جامعة بيرزيت

BIRZEIT UNIVERSITY

Electrical and Computer Engineering Department

ENCS4310, Digital Signal Processing | Assignment 1| Deadline 12/1/2023

Part 1

Q1. Generate and plot each of the following sequences over the indicated interval.

A) $x[n] = 2\delta(n+2) - \delta(n-4),\ -5 \le n \le 5.$

B) $y[n] = cos(0.04\pi n) + 0.2w(n),\ 0 \le n \le 50.$ where w(n) is a Gaussian random sequence with zero mean and unit variance

C) $z[n] = \{\ldots,\ 5,4,3,2,1,\underline{5},4,3,2,1,5,4,3,2,1,\ \ldots\};\ -10 \le n \le 9,$

Q2. Generate and plot each of the following sequences over the indicated interval.

$g(t) = cos(2\pi F_1 t) + 0.125 cos(2\pi F_2 t),\ F_1 = 5Hz,\ F_2 = 15,$, plot g[n] for one second.

A) For $Fs = 50Hz$

B) For $Fs = 30Hz$

C) For $Fs = 20Hz$

Comment on the results.

Useful MATLAB functions:

```
n=-5:1:5;
subplot(3,1,1)
stem(n,x); title('Question1_partA')
xlabel('n'); ylabel('x(n)');
randn(x,y)
size(n)
```

Q3. Generate the complex-valued signal

$x[n] = e^{(-0.1 + j0.3)n},\ -10 \le n \le 10$

and plot its magnitude, phase, the real part, and the imaginary part in four separate subplots.

Useful MATLAB functions:

```
real(x); imag(x); abs(x); (180/pi)*angle(x)
```

Part 2 Convolution

We introduced the convolution operation to describe the response of an LTI system. If arbitrary sequences are of infinite duration, then MATLAB cannot be used directly to compute the convolution. MATLAB does provide a built-in function called conv that computes the convolution between two finite-duration sequences. The conv function assumes that the two sequences begin at n = 0 and is invoked by

```
>> y = conv(x,h);
```

A simple modification of the conv function, called conv_m, which performs the convolution of arbitrary support sequences can now be designed.

```
function [y,ny] = conv_m(x,nx,h,nh) % Modified convolution routine for signal processing
 % -----------------------------------------------
% [y,ny] = conv_m(x,nx,h,nh)
% [y,ny] = convolution result
% [x,nx] = first signal
% [h,nh] = second signal
 nyb = nx(1)+nh(1);
nye = nx(length(x)) +  nh(length(h)); ny = [nyb:nye];
 y = conv(x,h);
```

Q5. For

$x[n] = [3, 11, 7, 0 , -1, 4, 2], \ -3 \le n \le 3;$

$h[n] = [2, 3 , 0, -5, 2, 1], \ -1 \le n \le 4$

Find and plot y[n].

Q6. Let the rectangular pulse x(n) = u(n) − u(n − 10) be an input to an LTI system with impulse response $h[n] = (0.9)^n u(n)$

Plot x{n], h[n], Find and plot the output y(n). Consider the interval [-5, 45].

Useful MATLAB functions: *ones()*

Part 3 Correlations of sequences

Correlation is an operation used in many applications in digital signal processing. It is a measure of the degree to which two sequences are similar. Given two real-valued sequences x[n] and y[n] of finite energy, the crosscorrelation of x[n] and y[n] is a sequence $r_{xy}(l)$ defined as

$$r_{x,y}(l) = \sum_{n=-\infty}^{\infty} x(n) y(n - l)$$

The index (*l*) is called the shift or lag parameter. The special case of crosscorrelation when y[n] = x[n] is called autocorrelation and is defined by

$$r_{x,x}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n - l)$$

It provides a measure of self-similarity between different alignments of the sequence.

Q7. To demonstrate one application of the crosscorrelation sequence.

Let $x[n] = [3, 11, 7, \underline{0}, -1, 4, 2]$ be a prototype sequence,

A) let y(n) be its noise-corrupted-and-shifted version

y[n]=x[n-2]+w[n]

where w[n] is Gaussian sequence with mean 0 and variance 1. Compute the crosscorrelation between y[n] and x[n] and comment on the results.

B) Repeat part (a) for y[n]=x[n-4]+w[n]

Useful MATLAB functions:

```
lx=[-3,3];
y= [0 0 x];
ly=[-5,5];
w = randn
z=xcorr(x,y)
l=lx+ly;
note that the xcorr function cannot provide the timing (or lag)
information.
```

Part 3 Difference Equation

An LTI discrete system can also be described by a linear constant coefficient difference equation of the form

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_k x[n-m], \ \forall n$$

A function called filter is available to solve difference equations numerically, given the input and the difference equation coefficients. In its simplest form this function is invoked by y = filter(b,a,x) where b = [b_0, b_1, ..., b_M]; a = [a_0, a_1, ..., a_N];

Q8. Given the following difference equation

$$y[n] - y[n-1] + 0.9y[n-2] \ = \ x[n]$$

A) Calculate and plot the impulse response h(n) at n = -5,..., 120.
B) Calculate and plot the unit step response s(n) at n = -5,..., 120.
C) Is the system specified by h(n) stable?

Useful MATLAB functions: *Ones()* *zeros()* *y = filter(b,a,x)* *sum(abs(h))*

Q9. A "simple" digital differentiator is given by

$$y[n] = x[n] - x[n-1]$$

which computes a backward first-order difference of the input sequence. Implement this differentiator on the following sequences, and plot the results. Comment on the appropriateness of this simple differentiator.

A) Rectangular pulse: $x[n]= 5[u(n) - u(n - 20)]$
B) Triangular pulse: $x[n] = n\,(u[n] - u[n - 10]) + (20-n)\,(u[n - 10] - u[n - 20])$
C) Sinusoidal pulse: $x[n] \ = \ sin\left(\frac{\pi n}{25}\right)(u[n] - u[n-100])$

Part 4 DTFT

The DTFT for n[n] is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-j\omega n} \tag{1}$$

If x[n]is of infinite duration, then MATLAB cannot be used directly to compute $X(e^{j\omega})$ from x[n]. However, we can use it to evaluate the expression $X(e^{j\omega})$ over [0,π] frequencies and then plot its magnitude and angle (or real and imaginary parts).

Q10. For $x[n] = (0.5)^n\ u[n]$. The corresponding DTFT is $X(e^{j\omega}) = \dfrac{e^{j\omega}}{e^{j\omega} - 0.5}$.

Evaluate $X(e^{j\omega})$ at 501 equispaced points between [0,π] and plot its magnitude, angle, real, and imaginary parts.

> *Useful MATLAB functions:*
> *w = [0:1:500]*pi/500; % [0, pi] axis divided into 501 points.*
> *>> X = exp(j*w) ./ (exp(j*w) - 0.5*ones(1,501));*
> *Note that to divided the w array by pi before plotting so that the frequency axes are in the units of π and therefore easier to read. This practice is strongly recommended.*

If x[n] is of finite duration, then MATLAB can be used to compute $X(e^{j\omega})$ numerically at any frequency ω. The approach is to implement (1) directly.

If, in addition, we evaluate $X(e^{j\omega})$ at equispaced frequencies between [0,π], then (1) can be implemented as a matrix-vector multiplication operation. To understand this, let us assume that the sequence x[n] has N samples between $n_1 \le n \le n_N$ (i.e., not necessarily between [0,N − 1]) and that we want to evaluate $X(e^{j\omega})$ at

$$\omega_k \triangleq \frac{\pi}{M}k, \quad k = 0,1,\ldots,\mathrm{M}$$

which are (M + 1) equispaced frequencies between [0,π]. Then (1) can be written as

$$X(e^{j\omega_k}) = \sum_{l=1}^{N} x[n_l]\mathrm{e}^{-j(\pi/M)kn_l}, \quad k = 0,1,\ldots,\mathrm{M}$$

When x[nl] and $X(e^{j\omega})$ are arranged as column vectors **x** and **X**, respectively, we have

$$X = Wx \tag{2}$$

where **W** is an (M + 1) × N matrix given by

$$W \triangleq \left\{\mathrm{e}^{-j(\pi/M)kn_l}; \quad k = 0,1,\ldots,\mathrm{M}, \qquad n_1 \le n \le n_2\right\}$$

In addition, if we arrange {k} and {n_l} as row vectors **k** and **n** respectively, then

$$W = \left[\exp\left(-j\frac{\pi}{M}\mathrm{k}^T\mathrm{n}\right)\right]$$

In MATLAB we represent sequences and indices as row vectors; therefore taking the transpose of (2), we obtain

$$\mathbf{X}^T = \mathbf{x}^T \left[\exp\left(-j \frac{\pi}{M} \mathbf{n}^T \mathbf{k} \right) \right] \qquad (3)$$

Note that $\mathbf{n}^T\mathbf{k}$ is an N $\times$(M + 1) matrix. Now (3) can be implemented in MATLAB as follows.

Useful MATLAB functions:
`>> k = [0:M]; n = [n1:n2];` `>> X = x * (exp(-j*pi/M)) .^ (n'*k);`

Q.11 Consider the sequence $x[n] = \{1, -\underset{\uparrow}{0}.5, -0.3, -0.1\}$

A) Numerically compute the discrete-time Fourier transform of at 501 equispaced frequencies between [0,π].

B) plot its magnitude, angle, real, and imaginary parts.

Q.12 Let $x[n] = \cos(\frac{\pi n}{2}), \quad 0 \le n \le 100$ and $y[n] = e^{j\pi n/4} x[n]$

A) Numerically compute the discrete-time Fourier transform of at 401 equispaced frequencies between [−2π,2π].

B) plot its magnitude, angle spectrum.

C) Comment on the relation between x[n] and y[n].