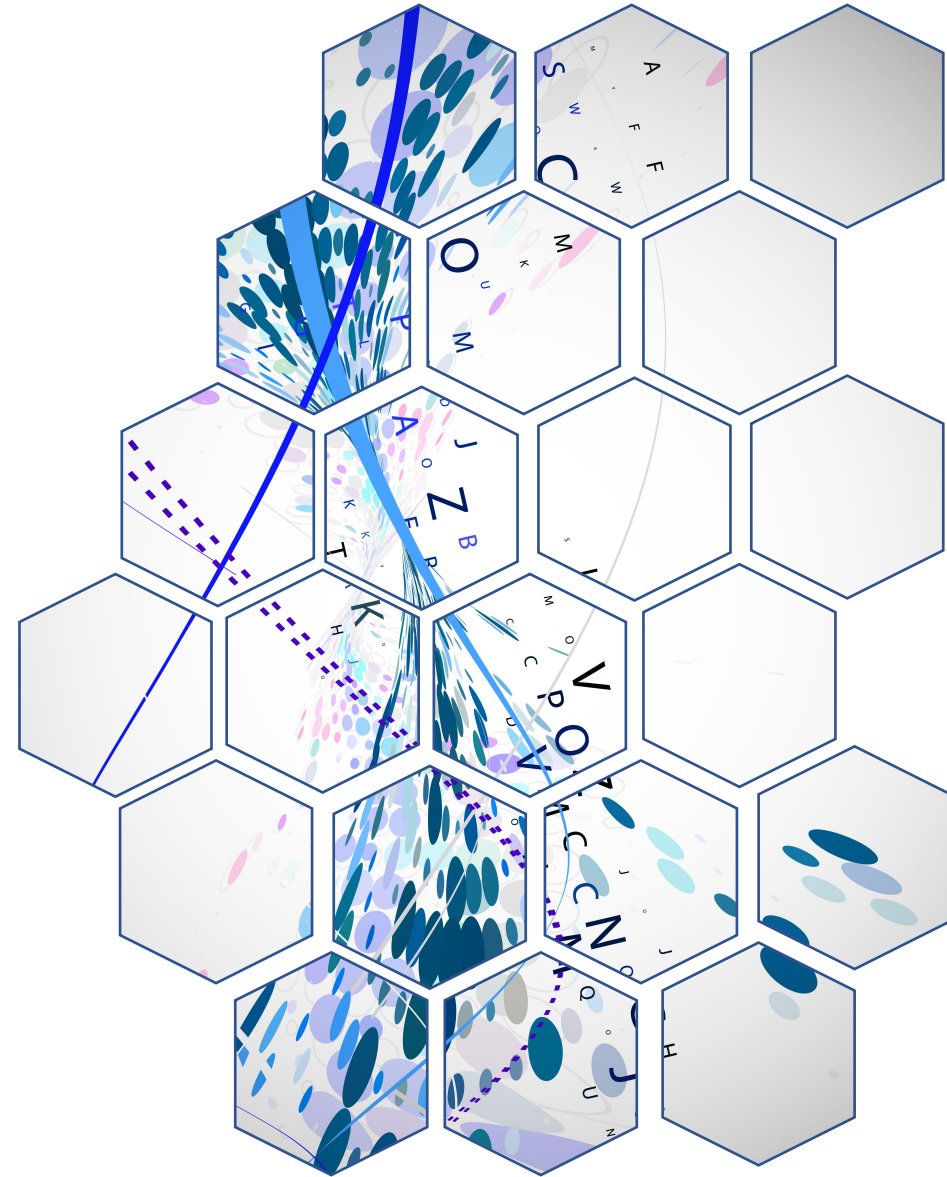


POLYNOMIAL

Abstract Algebra



Submitted by

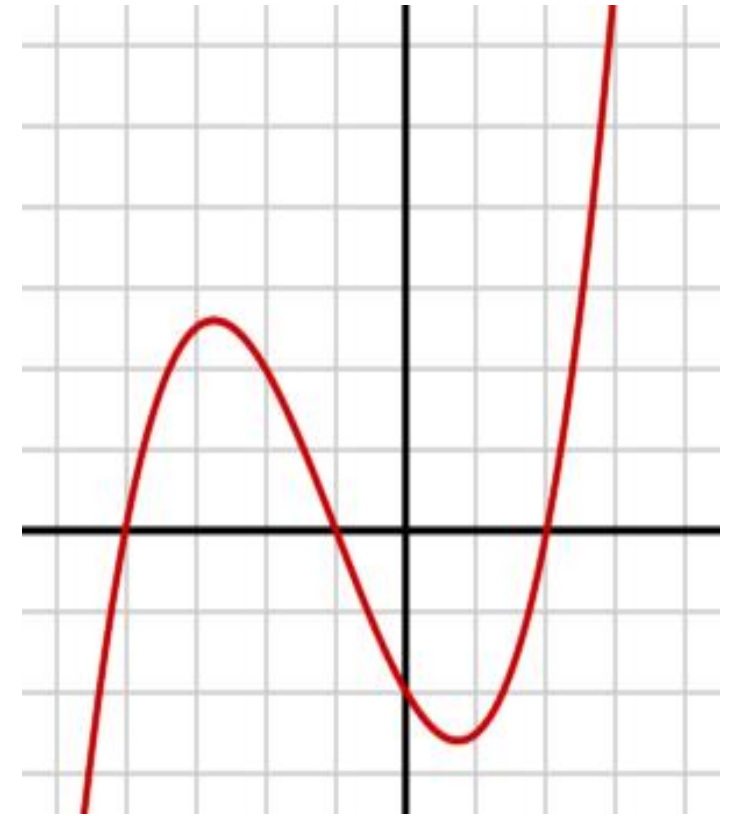
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OUTLINE :

- INTRODUCTION OF POLYNOMIALS
- TYPES OF POLYNOMIALS
- DEGREES OF POLYNOMIAL
- POLYNOMIAL RINGS
- IRREDUCIBLE POLYNOMIAL
- PSEUDO RANDOM GENERATOR FOR POLYNOMIAL
- Applications of Polynomials in Abstract Algebra



POLYNOMIALS :

- Polynomials are algebraic expressions that consist of variables and coefficients.
- Addition, subtraction, multiplication, and also positive integer exponents can be performed for polynomial expressions.

POLYNOMIALS

The diagram illustrates the components of the polynomial $5x^2 + 2y - 7$. The terms are color-coded: the coefficient 5 is purple, the exponent 2 is blue, the variable x is green, the coefficient 2 is purple, the variable y is green, and the constant 7 is pink. Brackets and labels identify the parts: 'Coefficient' points to the 5; 'Exponent' points to the 2; 'Variable' points to the x; 'Operator' points to the '+' sign; and 'Constant' points to the 7. A dashed line also connects the '-' sign to the 'Operator' label.

$$5x^2 + 2y - 7$$

Labels and connections:

- Coefficient** points to 5.
- Exponent** points to 2.
- Variable** points to x.
- Operator** points to + and -.
- Constant** points to 7.

TYPES OF POLYNOMIALS

Monomials:

A monomial is a polynomial expression that contains only one term. For example $4t$, $21x$, $2y$, $9pq$.

Binomials:

A binomial is a polynomial with two, unlike terms. For example $3x + 4x^2$

Trinomial :

A trinomial is a polynomial with three, unlike terms. For example, $3x + 5x^2 - 6x^3$ and $12pq + 4x^2 - 10$.

DEGREES OF POLYNOMIAL

Zero or constant
polynomial

Polynomials with 0 degree. Example :
 3 or $3x^0$

Linear polynomial

Polynomials with 1 as the degree. Example : $x + y - 4$, $5m + 7n$, $2p$

Quadratic polynomial

Polynomials with 2 as the degree . Example :
 $8x^2 + 7y - 9$, $m^2 + mn - 6$

Cubic polynomial

Polynomials with 3 as the degree. Example : $3x^3$,
 $p^3 + pq + 7$

POLYNOMIAL RINGS

A polynomial ring $R[x]$ consists of polynomials where the coefficients come from a ring R .

Basic operations in polynomial rings:

Addition:

Combine like terms.

Multiplication:

Distribute terms and multiply coefficients.

Factorization:

Break down polynomials into irreducible factors.

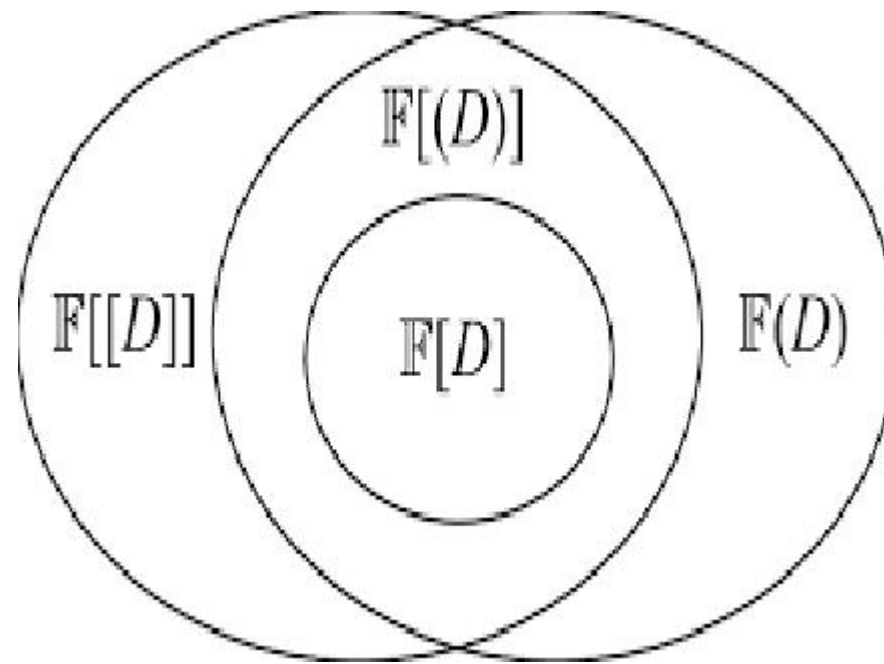


fig - Relationship among polynomial rings

IRREDUCIBLE POLYNOMIAL

- An irreducible polynomial is a polynomial that cannot be factored into the product of two non-constant polynomials.
- An irreducible polynomial is also called a prime polynomial.

The polynomial $x^2 - 2 \in \mathbb{Q}[x]$ is irreducible since

Solution :

it cannot be factored any further over the rational numbers.

$x^2 + 1$ is irreducible over the real numbers.

PSEUDO RANDOM GENERATOR FOR POLYNOMIAL

- Pseudorandom generators for low-degree polynomials are a particular instance of a Pseudo Random Number Generator (PRNG)
- In statistical tests, the tests are considered as evaluations of low degree polynomials.
- Efficient procedure that maps a short truly random seed to a longer pseudorandom string .

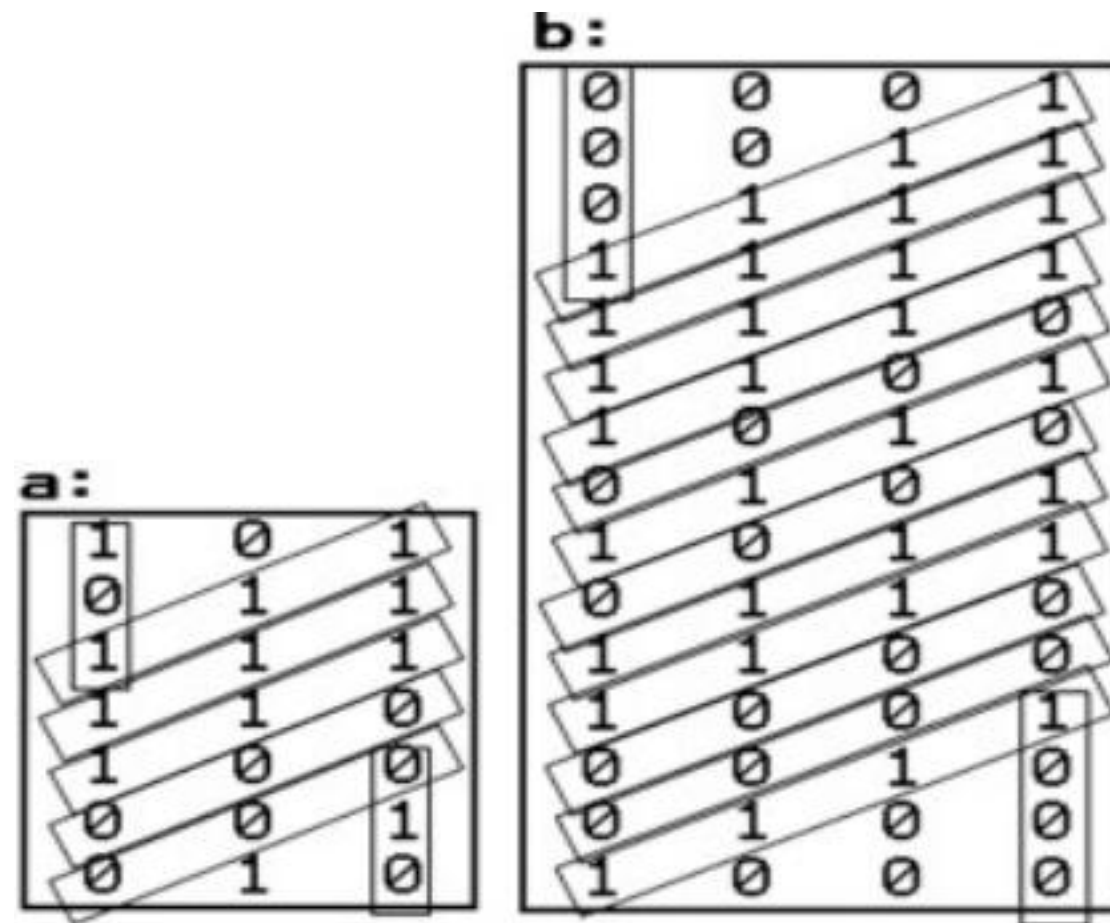


Figure : Pseudo random code generating

Applications of Polynomials in Abstract Algebra:

1. Coding Theory
2. Cryptography
3. Algebraic Geometry
4. Commutative Algebra

Reffenences :

1. Abstract Algebra : Theory and Applications

Thomas W. Judson

2. Polynomial pseudo random number generator via cyclic phase

Angelo Marchi, Alfonso Del Giudice, Antonio Liverani

Faculty of Economics

THANK YOU