Interpolation: it is a method used to estimate the value of a function at determined point, while extrapolation compute the value of the function outside the given range.

Many techniques are used to calculate the value of the function at specific point; however, we will talk about only three techniques Newtown General, Newtown Gregory and Lagrange.

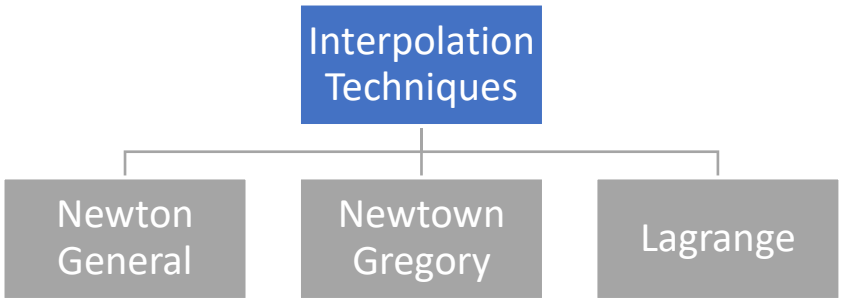

Figure1: Interpolation Techniques

1-**Newton General Interpolation:** is an interpolation technique that used when the step between points is not same.

Divided difference Formula: $\Delta \mathrm{yi} = \frac{\mathrm{yi+1-yi}}{xi+1-xi}$

Xi	F[xi]	1st order Differences	2st order Differences
X1	Y1		
		$\Delta 1 = \frac{y2 - y1}{x2 - x1}$	
X2	Y2		$\Delta^2 1 = \frac{\Delta 2 - \Delta 1}{x3 - x1}$
		$\Delta 2 = \frac{y3 - y2}{x3 - x2}$	
X3	Y3		

Table1: Divided Difference For 3points

Newton's General Interpolation Formula

$$Pn(x) = y0 + \frac{\Delta y0\ (x - x0)}{h} + \frac{\Delta 2\ y0\ (x - x0)(x - x1)}{2!\ h2} + \cdots\ldots\ldots\ldots + \frac{\Delta n\ y0\ (x - x0)\ (x - x1)..(x - x(n-1))}{n!\ hn}$$

Residual Error for Newton's General Interpolation: if we stop at (n-1) term before stopping term then the residual error will be (n) term.

2-**Newton Gregory Interpolation:** is a special case of newton general that used when the step between points is same for all sequence.

Forward Difference Formula: $\Delta yi = \frac{yi+1-yi}{xi+1-xi}$

Xi	F[xi]	1st Differences	2st Differences
X1	Y1		
		ΔY_0=Y2-Y1	
X2	Y2		$\Delta^2 Y_0 = \Delta Y_1 - \Delta Y_0$
		ΔY_1=Y3-Y2	
X3	Y3		

Table2: Forward Difference For 3points

Newton's Gregory Interpolation Formula

$$Y = Y0 + \alpha\,\Delta y0 + \Delta^2 y0\,\frac{(\alpha)(\alpha-1)}{2!} + \Delta^3 y0\,\frac{(\alpha)(\alpha-1)(\alpha-1)}{3!} + \cdots \ldots\ldots\ldots\ldots \text{ where } \alpha\,\frac{x-x0}{h}$$

- **Newton's Gregory can also used to compute derivatives**

$$\frac{dy}{dx} = \frac{1}{h}\left(\Delta y0 + \frac{(2\alpha-1)\Delta^2 y0}{2!} + \frac{(3\alpha^2-6\alpha+2)\Delta^3 y0}{3!} \ldots.\right)$$

- In case this derivative is being evaluated at a point on the table

$$\frac{dy}{dx} = \frac{1}{h}\left(\Delta y0 + \frac{\Delta^2 y0}{2!} + \frac{2\Delta^3 y0}{3!} \ldots..\right)$$

3-Lagrange polynomial: For a given set of (X_i , Y_i) points with no two X_i values equal it used to get function of power (n-1) from n points.

- **Lagrange Interpolation Formula**

$$Pn(x) = \sum_{i=1}^{n}\{\,(\prod_{\substack{j=1\\ j\neq i}}^{n}\frac{x-xj}{xi-xj})yi\,\}$$

- An advantage to Lagrange Interpolation to compute $F^{-1}(x)$ easily by for a few numbers of points