We have designed a program that computes the derivative of a function at a point using finite central difference formulas, then improved the results using Richardson's extrapolation method.

Method used:

A finite difference formula is a mathematical expression of the form $f(x + b) - f(x + a)$. If we divide this expression by $b - a$, we get the difference quotient. Approximating derivatives using finite differences plays an important role in numerically solving differential problems.

We've designed the program to compute the first, second, and third derivatives of polynomial functions using the following formulas central difference formulas respectively:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f'''(x) \approx \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3}$$

Where h is the step size = $b - a$, note that the error of all three previous equations is of order $O(h^2)$.

We then applied Richardson extrapolation which is a method used to improve the performance of any iterative formula by reducing the step size h while increasing the order of error $O(h)$. Richardson extrapolation basically uses the estimates of two derivatives to compute a third, more accurate approximation.

$$A_{k,j} = \frac{4^{j-1}A_{k,j-1} - A_{k-1,j-1}}{4^{j-1} - 1}$$

This iterative formula can be represented in the following tabular form

k	h_k	$A_{k,0}$	$A_{k,1}$	$A_{k,2}$	$A_{k,3}$	$A_{k,4}$
0	h	$A_{0,0}$				
1	$h/2$	$A_{1,0}$	$A_{1,1}$			
2	$h/4$	$A_{2,0}$	$A_{2,1}$	$A_{2,2}$		
3	$h/8$	$A_{3,0}$	$A_{3,1}$	$A_{3,2}$	$A_{3,3}$	
4	$h/16$	$A_{4,0}$	$A_{4,1}$	$A_{4,2}$	$A_{4,3}$	$A_{4,4}$

Where k is the index of the vertical dimension representing the step sizes $h_k = h/2^k$ and j is the index of the horizontal dimension representing the method obtained the j level of iteration

Applications in Computer Engineering:

- Graphs and Visuals

Numerical differentiation and calculus in general are used in creating 3D graphs and visuals, which are later used in video games and physics engines. Physics engines are used to define the physics in a video game like momentum, collision, and gravity for example. Visuals are also used by the military to simulate flights, artillery paths, and satellite images. Architects may also use them to graph buildings.

- Programs to solve mathematical problems

Numerical calculus is used in general problem solving programs and simulations. For instance, programs that compute integrals and derivatives like the ones used in a scientific calculator. Simulations in videogames calculate probabilities which are then solved by programs rather than the programmer himself.