

Final Project Report:

Autonomous Highway Overtaking with Reduced Lane Widths

Reproduction of Collision Metric C and STPA Safety Analysis

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December 17, 2025

Abstract

This report combines two complementary perspectives on the safety of an autonomous overtaking scenario under reduced lane widths. First, we reproduce and verify the collision metric C introduced in the paper *An Investigation into the Safety of Autonomous Vehicles on Highways with Reduced Lane Widths* [1] by reimplementing the described methodology in MATLAB/Simulink and comparing the resulting time histories against the published straight and curved figures. Second, we apply System-Theoretic Process Analysis (STPA) [2, 3] to the same scenario to identify hazards, Unsafe Control Actions (UCAs), and controller-level safety constraints, and we close the loop by injecting a representative UCA into simulation.

Across both parts, the central focus is how reduced lane width, Virtual Boundary (VB) logic, and control decisions interact to create or mitigate unsafe conditions in overtaking.

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1 Introduction

Reduced lane widths are increasingly considered in road design and traffic management, but they also reduce safety margins for automated driving functions such as overtaking. This project studies an autonomous overtaking scenario with two vehicles: a lead vehicle (AV_a) and an overtaking vehicle (AV_b), operating on highway segments whose lane width is reduced from a nominal 4.0 m to narrower values (e.g. 3.75 m, 3.50 m, 3.25 m, 3.00 m).

The work is organised in two parts:

- **Part I (Reproduction and verification)** reimplements the collision metric C and VB logic described in the reference paper, reproduces straight and curved scenarios, and checks analytically whether a strong lane-width dependence should arise under the stated definitions.
- **Part II (STPA and fault injection)** applies STPA to the same control structure, derives UCAs and safety constraints, and injects one UCA in simulation to observe safety degradation in measurable signals (separation, VB overlap, collision metric).

Throughout the report, we use AV_a / AV_a for the lead vehicle and AV_b / AV_b for the overtaking vehicle; both notations refer to the same two vehicles, depending on whether the discussion is equation-based (Part I) or control-structure-based (Part II).

Part I

Reproducing the Collision Metric Under Reduced Lane Widths

2 Project objectives and scope

The objectives of this project are:

- to reimplement the collision metric $C(t)$ and Virtual Boundary (VB) overlap logic described in the reference paper;
- to reproduce the straight & curved-highway scenario for lane widths $w \in \{4.0, 3.75, 3.5, 3.25, 3.0\}$ m;
- to validate analytically whether $C(t)$ should depend on w under the published definitions.

3 Methodology from the paper: Virtual Boundaries and collision metric

We consider two autonomous vehicles, AV_a and AV_b , travelling on a straight, two-lane highway of lane width w . Their lateral and longitudinal positions are

$$(x_a(t), y_a(t)), \quad (x_b(t), y_b(t)).$$

The relative longitudinal and lateral separations (equations (1)–(2) in the paper [1]) are

$$y_s(t) = y_b(t) - y_a(t), \tag{1}$$

$$x_s(t) = x_b(t) - x_a(t). \tag{2}$$

The paper introduces longitudinal VBs (LL, ML, UL) in front of and behind each vehicle, with distances such as U_{af} and U_{br} obtained from a speed-dependent table. In addition, it defines lateral side VBs on the left and right. Crucially, the text states that for each lane

$$S_{nl} = S_{nr} = \frac{w}{2}, \quad (3)$$

so that between the two vehicles

$$S_{bl} + S_{ar} = \frac{w}{2} + \frac{w}{2} = w. \quad (4)$$

The relative collision metric C (equation (3) in the paper) is then

$$C = \left[1 - \frac{x_s}{U_{af} + U_{br}}\right] \left[1 - \frac{y_s}{S_{bl} + S_{ar}}\right]. \quad (5)$$

In practice we work with absolute values and clamp the terms to the interval $[0, 1]$:

$$C(t) = \left[1 - \frac{|x_s(t)|}{U_{af}(v_a) + U_{br}(v_b)}\right]^+ \left[1 - \frac{|y_s(t)|}{S_{bl} + S_{ar}}\right]^+, \quad [z]^+ := \max(0, \min(1, z)), \quad (6)$$

and set $C = 0$ when there is no VB overlap.

Deontological Rules (DRs). The paper also specifies Deontological Rules for longitudinal speed:

- AV_a : nominal 60 mph (26.82 m/s), reduced to 55 mph when UL VBs overlap;
- AV_b : 70 mph (31.29 m/s) before and during the overtake, reduced to 60 mph once the overtake is complete.

In our Simulink model these DRs are enforced for all lane widths.

4 Simulink implementation

We reimplemented the scenario in MATLAB/Simulink with the following choices:

- lateral reference paths of the form (7)–(8) for each w ;
- side VBs $S_a = S_b = w/2$ exactly as written in the paper;
- longitudinal speeds governed by the Deontological Rules (DRs) for all lane widths, in a MATLAB Function block `VelocityControl`;
- computation of VB overlaps and the collision metric $C(t)$ in a MATLAB Function block `VirtualBoundaries` using (6).

Substituting (7)–(8) into (1) gives

$$y_s(t) = y_b(t) - y_a(t) = f(t)w - w = (f(t) - 1)w. \quad (9)$$

Using (4), the lateral factor in (6) becomes

$$\begin{aligned} \gamma_{\text{lat}}(t, w) &= 1 - \frac{|y_s(t)|}{S_{bl} + S_{ar}} = 1 - \frac{|(f(t) - 1)w|}{w} \\ &= 1 - |f(t) - 1|. \end{aligned} \quad (10)$$

The lane width w cancels out exactly. Thus, *under the side-VB definition (3) and lane-centre trajectories, the lateral contribution to C is mathematically independent of w .*

The longitudinal factor is

$$\gamma_{\text{long}}(t) = 1 - \frac{|x_s(t)|}{U_{af}(v_a(t)) + U_{br}(v_b(t))}. \quad (11)$$

Combining both factors,

$$C(t, w) = \gamma_{\text{long}}(t) \gamma_{\text{lat}}(t, w) = \gamma_{\text{long}}(t) [1 - |f(t) - 1|]. \quad (12)$$

Therefore, as long as the model follows the paper’s geometry and VB definitions, $C(t)$ should be practically the same for all w .

6 Results

6.1 Straight highway: published figure

Figure 3 is the the straight-highway figure from the original paper. The top row shows constant velocities for both vehicles, independent of time; the middle row shows the lateral positions of AV_a and AV_b for five lane widths; and the bottom panel shows the relative collision metric $C(t)$.

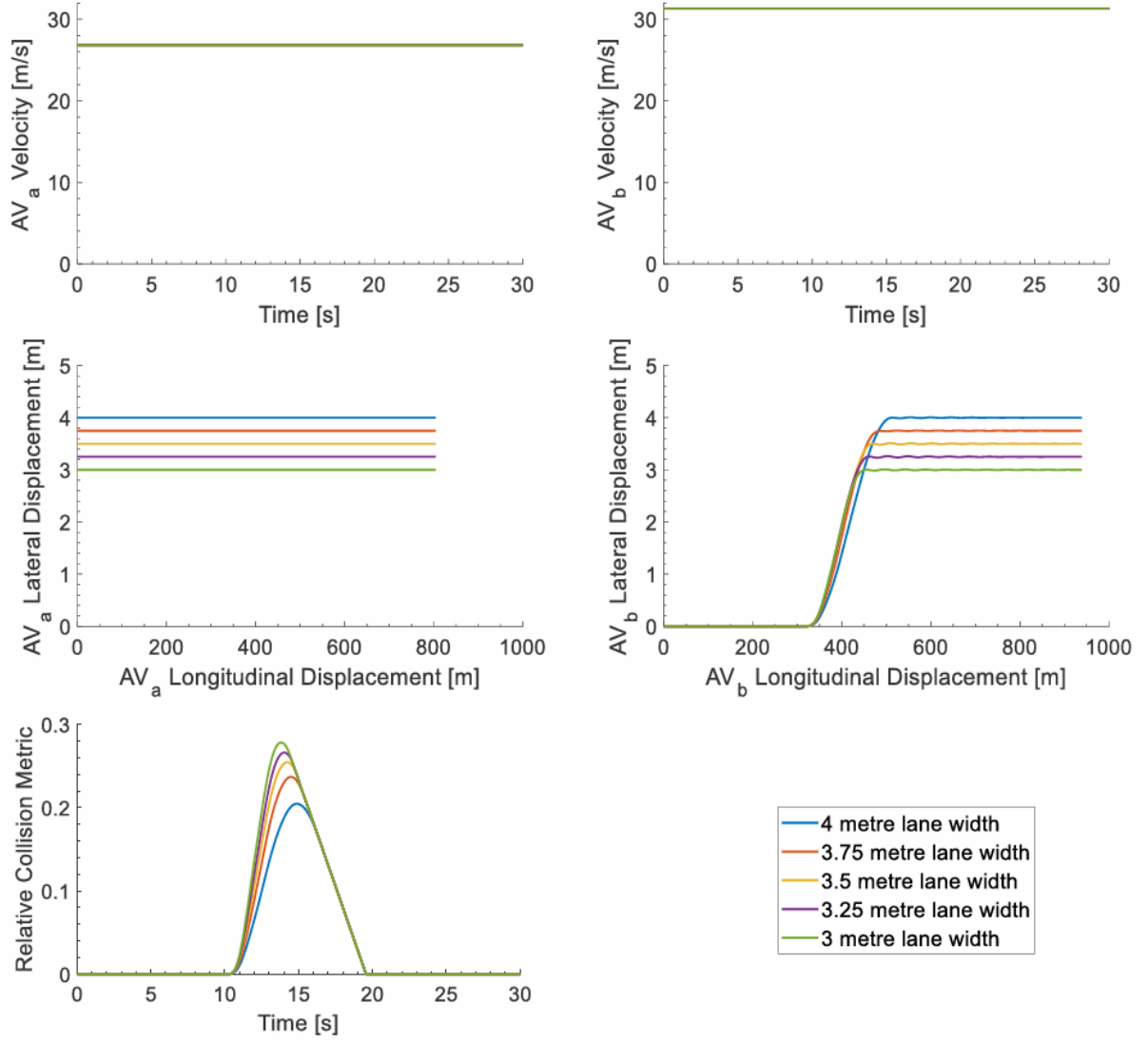


Figure 3: Straight-highway results as presented in the original paper [1]. Top: constant velocities. Middle: lateral motion of AV_a (left) and AV_b (right) for lane widths $w \in \{4.0, 3.75, 3.5, 3.25, 3.0\}$ m. Bottom: relative collision metric $C(t)$. The peak of $C(t)$ clearly increases as the lane width decreases.

The key observation is that the lower-lane curve ($w = 4.0$ m) has the smallest maximum, while the narrowest lane ($w = 3.0$ m) produces the largest C_{\max} , with an increase of roughly 20%.

6.2 Straight highway: Simulink reproduction

The resulting time histories are shown in Figure 4.

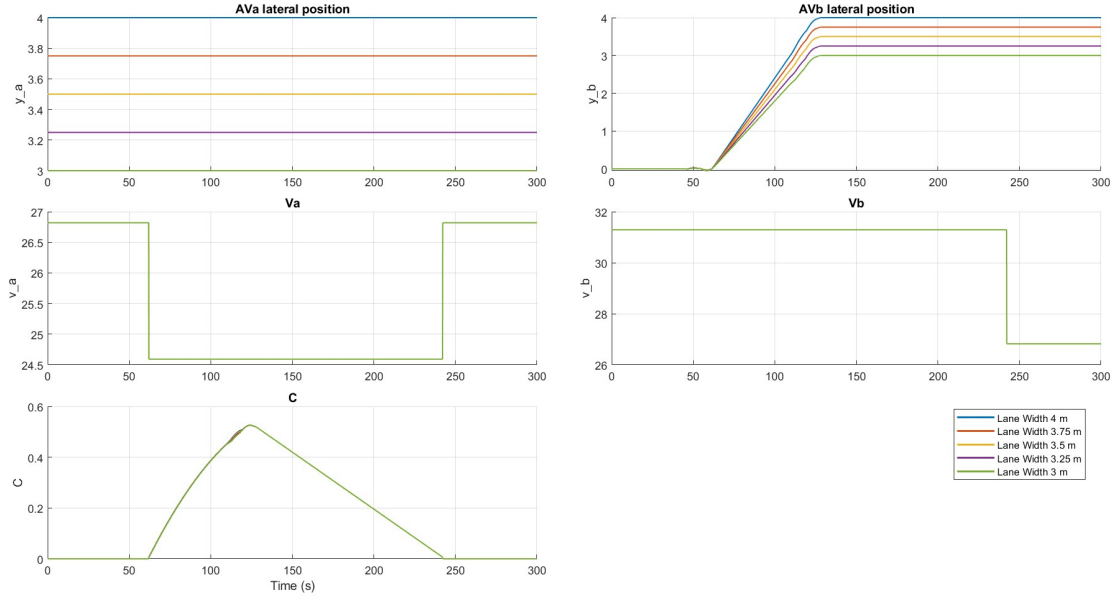


Figure 4: Simulink results using the methodology described in the paper. Top row: lateral positions of AV_a (left) and AV_b (right) for the five lane widths. Middle row: velocities of AV_a (left) and AV_b (right) generated by the Deontological Rules. Bottom-left: collision metric $C(t)$; all five curves essentially lie on top of each other, with almost identical C_{\max} .

6.3 Curved highway: published figure

Figure 5 is the curved-highway figure from the original paper. The top row shows the desired longitudinal velocities for both vehicles; the middle row shows the trajectories of AV_a and AV_b in the (x, y) plane for five different lane widths; and the bottom panel shows the relative collision metric $C(t)$.

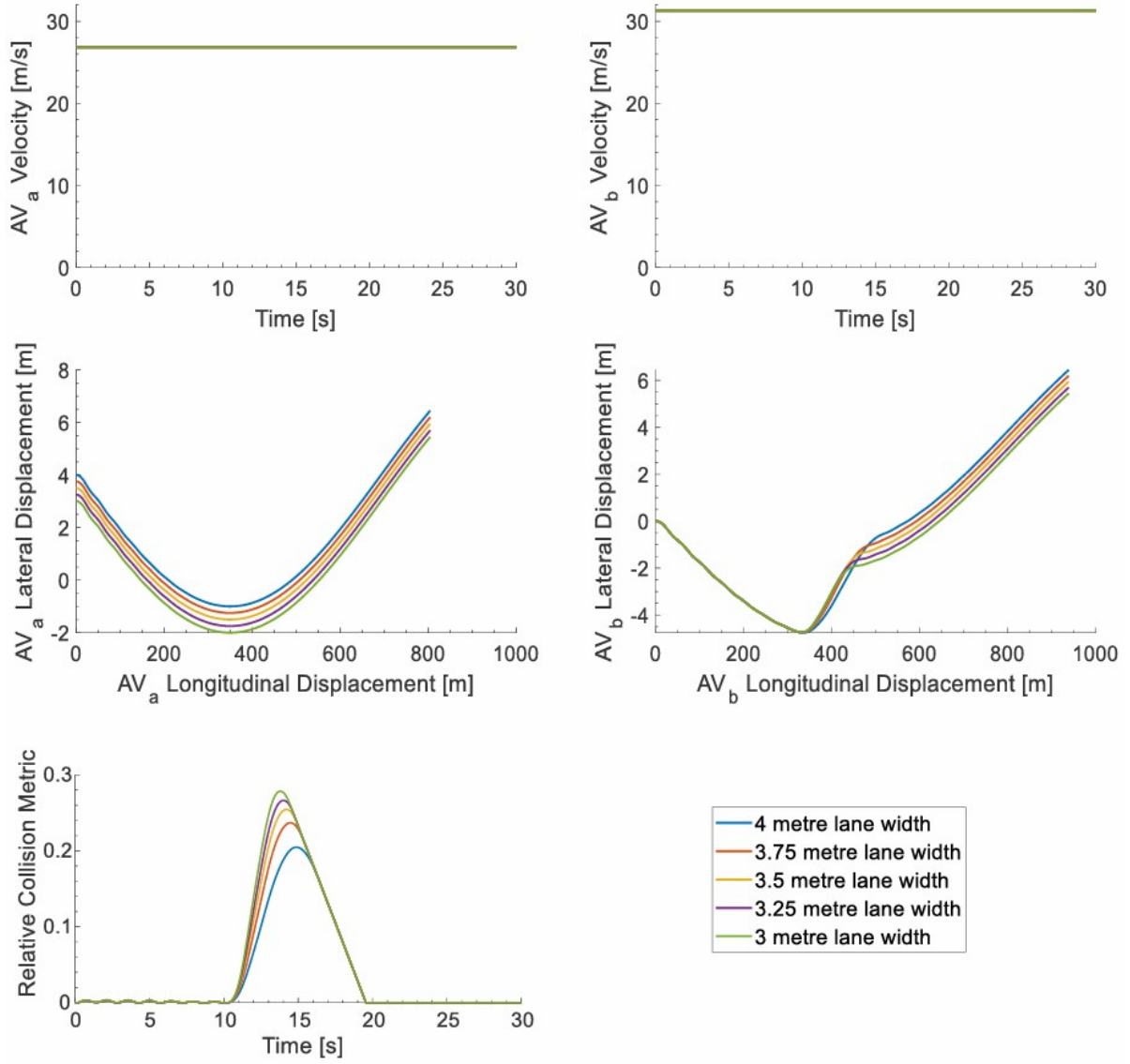


Figure 5: Curved-highway results as presented in the original paper [1]. Top: desired velocities of AV_a and AV_b . Middle: vehicle trajectories in the (x, y) plane for lane widths $w \in \{4.0, 3.75, 3.5, 3.25, 3.0\}$ m. Bottom: relative collision metric $C(t)$. The peak value of $C(t)$ increases as the lane width decreases.

As in the straight-highway case, the widest lane ($w = 4.0$ m) yields the smallest maximum collision metric, while progressively narrower lanes lead to larger values of C_{\max} .

6.4 Curved road section: Simulink results

We also simulated the *curved road section* scenario using the same Simulink model (Figure 1), but with curved reference trajectories/waypoints for the road geometry. The corresponding outputs are shown in Figure 6.

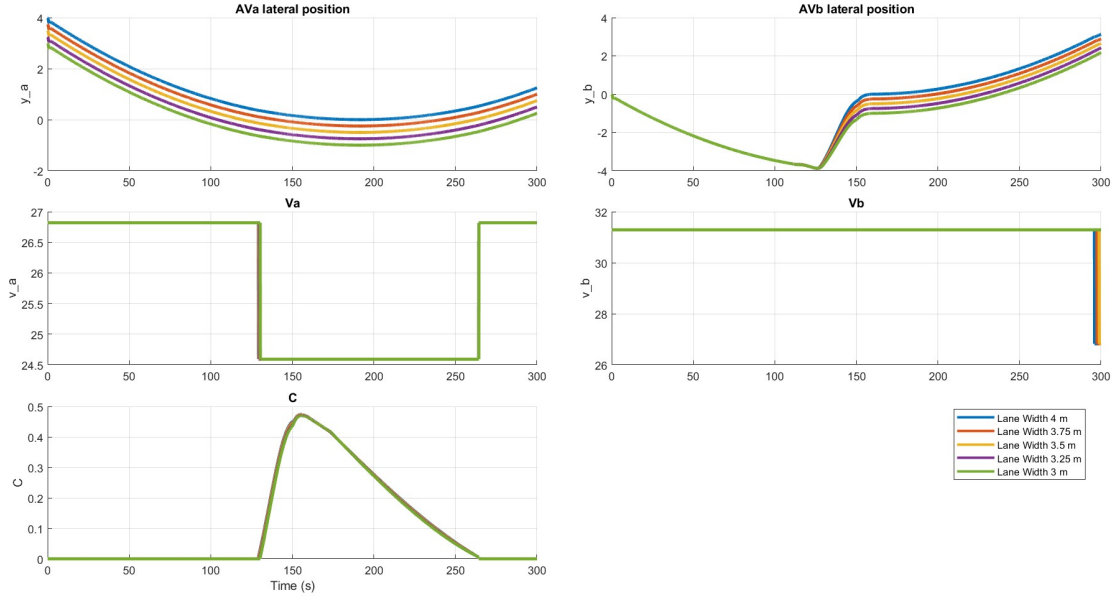


Figure 6: Simulink results for the curved road section scenario.

7 Reproducibility and execution of the simulations

All simulations presented in Part I can be reproduced directly from the provided MATLAB project directory, which is also made available through a public **GitHub repository** accompanying this report [5]. The repository contains the complete MATLAB/Simulink project, the automation scripts, and the figures used in the analysis, and is intended to facilitate inspection and independent reproduction of the results.

To run the complete set of scenarios (straight and curved road sections, for all lane widths), it is sufficient to execute the script

RUNME_CAS781_Project.m

from the root directory of the project.

This script serves as a single automated entry point: it configures the MATLAB path, initializes all required workspace variables, and sequentially executes all simulation scenarios without user intervention, generating the outputs used throughout Part I of this report.

8 Discussion and limitations (reproduction study)

From the above we can draw the following conclusions:

- Under the paper's own geometric assumptions and VB definitions (notably $S_{nl} = S_{nr} = w/2$), the collision metric $C(t)$ should not depend strongly on lane width for lane-centre tracking, due to cancellation of w in the lateral term.
- A Simulink implementation that respects those assumptions and enforces the Deontological Rules produces $C(t)$ curves that are essentially invariant with lane width (numerical tolerances aside).
- The published figures show a pronounced increase of C_{\max} as w decreases; such a trend cannot be obtained from the written methodology alone and suggests additional, undocumented modelling choices.

Part II

System-Theoretic Process Analysis (STPA) and Fault Injection

9 System and STPA Objective

Two autonomous vehicles drive on a two-lane highway. AVb overtakes AVa on straight and curved segments while the lane width is reduced from a nominal 4.0 m to narrower values (e.g. 3.75, 3.50, 3.25, 3.00 m). Safety is assessed using Virtual Boundaries (VBs) and a collision metric C .

The STPA focuses on:

- high-level overtaking planning and VB logic,
- lateral and longitudinal motion control,
- perception of lane geometry and relative states.

Goal: find unsafe control conditions introduced or amplified by reduced lane widths and turn them into safety constraints and test cases.

10 Step 1 – Losses, Boundaries, Hazards

In STPA, Step 1 clarifies what is inside the scope of the analysis, what we want to avoid (losses), the system states that may lead to those losses (hazards), and the corresponding high-level safety constraints.

10.1 System Boundary

The analysis covers the AV automation and its direct interface with the physical highway process, as later summarised in the control structure of Fig. 7. The following elements are treated as part of the system under design:

- high-level overtaking planner for AVb and VB-based safety layer,
- motion control for both AVs (lateral MPC and longitudinal speed controllers),
- perception and estimation (vehicle poses, lane centre, lane width),
- V2V communication between AVa and AVb,
- steering, throttle and brake actuators,
- physical highway process for AVa and AVb, including vehicle dynamics and interaction with the lane geometry.

Several factors influence safety but are not directly controlled by the design and are therefore treated as part of the environment:

- road authority policies and traffic rules,
- road markings quality,
- other surrounding traffic not explicitly modelled,
- weather and surface conditions.

10.2 Losses

Table 1: System-level losses

ID	Loss
L1	Collision between AVa and AVb.
L2	AV leaves its lane and hits infrastructure or another vehicle.
L3	Severe emergency manoeuvre (harsh braking or steering).
L4	Violation of VB-based ethical safety rules.

10.3 Hazards

Table 2: Main hazards related to reduced lane width

ID	Hazard
H1	Lateral separation between AVa and AVb becomes insufficient.
H2	Longitudinal separation becomes insufficient during overtaking.
H3	Controllers use an incorrect lane width.
H4	Lateral position in the lane is biased or poorly estimated.

10.4 System-Level Safety Constraints

Table 3: System-level safety constraints

ID	Constraint
SC1	Maintain sufficient lateral separation for all lane widths.
SC2	Maintain sufficient longitudinal separation during overtaking.
SC3	Use lane width estimates consistent with the actual road.
SC4	Inhibit or abort overtaking if lateral position uncertainty becomes too large.

11 Step 2 – Control Structure

Figure 7 shows the control structure used as the basis for the STPA of the autonomous overtaking scenario.

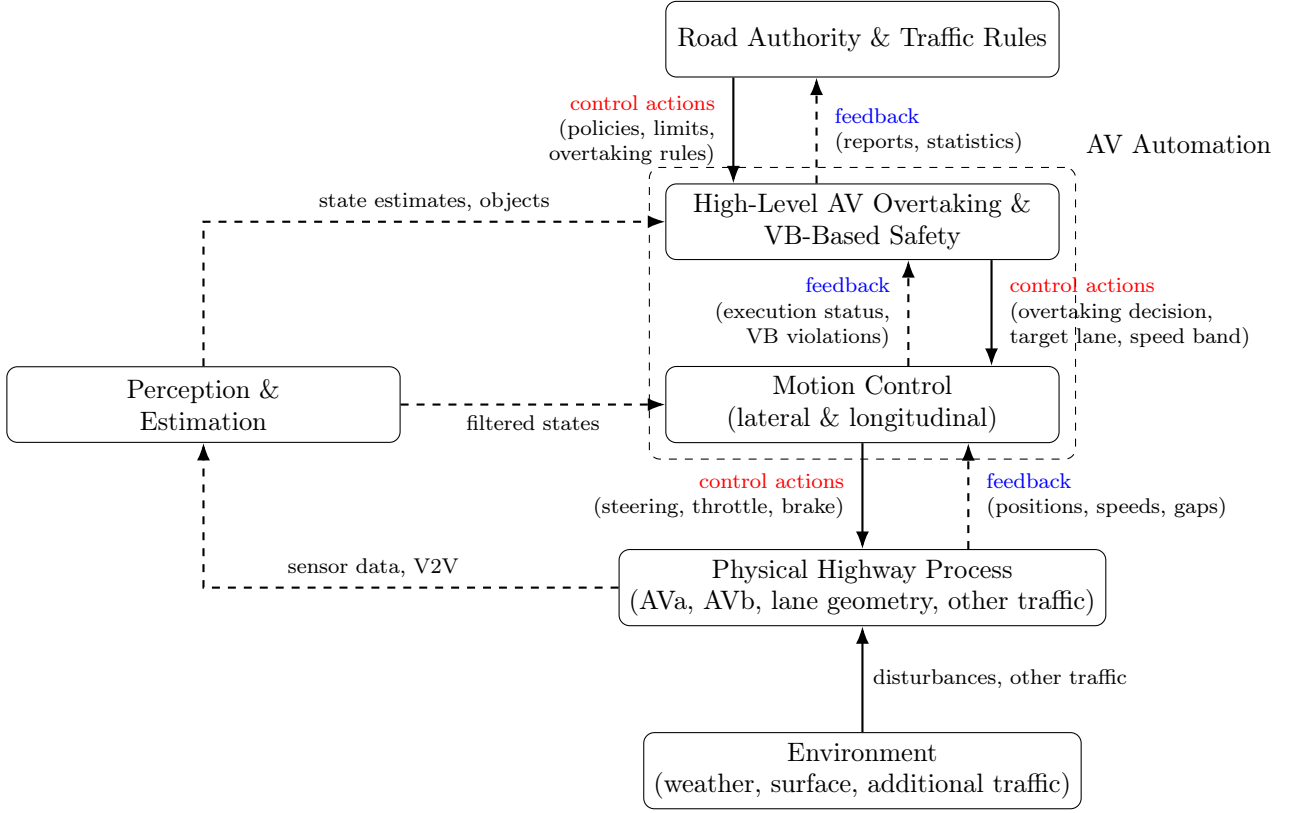


Figure 7: STPA control structure for the autonomous overtaking scenario.

12 Step 3 – Unsafe Control Actions

In STPA, an Unsafe Control Action (UCA) is a control action that, in a particular context and worst-case environment, can lead directly to one of the hazards. Only combinations that can lead to hazards H1–H4 are retained.

12.1 Technical UCAs for the AV overtaking scenario

Table 4: Unsafe control actions (UCAs) for the autonomous overtaking scenario.

Control action	Not providing causes hazard	Providing causes hazard	Too early / too late / out of order	Stopped too soon / applied too long
CA1: Overtaking authorisation (high-level overtaking and VB safety)	No direct hazard: AVb stays behind AVa (loss of performance only).	UCA1-1: Overtaking is authorised although, for the current lane width, the predicted lateral and/or longitudinal clearance is below the VB limits (H1, H2, H3).	UCA1-2: Authorisation is issued so late that the lane change must be very aggressive in a narrow segment, reducing lateral and longitudinal margins (H1, H2).	Not applicable for this discrete decision.
CA2: Lane-change trajectory for AVb (planner → motion control)	AVb continues to follow AVa in its lane; no additional hazard beyond the baseline car-following behaviour.	UCA2-1: Trajectory is generated for a wider lane than is actually available, or with an overly optimistic VB envelope, so the planned path violates lateral separation limits (H1, H3).	UCA2-2: Trajectory is generated too late, so high lateral acceleration and rate of change of lateral acceleration are needed in a narrow segment, reducing lateral and longitudinal margins during the lane change (H1, H2).	UCA2-3: Trajectory tracking is stopped before AVb has safely returned to the lane, or kept active after an abort is requested, reducing separation (H1, H2).
CA3: Cooperative speed reduction of AVa (motion control)	UCA3-1: AVa does not reduce speed when VB intrusion by AVb is detected, leading to insufficient longitudinal separation during overtaking (H2).	UCA3-2: AVa initiates VB cooperative braking although VB intrusion is no longer present, creating a rear-end risk for following traffic (H2).	UCA3-3: AVa starts cooperative braking too late; remaining headway is too short in narrow segments (H2).	UCA3-4: AVa maintains braking longer than necessary, creating unnecessary risk for following traffic (H2).
CA4: Lane-width and VB update (Perception and estimation → VB safety)	UCA4-1: Lane-width estimates are not updated when the road narrows, so VBs and controllers keep using a larger, outdated lane width (H3 → H2). UCA4-2: Lane width is systematically overestimated and VB parameters are relaxed too much (H3 → H1, H1, H2).	UCA4-3: Updates are triggered too late with respect to the actual narrowing, leaving a window where constraints do not match the true lane width (H3 → H1, H2).	UCA4-4: Lane width is updated too frequently or with high noise, causing oscillating VB limits that are difficult for the motion controller to follow (comfort and potentially H1/H2).	
CA5: Lateral lane keeping for AVa (motion control)	Controllers revert to an open-loop or degraded mode and no longer enforce VB lateral limits, increasing collision risk if AVb drifts laterally (H1).	UCA5-1: Lane centre is biased so that the steady-state lateral position is offset towards the other lane or vehicle (H1, H4).	UCA5-2: Lateral control is tuned too aggressively, causing overshoot and oscillations that cross VB boundaries in narrow lanes (H1).	UCA5-3: High lateral effort is maintained after the lane change is completed, causing oscillations and transient VB violations (H1).

12.2 Derived controller constraints

The UCAs in Table 4 are converted into controller-level safety constraints that refine the system-level constraints SC1–SC4.

Table 5: Controller-level safety constraints derived from the UCAs (Step 3).

ID	Controller constraint
CC1	The overtaking planner shall authorise overtaking only when the predicted lateral and longitudinal clearances satisfy the VB limits for the current lane width (SC1, SC2, H1, H2, H3).
CC2	The overtaking planner shall issue overtaking decisions early enough that the resulting lane-change trajectory respects specified limits on lateral acceleration and its rate of change, even in narrow-lane segments (SC1, H1, H2).
CC3	The cooperative controller for AVa shall reduce speed whenever VB intrusion by AVb is detected and shall maintain a minimum headway consistent with SC2 (H2).
CC4	The VB-based safety layer shall update its parameters whenever lane-width estimates decrease and shall use conservative bounds when perception is uncertain, so that VB limits remain safe with respect to the true lane width (SC1–SC3, H1–H3).
CC5	The lateral motion control of AVa shall account for lane-centre uncertainty and limit lateral offset and control effort so that VB boundaries are not violated in steady state or during transients (SC1, SC4, H1, H4).

13 Step 4 – Loss Scenarios and Constraints

This step explains *how* the UCAs identified in Step 3 can occur. We focus on a representative scenario (lane-width misestimation) and then summarise additional key scenarios.

14 Simulation-based injection of an Unsafe Control Action

Steps 1–4 identified losses, hazards, UCAs, and controller-level constraints. We now close the loop by *injecting* one representative UCA in the simulation and observing how it propagates into measurable safety degradation during the overtaking manoeuvre.

14.1 Selected UCA

We implement **UCA3-1** (Table 4): *AVa does not reduce speed when VB intrusion by AVb is detected*, which can reduce longitudinal separation during overtaking and lead to Hazard H2.

This UCA targets the cooperative longitudinal behaviour expected from AVa. In the model, it is injected by disabling the cooperative speed-reduction mechanism in AVa’s longitudinal controller, while keeping all other components unchanged.

14.2 Fault injection and observed effects

We compare a baseline run (nominal cooperative behaviour enabled) against a faulty run (UCA3-1 injected) under identical initial conditions.

Figure 8 summarises the outcome. The top panels show that the lateral motion of both vehicles is essentially unchanged between runs: AVa follows the same road geometry and AVb performs the same lateral manoeuvre. This indicates that the injected fault is not a lateral-control artefact; the difference between runs is dominated by longitudinal behaviour.

The middle panels illustrate the fault mechanism itself. In the baseline case, AVa temporarily reduces its speed during the critical interval of the manoeuvre, consistent with a cooperative response to

a VB-intrusion condition. With UCA3-1 injected, this temporary reduction does not occur and v_a remains near its nominal value, while AVb’s speed profile remains comparable.

This effect is reflected in the collision metric in the bottom panel. When the fault is injected, $C(t)$ reaches a higher peak and decays more slowly, remaining non-zero for longer than in the baseline run. This provides direct simulation evidence that UCA3-1 amplifies longitudinal risk during overtaking and supports constraints requiring timely cooperative speed reduction by AVa in critical VB zones (e.g., **CC3** and **CC5**).

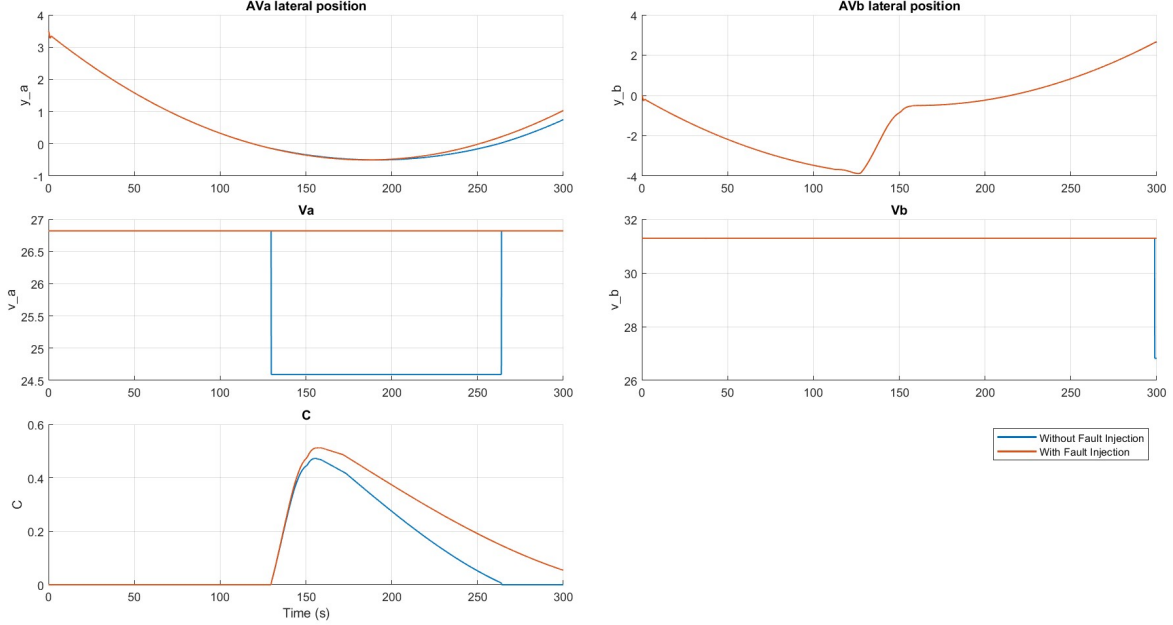


Figure 8: Baseline vs. UCA3-1 fault injection. Top: lateral positions of AVa and AVb. Middle: longitudinal speeds v_a and v_b (baseline shows a temporary cooperative speed reduction of AVa; fault case keeps nominal speed). Bottom: collision metric $C(t)$, showing a higher peak and a longer persistence under UCA3-1.

Part III

Overall Conclusion

15 Conclusion

This final report combined (i) a reproduction and verification of the Virtual Boundary collision metric C under lane-width reduction, and (ii) an STPA-based safety analysis of the same autonomous overtaking scenario, complemented by simulation-based injection of a representative Unsafe Control Action.

In Part I, an analytic check showed that, under the paper’s stated side-VB definition ($S_{nl} = S_{nr} = w/2$) and lane-centre tracking, the lane width cancels out of the lateral term of $C(t)$. A Simulink reimplementa-tion consistent with those definitions produced collision-metric curves that are nearly invariant across lane widths. This contrasts with the strong lane-width dependence shown in the published figures and suggests that additional, undocumented modelling choices are required to reproduce the paper’s trend.

In Part II, STPA provided a structured account of how hazards can arise through inadequate control or incorrect feedback. The derived UCAs and controller-level constraints clarify what the system

must enforce to avoid loss conditions under reduced lane width. Finally, injecting UCA3-1 (lack of cooperative deceleration by AVa upon VB intrusion) showed a clear degradation in the collision metric behaviour, illustrating how a single unsafe control decision can amplify longitudinal risk during overtaking.

Taken together, the reproduction study and the STPA analysis support the same message: lane-width reduction primarily becomes safety-critical when it interacts with modelling/estimation choices and control decisions (VB parameterisation, timely updates, cooperative behaviours). These elements should therefore be made explicit, conservative, and testable in any safety argument for reduced-width highway operations.

References

- [1] J. D’Souza, K. Burnham, and J. E. Pickering. An Investigation into the Safety of Autonomous Vehicles on Highways with Reduced Lane Widths. In *Proceedings of the 10th International Conference on Control, Decision and Information Technologies (CoDIT)*, 2024. DOI: 10.1109/CoDIT62066.2024.10708388.
- [2] N. G. Leveson. *Engineering a Safer World: Systems Thinking Applied to Safety*. MIT Press, 2012.
- [3] J. P. Thomas and N. G. Leveson. *STPA Handbook*. March 2018. Available at https://psas.scripts.mit.edu/home/get_file.php?name=STPA_handbook.pdf.
- [4] MathWorks, “Model Predictive Control (MPC) Virtual Lab,” MATLAB Central File Exchange, 2024. Available: <https://fr.mathworks.com/matlabcentral/fileexchange/158356-model-predictive-control-mpc-virtual-lab>
- [5] Github repository of the project
<https://github.com/TaharHERRI/Edu-Safety-Analysis-Autonomous-Overtaking>.