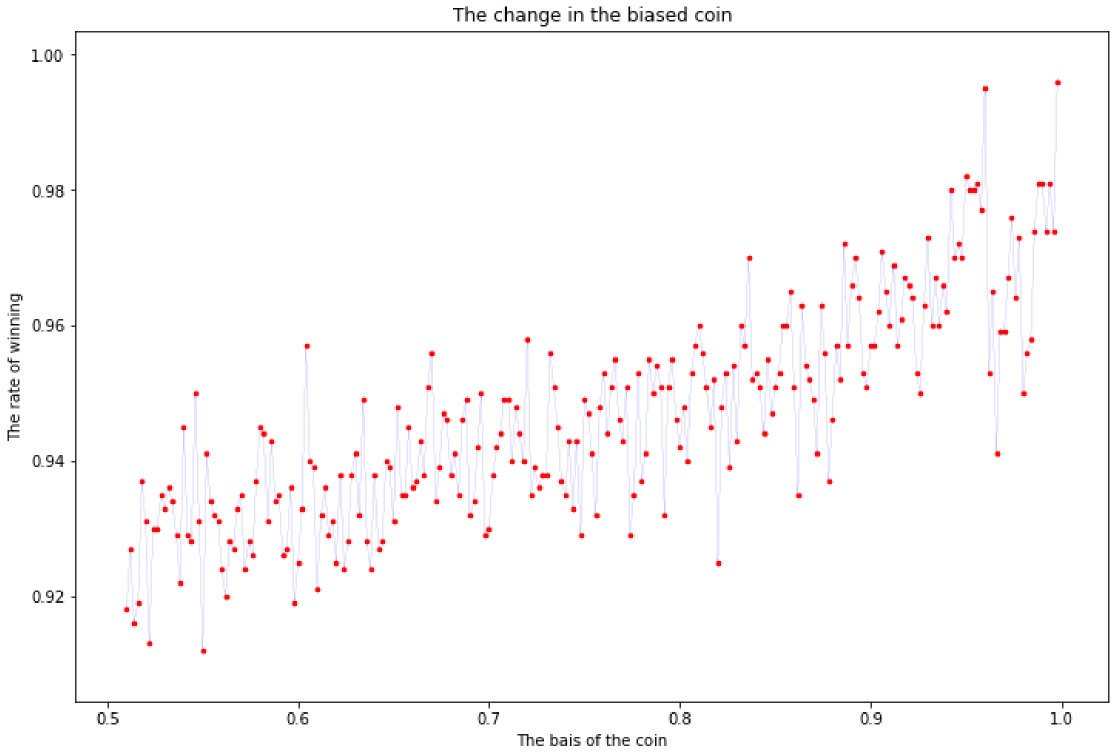
```
In [80]: # Playing the optimal strategy
         """
         Following the rule of Kelly at each trial, the participant
         should bet 20% of the balance on heads every time.
         This method would maximize the outcome of the play.
         """

         def flip_coin(p):
             return True if rn.random() < p else False

         N, pr, tracks = 10, .51, []
         for _ in range(N):
             track = []
             balance = 25
             while balance > 2 and balance < 250:
                 bet = (2*pr-1) * balance
                 if flip_coin(pr):
                     balance += bet
                 else:
                     balance -= bet
                 track.append(balance)
             tracks.append(track)

         won, bust = 0, 0
         for _ in tracks:
             if _[-1] < 2:
                 bust +=1
             else:
                 won +=1
         print('\nThe number of busts: ', bust,
               '\nThe number of wins: ', won)

         plt.figure(figsize=(12, 8))
         for _ in tracks:
             plt.plot(_, linewidth=.8)
         plt.title('Simulation of the optimal strategy')
         plt.show()
```

```
The number of busts:  1
The number of wins:  9
```



```
In [133]: # An attempt to generalize the idea:

          # I. Changing the bias of the coin to test the Kelly rule

          def flip_coin(p):
              return True if rn.random() < p else False

          N = 1000
          results = []
          for pr in range(510, 1000, 2):
              bust, win = 0, 0
              for _ in range(N):
                  balance = 25
                  while balance > 2 and balance < 250:
                      bet = (2*(pr/1000)-1) * balance
                      if flip_coin(pr/1000):
                          balance += bet
                      else:
                          balance -= bet
                  if balance < 2:
                      bust += 1
                  else:
                      win += 1
```

```
    results.append([pr/1000, bust, win, win/(win+bust)])

results = pd.DataFrame(results)
results.columns=['Bias', 'Bust', 'Win', 'Rate']

plt.figure(figsize=(12, 8))
plt.scatter(results['Bias'], results['Rate'], s=7, color='red')
plt.plot(results['Bias'], results['Rate'], linewidth=.1, color='blue')
plt.title('The change in the biased coin')
plt.xlabel('The bais of the coin')
plt.ylabel('The rate of winning')
plt.show()
```

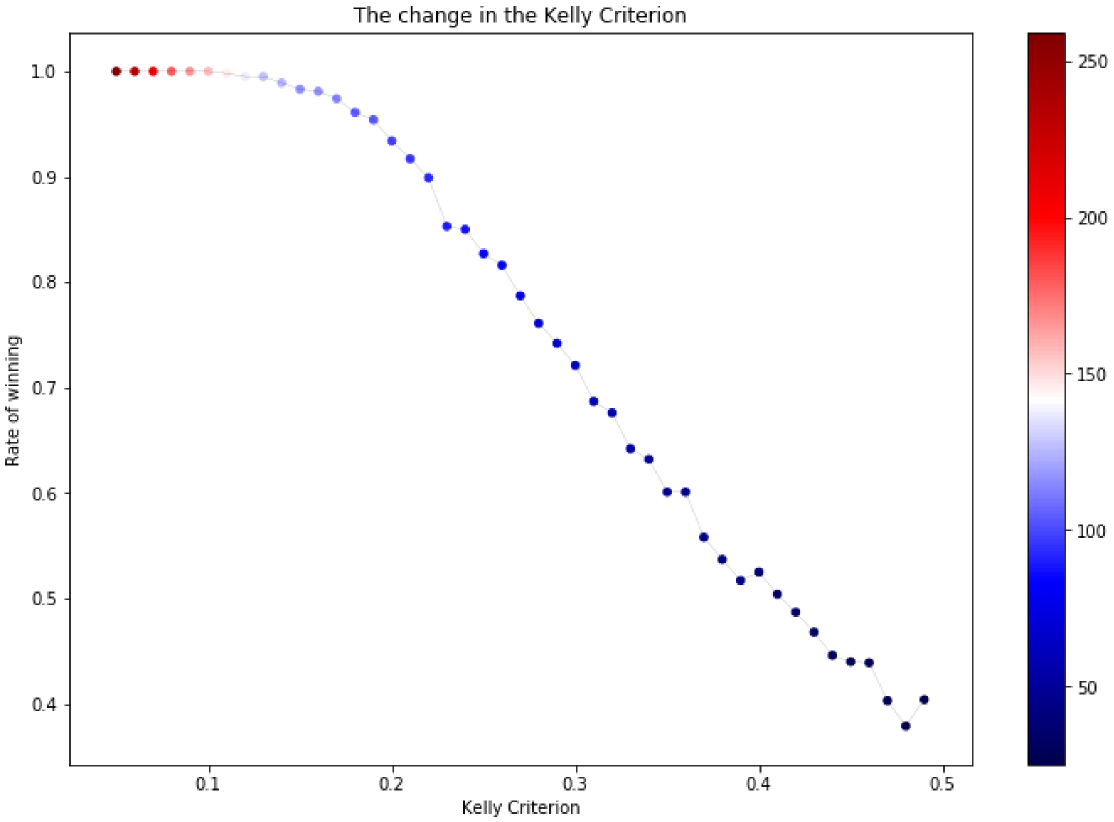


```
In [ ]: # TODO:
        # re-visualize the performance using the Kelly criterion
        # tune the bias of the coin
        # tune the number of tries before maxing out
        # tune the interval of the kelly criterion

In [171]: # Tunning the interval of the Kelly Criterion
          """
          Holding the bias at .6 for the coin to land head
          Changing the Kelly Criterion between 10 to 30%
          """

          N, pr = 1000, 0.6
          result = []

          for i in range(5, 50):
              bust, win = 0, 0
              record = []
              for _ in range(N):
                  balance = 25
                  trials = 0
                  while balance > 2 and balance < 250:
                      trials += 1
                      bet = (i/100) * balance
                      if flip_coin(pr):
                          balance += bet
                      else:
                          balance -= bet
                  if balance < 2:
                      bust += 1
                  else:
                      win += 1
                  record.append(trials)
              result.append([i/100, int(np.mean(record)), bust, win, win/(win+bust)])
          result = pd.DataFrame(result)
          result.columns=['Percentage', 'Average trials',
                          'Bust', 'Win', 'Rate']
          result.head(10)
```

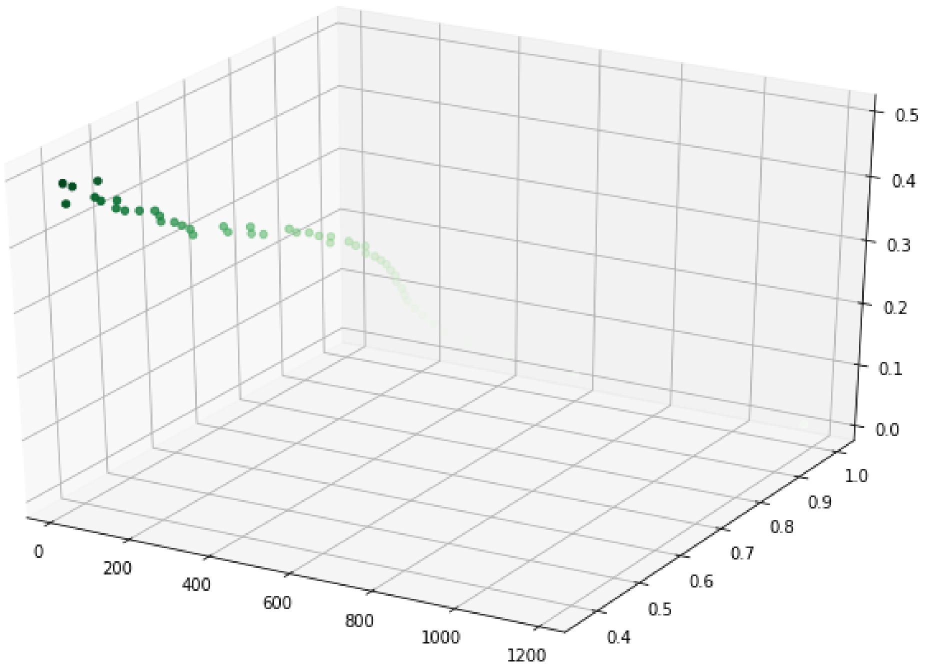
```
Out[171]:    Percentage  Average trials  Bust   Win   Rate
          0        0.05             259     0  1000  1.000
          1        0.06             230     0  1000  1.000
          2        0.07             206     0  1000  1.000
          3        0.08             179     0  1000  1.000
          4        0.09             167     0  1000  1.000
          5        0.10             157     0  1000  1.000
          6        0.11             146     2   998  0.998
          7        0.12             138     5   995  0.995
          8        0.13             128     5   995  0.995
          9        0.14             125    11   989  0.989
```

```
In [172]: plt.figure(figsize=(12, 8))
          jet=plt.get_cmap('seismic')

          plt.scatter(result['Percentage'], result['Rate'],
                      c= result['Average trials'], s=20, cmap=jet)
          plt.plot(result['Percentage'], result['Rate'],
                  linewidth=.1, color='black')
```

```
plt.title('The change in the Kelly Criterion')
plt.ylabel('Rate of winning')
plt.xlabel('Kelly Criterion')
plt.colorbar()
plt.show()
```



```
In [149]: from mpl_toolkits import mplot3d
          plt.figure(figsize=(12, 8))
          ax = plt.axes(projection='3d')
          zdata = result['Percentage']
          xdata = result['Average trials']
          ydata = result['Rate']
          ax.scatter3D(xdata, ydata, zdata, c=zdata, cmap='Greens')
          plt.show()
```


0.5
0.4
0.3
0.2
0.1
0.0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0
200
400
600
800
1000
1200