$\boxed{1.6}$

(a) * $\mu = 0.05$

$P(\text{not red}) = (1-0.05)^{10} = 0.59874$

* $\mu = 0.5$

$P(\text{not red}) = (1-0.5)^{10} = 9.76563 * 10^{-4}$

* $\mu = 0.8$

$P(\text{not red}) = (1-0.8)^{10} = 1.024 * 10^{-7}$

(b) we want to calalate the probability of having at least one sample with no red marbles.

let X the event of at least having one sample with zero red marbles

$P(X) = 1 - P(\bar{X})$

where $\bar{X}$ here represent having no sample that is free from red marbles

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* To get a certain number of success out of n trails we will use the binomial distribution

$P(K) = \binom{n}{k} P^{k} (1-P)^{n-k}$ where k is the number of success and p is the probability of getting a success

$$P(X) = 1 - P(\bar{X})$$

$$= 1 - \binom{n}{0}^{1} \cancel{(P(\text{no red}))^{0}} (1 - P(\text{no red}))^{n}$$

$$= 1 - (1 - P(\text{no red}))^{n}$$

* $\mu = 0.05$ $P(X) = 1 - (1 - 0.59874)^{1000}$

$= 1$

* $\mu = 0.5$ $P(X) = 1 - (1 - 9.76563 * 10^{-4})^{1000}$

$= 0.62358$

* $\mu = 0.8$ $P(X) = 1 - (1 - 1.024 * 10^{-7})^{1000}$

$= 1.02395 * 10^{-4}$

(C) Assume that X is the event of at least having one sample with zero red marbles

$$P(X) = 1 - P(\bar{X})$$
$$= 1 - \cancel{\binom{n}{0}} (P(\text{no red}))^0 (1 - P(\text{no red}))^n$$
$$= 1 - (1 - P(\text{no red}))^n$$

$*\mu = 0.05$ $P(X) = 1 - (1 - 0.59874)^{1000,000}$
$= 1$

$*\mu = 0.5$ $P(X) = 1 - (1 - 9.76563 * 10^{-4})^{1000,000}$
$= 1$

$*\mu = 0.8$ $P(X) = 1 - (1 - 1.024 * 10^{-7})^{1000,000}$
$= 0.09733$

2.5

Prone by induction:

steps
① Validate the base Case
② check if it is working for N, does it work for $N+1$?

① • For $N=1$ and $D=0$

$$\sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} = 1 \leq N^{D}+1$$
$$\leq 1^{0}+1$$
$$\leq 2$$

• For $N=1$ and $D=1$

$$\sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} + \binom{1}{1} = 2 \leq N^{D}+1$$
$$\leq 1^{1}+1$$
$$\leq 2$$

∴ The inequality holds for the base Case

2 Assume that $\sum_{i=0}^{D} \binom{N}{i} \leq N^{D}+1 \rightarrow 1$

does $\sum_{i=0}^{D} \binom{N+1}{i} \leq (N+1)^{D}+1$??

* we will use the property that say

$$\binom{N}{i-1} + \binom{N}{i} = \binom{N+1}{i} \rightarrow 2$$

* $$\sum_{i=0}^{D} \binom{N+1}{i} = \sum_{i=0}^{D} \left[\binom{N}{i} + \binom{N}{i-1} \right] \text{ From } 2$$

$$= \sum_{i=0}^{D} \binom{N}{i} + \sum_{j=0}^{D-1} \binom{N}{j}$$

$$\leq N^{D}+1 + N^{D-1}+1 \quad \text{From } 1$$

$$\leq (N^{D} + N^{D-1} + 1) + 1$$

$$\leq (N+1)^{D} + 1$$

* still valid as we removed a term and replaced it with a bigger term.

$(N+1)^{D} = N^{D} + DN^{D-1} + \cdots + 1$

$$\therefore \sum_{i=0}^{D} \binom{N}{i} \leq N^{D}+1$$

$\because m_H(N) \leq \sum_{i=0}^{d_{VC}} \binom{N}{i}$ and $\sum_{i=0}^{d_{VC}} \binom{N}{i} \leq N^{d_{VC}}+1$

$\therefore m_H(N) \leq N^{d_{VC}}+1$