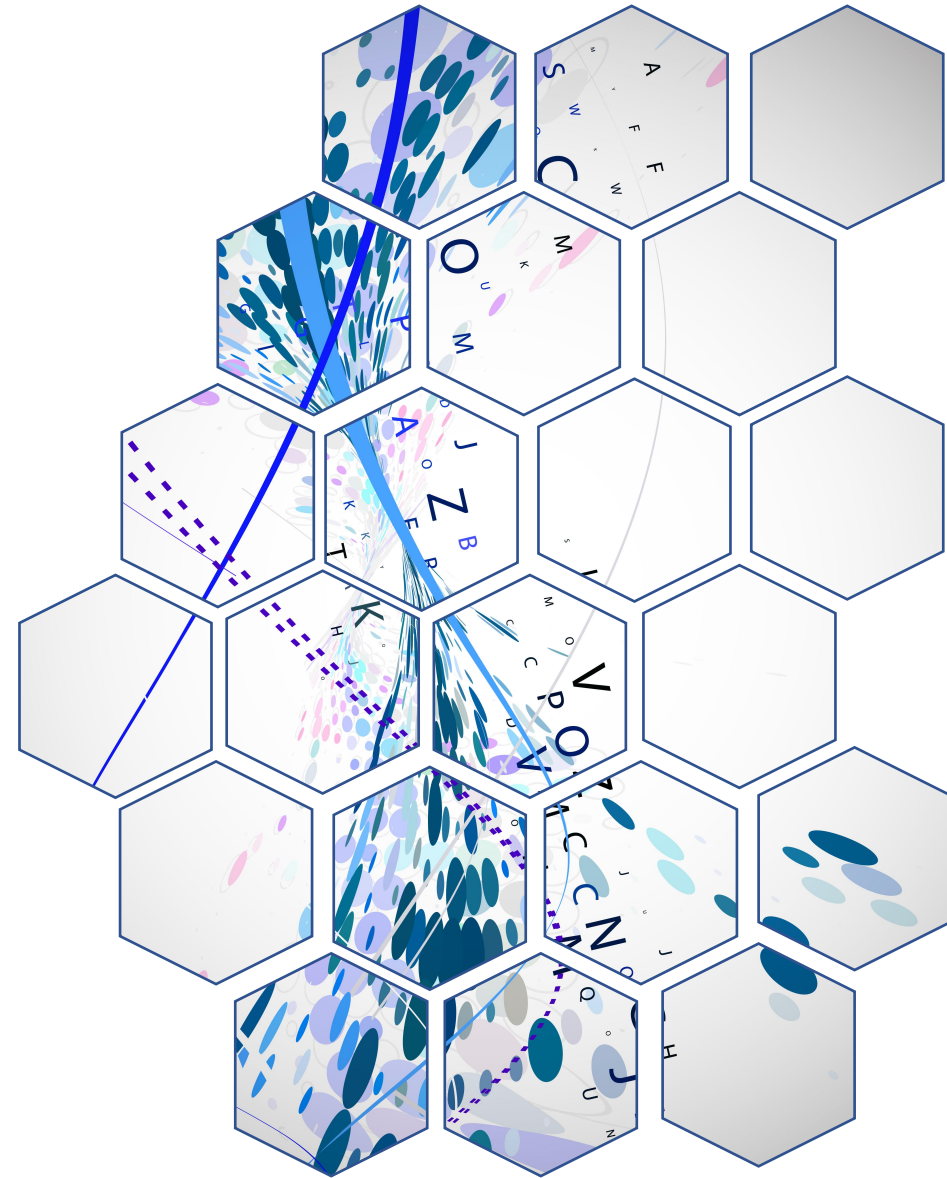


# POLYNOMIAL

## Abstract Algebra

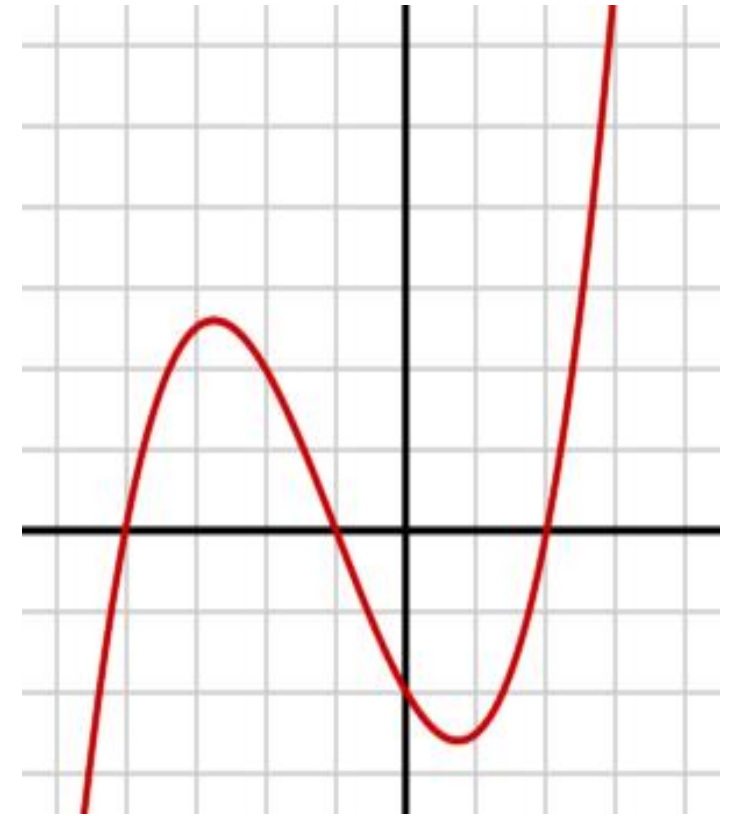


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## OUTLINE :

- INTRODUCTION OF POLYNOMIALS
- TYPES OF POLYNOMIALS
- POLYNOMIAL RINGS
- IRREDUCIBLE POLYNOMIAL
- PSEUDO RANDOM GENERATOR FOR POLYNOMIAL
- POYNOMIAL PSEUDO-RANDOM NUMBER GENERATOR VIA CYCLIC PHASE



# POLYNOMIALS :

- Polynomials are algebraic expressions that consist of variables and coefficients.
- Variables are also sometimes called indeterminates.
- We can perform arithmetic operations such as addition, subtraction, multiplication, and also positive integer exponents for polynomial expressions.

# POLYNOMIALS

The diagram illustrates the components of the polynomial  $5x^2 + 2y - 7$ . The terms are color-coded: the coefficient 5 is purple, the exponent 2 is blue, the variable x is green, the coefficient 2 is purple, the variable y is green, the operator - is black, and the constant 7 is pink. Brackets and lines connect the labels to the corresponding parts of the expression.

**Coefficient**  
**Exponent**  
**Constant**  
**Variable**  
**Operator**

$$5x^2 + 2y - 7$$

# TYPES OF POLYNOMIALS

## Monomials:

A monomial is a polynomial expression that contains only one term. For example  $4t$ ,  $21x$ ,  $2y$ ,  $9pq$ .

## Binomials:

A binomial is a polynomial with two, unlike terms. For example  $3x + 4x^2$

## Trinomial :

A trinomial is a polynomial with three, unlike terms. For example,  $3x + 5x^2 - 6x^3$  and  $12pq + 4x^2 - 10$ .

# DEGREES OF POLYNOMIAL

Zero or constant  
polynomial

Polynomials with 0 degree. Example :  
 $3$  or  $3x^0$

Linear polynomial

Polynomials with 1 as the degree. Example :  $x + y - 4$ ,  $5m + 7n$ ,  $2p$

Quadratic polynomial

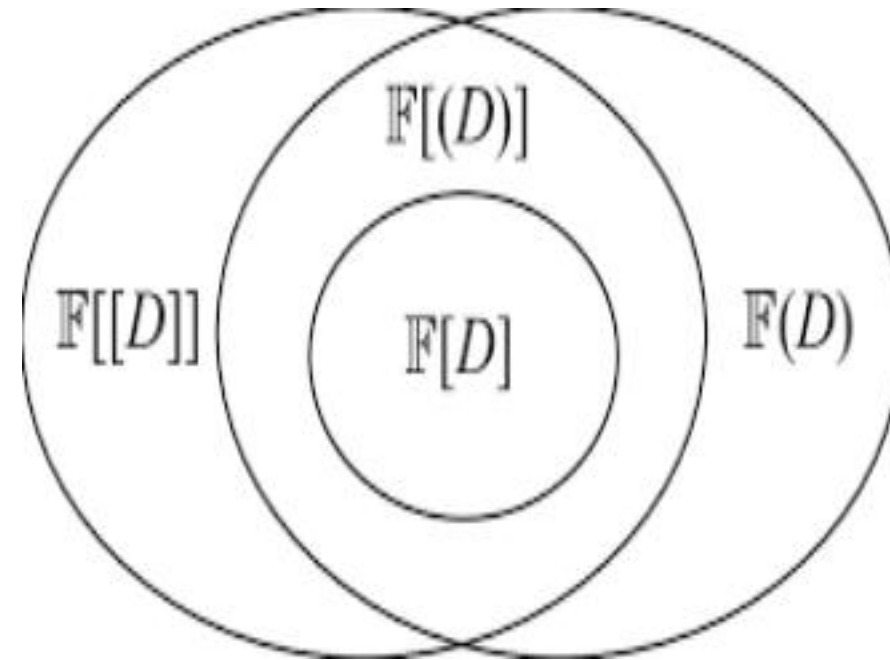
Polynomials with 2 as the degree . Example :  
 $8x^2 + 7y - 9$ ,  $m^2 + mn - 6$

Cubic polynomial

Polynomials with 3 as the degree. Example :  $3x^3$ ,  
 $p^3 + pq + 7$

# POLYNOMIAL RINGS

A polynomial ring or polynomial algebra is a ring formed from the set of polynomials in one or more indeterminates (traditionally also called variables) with coefficients in another ring, often a field.



**fig - Relationship among polynomial rings**



# IRREDUCIBLE POLYNOMIAL

- An irreducible polynomial is a polynomial that cannot be factored into the product of two non-constant polynomials.
- An irreducible polynomial is also called a prime polynomial.

The polynomial  $x^2 - 2 \in \mathbb{Q}[x]$  is irreducible since

**Solution :**

it cannot be factored any further over the rational numbers.

$x^2 + 1$  is irreducible over the real numbers.

# PSEUDO RANDOM GENERATOR FOR POLYNOMIAL

- Pseudorandom generators for low-degree polynomials are a particular instance of a Pseudo Random Number Generator ( PRNG )
- In statistical tests, the tests are considered as evaluations of low degree polynomials.
- Efficient procedure that maps a short truly random seed to a longer pseudorandom string .

# POLYNOMIAL PSEUDO-RANDOM NUMBER GENERATOR VIA CYCLIC PHASE

Polynomials  $f(x)$ :  $f(x) = f_0 + f_1 x + f_2 x^2 + \dots + f_n x^n$ ;  $f_i \in GF(p)$   
are assigned to a Galois field  $GF(p)$ .

If a generic algebra  $A(n)$  is defined in  $GF(p)$  and modular arithmetic modulo  $p$  [12] is used, the algebraic field  $Z(p)$  is the ring of quotients modulo the prime  $p$  (at least for the multiplication operator).

For example, addition of two polynomials in  $GF(2)$  and  $Z_3(2)$   
modulo  $(x^3 + x + 1)$  is:  $f(x) = x^2 + 1$ ;  $f_2(x) = x^2 + x$   $f(x) = \sum f_i(x)$   
 $= f_1(x) + f_2(x) = x + 1$   
This is the polynomial equation .

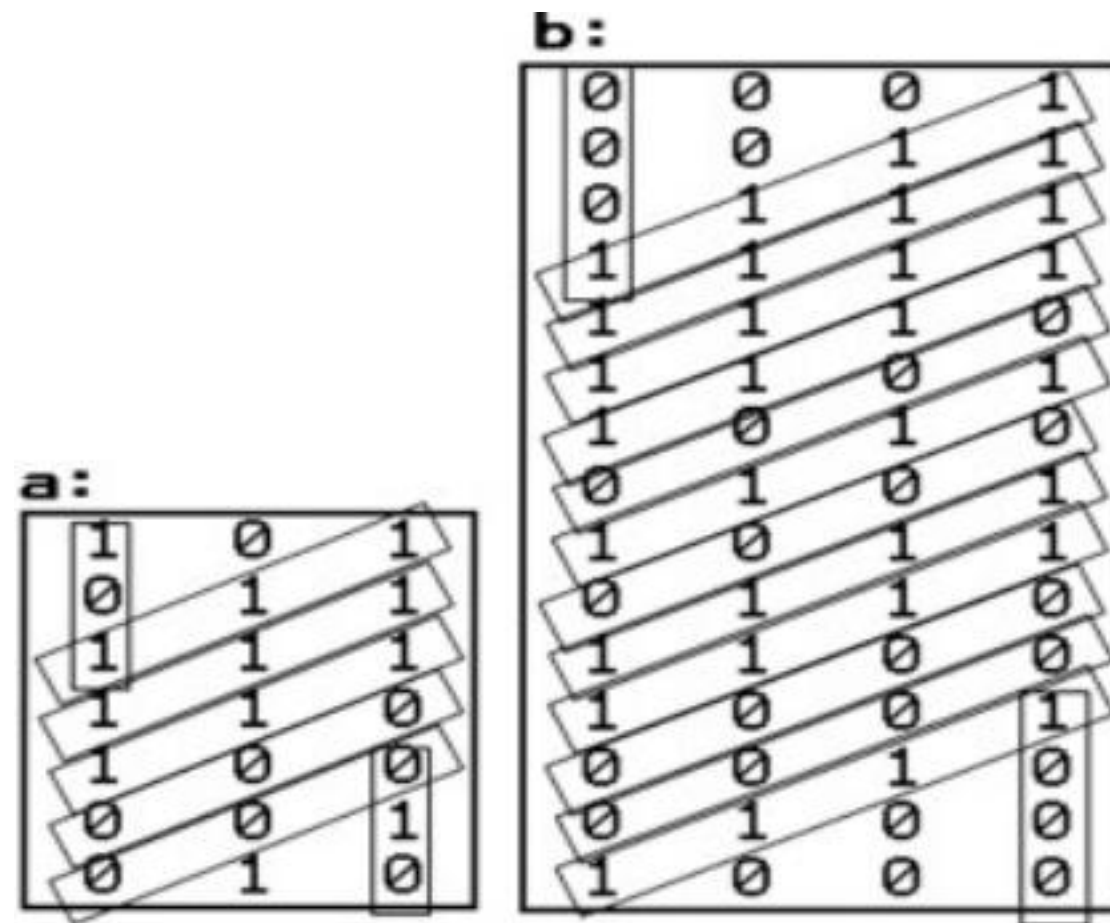


Figure : Pseudo random code generating