C. Surface Fitting

The third module in our program is **Surface_Fit_beta()** and it deals with three dimensional forms that satisfy the general regression formula, this includes but is not limited to planes, paraboloids (as shown in [Fig. 3,3]) and other surfaces that represent a quadratic form.

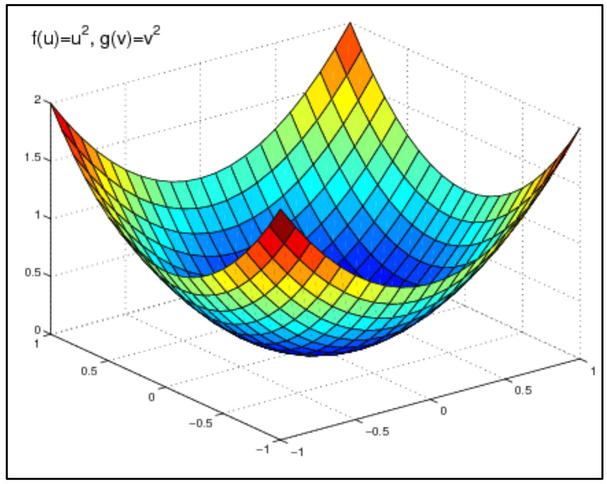

Fig. 3 Three-dimensional surface

Very analogous to curve fitting of linear forms, the approach to minimize the sum of squared errors here comprises in solving the linear system that corresponds to zero partial derivatives for each of the constants, the general formula for the relevant matrix equation is shown in the appendix.

Inputs:

- The decimal fixing constant
- The data set (x, y, z)
- The set of functions that correspond to the surface that satisfies the general regression formula (limitless number)

Outputs:

- The surface equation with substituted constants
- The regression error

Additional Features:

- A plot involving the best fit of the surface is shown through the **Plot_3D_RHS()** module

V. References

[1] *Least-Squares Regression*. https://www.ck12.org/statistics/least-squares-regression/. Accessed 14 May 2020.

[2] Shetye, Sweta. *Gradient Descent*. 2020, https://datavyom.com/2020/05/02/gradient-descent/. Accessed 14 May 2020.

[3] Munteanu, Marian. *The Paraboloid Of Revolution*. 2008, https://www.researchgate.net/figure/The-paraboloid-of-revolution_fig1_2207021. Accessed 14 May 2020.

VI. Appendix

The General Regression Formula:

For:

$$Y = C_0\emptyset_0 + C_1\emptyset_1 + C_2\emptyset_2 + C_3\emptyset_3 + \cdots + C_n\emptyset_n$$

Solve the matrix equation:

$$\begin{pmatrix} \sum \emptyset_0\emptyset_0 & \sum \emptyset_0\emptyset_1 & \sum \emptyset_0\emptyset_2 & \cdots & \sum \emptyset_0\emptyset_n \\ \sum \emptyset_1\emptyset_0 & \sum \emptyset_1\emptyset_1 & \sum \emptyset_1\emptyset_2 & \cdots & \sum \emptyset_1\emptyset_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum \emptyset_n\emptyset_0 & \sum \emptyset_n\emptyset_1 & \sum \emptyset_n\emptyset_2 & \dots & \sum \emptyset_n\emptyset_n \end{pmatrix} \begin{pmatrix} C_0 \\ C_1 \\ \vdots \\ C_n \end{pmatrix} = \begin{pmatrix} \sum \mathrm{Y}\emptyset_0 \\ \sum \mathrm{Y}\emptyset_1 \\ \vdots \\ \sum \mathrm{Y}\emptyset_n \end{pmatrix}$$

For surface fitting replace Y with Z and $\emptyset$ becomes a function in both x and y.

Gradient Descent Iterative Scheme:

$$x_{i+1} = x_i - \lambda \nabla f(x_i)$$

Gauss-Newton Iterative Scheme:

$$x_{i+1} = x_i - \left(\nabla^2 f(x_i)\right)^{-1} \nabla f(x_i)$$

Livenberg-Marquardt Iterative Scheme:

$$x_{i+1} = x_i - (H + \lambda I)^{-1} \nabla f(x_i)$$