Heuristic Function

The calculation of the heuristic function for a Node includes the minimum possible distance from the current Node to the goal node and the minimum possible cost from the Node to one of its children.

Formally:

H(G) = 0

If n is the parent of the goal node : H(n) = 5

Otherwise: H(n) = max(dx, dy) – 2 + c + 5

where:

- n is the current state
- dx is the horizontal distance from the current state to the goal state.
- dy is the vertical distance from the current state to the goal state.
- c is the cost of the cheapest possible move for n.
- -2 : subtracting the cost to the goal node and the minimum cost to the child of the current node. (we will add c and 5 instead).
- +5 : adding the cost of the last step that leads to the goal state.

*Note: there is no reference to the start state in the calculation of the heuristic function because we implemented the algorithm with an open list, so it is not possible to expands the start state again.

Claim: The above heuristic function is consistent.

That is, we will prove that for each node n, and for each of its children m, that:

H(n) <= C(n, m) + H(m).

*We note that at the very least, it holds that D(m) = D(n) - 1, where D is the max(dx, dy) distance from the current state to the goal state.

* In the following calculations, we will omit the constant numerical values since they do not affect the inequality.

We will divide the proof into four cases:

1) C(n, m) = 1:
   So, the cheapest possible move for both n and m will be 1 and we get that the following equation holds:
   H(n) = d + 1
   H(m) = (d-1) + 1
   ➔ H(n) <= 1 + H(m).

2) C(n, m) = 3:

H(n) = d + 3 (at most)
H(m) = (d-1) + 1 (at least)
➔ H(n) <= 3 + H(m).

- A case where c_n (the cheapest possible move n) > 3 is not possible because it is known that there is a path with a lower cost than it (cost 3).

3) C(n, m) = 5:

H(n) = d + 5 (at most)
H(m) = (d-1) + 1 (at least)
➔ H(n) <= 5 + H(m).

- A case where c_n (the cheapest possible move n) > 5 is not possible because it is known that there is a path with a lower cost than it (cost 5).

4) C(n, m) = 10:

H(n) = d + 10 (at most)
H(m) = (d-1) + 1 (at least)
➔ H(n) <= 10 + H(m).

We have covered all possible cases, and for each case the heuristic function maintains consistency.

Thus, we have proved that the function is consistent.

Since the function is consistent, it can also be said to be admissible.

QED.