

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$= 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$$

$$T(n) = \begin{cases} 1, & n=1 \\ 2T\left(\frac{n}{2}\right) + n, & \end{cases}$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2}$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + \frac{2}{2}n + n$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + n + n$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

$$= 2^2 \left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + \frac{2^2}{2^2} \cdot n + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + n + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{2^k}\right) + k \cdot n$$

$$= 2^k T(1) + k \cdot n$$

$$\frac{n}{2^k} = 1$$

$$= 2^k \cdot 1 + k \cdot n$$

$$\Rightarrow n = 2^k$$

$$= 2^k + k \cdot n$$

$$\Rightarrow k = \log_2 n$$

$$= n + n \log n$$

$$= O(n \log n)$$

Recurrence Relation
Substitution Method