

$$F_{xy} = \frac{\partial^2 f}{\partial y \partial x}$$

$$f(x, y) = 2x^3 + 3xy^2$$

$$\frac{\partial f}{\partial x} = 6x^2 + 3y^2$$

$$\frac{\partial^2 f}{\partial x^2} = 12x$$

$$\frac{\partial^3 f}{\partial x^3} = 12$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial y} = 6xy$$

$$6xy \frac{\partial}{\partial x} = 6y$$

$$\frac{\partial f}{\partial x} = 6xy$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 f}{\partial x^2} = 6x$$

$$\frac{\partial^3 f}{\partial y^3} = 0$$

$$\frac{\partial f}{\partial x} = 6x^2 + 3y^2$$

$$6x^2 + 3y^2 \frac{\partial}{\partial y} = 6y$$

F_{xy} and F_{yx} don't need to be equal. If there are partial derivatives for every xy then $F_{xy} = F_{yx}$

Laplace

$$v = (x^2 + y^2 + z^2)^{-\frac{1}{2}} \Rightarrow \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

$$\frac{\partial v}{\partial x} = 2x \cdot \frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -x (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial x^2} = - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \times 2x \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\frac{\partial v}{\partial y} = 2y \cdot \frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -y (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial y^2} = - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \times 2y \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\frac{\partial v}{\partial z} = 2z \cdot \frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -z (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial z^2} = - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \times 2z \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$z = x^2 \arctan\left(\frac{y}{x}\right) \Rightarrow \frac{\partial^2 z}{\partial x \partial y} \Big|_{(1,1)} = ? \quad \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = z_{yx}$$

$$\frac{\partial z}{\partial y} = x^2 \left(\frac{\frac{1}{x}}{1 + \left(\frac{y}{x}\right)^2} \right) = \frac{x \cdot x^2}{x^2 + y^2} = \frac{x^3}{x^2 + y^2}$$

$$\frac{\partial}{\partial x} \left(\frac{x^3}{x^2 + y^2} \right) = \frac{3x^2(x^2 + y^2) - 2x^4}{(x^2 + y^2)^2} = \frac{x^4 + 3x^2 y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} \Big|_{(1,1)} = \frac{1+3}{(1+1)^2} = 1$$

Let's assign Δx as changes for x and Δy for y
 $f(x, y)$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$$

$$\Delta x \approx dx \quad \Delta y \approx dy$$

$$\Delta z = \frac{df}{dx} \cdot \Delta x + \frac{df}{dy} \cdot \Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

where ε_1 is partial derivative for Δx and ε_2 for Δy

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$$

$$\Delta z = \frac{df}{dx} dx + \frac{df}{dy} dy + \varepsilon_1 dx + \varepsilon_2 dy$$

Important part = Differentiation of f

$$z = f(x, y) = x^2y - 3y \Rightarrow \Delta z = ? \quad dz = ?$$

$$\Delta x = -0,01 \quad \Delta y = 0,02$$

$$\Delta z = \left[(x + \Delta x)^2 (y + \Delta y) - 3(y + \Delta y) \right] - (x^2 y - 3y)$$

$$\left[\left(x^2 + 2x\Delta x + (\Delta x)^2 \right) (y + \Delta y) - 3(y + \Delta y) \right] - y(x^2 - 3)$$

$$y + \Delta y \left(x^2 + 2x\Delta x + (\Delta x)^2 - 3 \right)$$

$$\Delta z = 2x\Delta x + (x^2 - 3)\Delta y$$

$$d_2 = 2xy \, dx + x^2 - 3 \, dy$$

As we seen here when Δx and Δy go to 0 then $d_2 \approx \Delta_2$

$$v = x^2 e^{yx} \Rightarrow dv = ? \quad dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy \dots$$

$$dx = \left(2x e^{yx} + \frac{y}{x^2} e^{yx} x^2 \right) dx$$

$$dy = (x e^{yx}) dy$$

$$dv = e^{yx} (2x + y) dx + e^{yx} dy$$

Chain Rule

$$z = f(x, y) \quad x = g(r, s) \quad y = h(r, s)$$

$$z = \begin{matrix} x \\ y \\ \downarrow s \end{matrix} \quad \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$z = e^{x \sin y}, \quad x = \ln t, \quad y = t^2$$

$$\frac{\partial z}{\partial x} = e^{x \sin y}$$

$$\frac{\partial z}{\partial t} \quad \boxed{?}$$

$$\frac{\partial z}{\partial y} = e^x \cos y$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial x}{\partial t} = \frac{1}{t}$$

$$\frac{\partial y}{\partial t} = 2t$$

$$e^{x \sin y} \cdot \frac{1}{t} + e^x \cos y \cdot 2t$$

$$e^x \left(\frac{\sin y}{t} + \cos y 2t \right)$$

$$e^{\ln t} \left(\frac{\sin(t)}{t} + \cos(t) 2t \right)$$

$$\sin(t^2) + 2t^2 \cos(t^2)$$

$$T = \left\{ \begin{array}{l} x - xy + y^3 \\ x = r \cos \theta \quad y = r \sin \theta \end{array} \right.$$

$$\frac{\partial T}{\partial r} = ?$$

$$\overline{T}$$

$$\frac{\partial T}{\partial \theta} = ?$$

$$\begin{array}{c} / \backslash \\ x \quad y \end{array}$$

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$\begin{array}{c} / \backslash \\ r \quad \theta \end{array}$$

$$\frac{\partial T}{\partial r} = (x^2 - y) \cos \theta + (3y^2 - x) \sin \theta$$

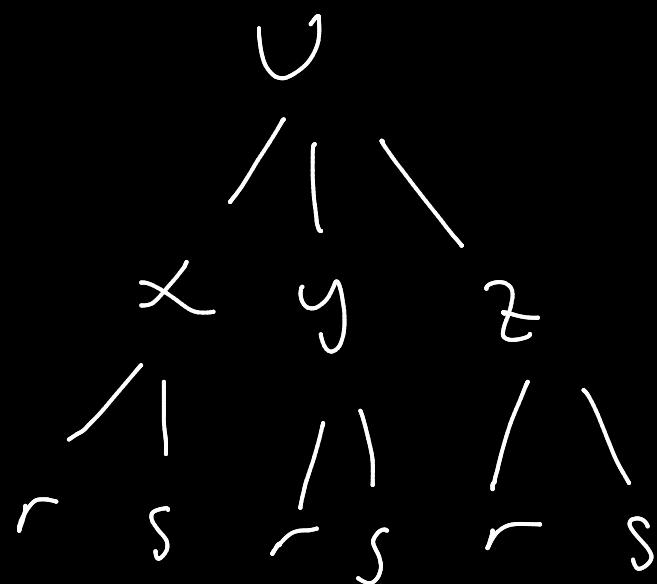
∂_r

$$\frac{\partial T}{\partial \theta} = (x^2 - y) \cdot -r \sin \theta + (3y^2 - x) r \cos \theta$$

∂_θ

$$v = \sin\left(\frac{s}{r}\right), x = 3r^2 + s, y = 4r - 2s^3, z = 2r^2 - 3s^2$$

$$\frac{dv}{ds} = ?, \quad \frac{dv}{dr} = ?$$



$$\frac{dv}{ds} = \frac{dv}{dx} \cdot \frac{dx}{ds} + \frac{dv}{dy} \cdot \frac{dy}{ds} + \frac{dv}{dz} \cdot \frac{dz}{ds}$$

$$\frac{dv}{dr} = \frac{dv}{dx} \cdot \frac{dx}{dr} + \frac{dv}{dy} \cdot \frac{dy}{dr} + \frac{dv}{dz} \cdot \frac{dz}{dr}$$