

$$T(n) = \begin{cases} 1 & , n=1 \\ 2T\left(\frac{n}{2}\right) + n & \end{cases}$$

$$\begin{aligned}
 T(n) &= 2T\left(\frac{n}{2}\right) + n \\
 &= 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n \\
 &= 2^2 T\left(\frac{n}{2^2}\right) + \cancel{\frac{2^2}{2}} \cdot n + n \\
 &= 2^2 T\left(\frac{n}{2^2}\right) + n + n \\
 &= 2^2 T\left(\frac{n}{2^2}\right) + 2n \\
 &= 2^2 \left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + 2n \\
 &= 2^3 T\left(\frac{n}{2^3}\right) + \cancel{\frac{2^2}{2^2}} \cdot n + 2n \\
 &= 2^3 T\left(\frac{n}{2^3}\right) + n + 2n \\
 &= 2^3 T\left(\frac{n}{2^3}\right) + 3n \\
 &\vdots \\
 &= 2^k T\left(\frac{n}{2^k}\right) + k \cdot n
 \end{aligned}$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2}$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$$

Recurrence Relation  
Substitution Method

$$\begin{aligned}
 &= 2^k T(1) + k \cdot n \\
 &= 2^k * 1 + k \cdot n \\
 &= 2^k + k \cdot n \\
 &= n + n \log n \\
 &= O(n \log n)
 \end{aligned}$$