

$$F_{xy} = \frac{\partial^2 f}{\partial y \partial x}$$

$$f(x, y) = 2x^3 + 3xy^2$$

$$\frac{\partial f}{\partial x} = 6x^2 + 3y^2$$

$$\frac{\partial^2 f}{\partial x^2} = 12x$$

$$\frac{\partial^3 f}{\partial x^3} = 12$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial y} = 6xy$$

$$6xy \frac{\partial}{\partial x} = 6y$$

$$\frac{\partial f}{\partial x} = 6xy$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 f}{\partial x^2} = 6x$$

$$\frac{\partial^3 f}{\partial y^3} = 0$$

$$\frac{\partial f}{\partial x} = 6x^2 + 3y^2$$

$$6x^2 + 3y^2 \frac{\partial}{\partial y} = 6y$$

F_{xy} and F_{yx} don't need to be equal. If there are partial derivatives for every xy then $F_{xy} = F_{yx}$

Laplace

$$v = (x^2 + y^2 + z^2)^{-\frac{1}{2}} \Rightarrow \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

$$\frac{\partial v}{\partial x} = 2x \cdot \frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -x (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial x^2} = - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \times 2x \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\frac{\partial v}{\partial y} = 2y \cdot \frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -y (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial y^2} = - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \times 2y \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\frac{\partial v}{\partial z} = 2z \cdot \frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -z (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial z^2} = - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \times 2z \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$z = x^2 \arctan\left(\frac{y}{x}\right) \Rightarrow \frac{\partial^2 z}{\partial x \partial y} \Big|_{(1,1)} = ? \quad \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = z_{yx}$$

$$\frac{\partial z}{\partial y} = x^2 \left(\frac{\frac{1}{x}}{1 + \left(\frac{y}{x}\right)^2} \right) = \frac{x \cdot x^2}{x^2 + y^2} = \frac{x^3}{x^2 + y^2}$$

$$\frac{\partial}{\partial x} \left(\frac{x^3}{x^2 + y^2} \right) = \frac{3x^2(x^2 + y^2) - 2x^4}{(x^2 + y^2)^2} = \frac{x^4 + 3x^2 y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} \Big|_{(1,1)} = \frac{1+3}{(1+1)^2} = 1$$

Let's assign Δx as changes for x and Δy for y
 $f(x, y)$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$$

$$\Delta x \approx dx \quad \Delta y \approx dy$$

$$\Delta z = \frac{df}{dx} \cdot \Delta x + \frac{df}{dy} \cdot \Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

where ε_1 is partial derivative for Δx and ε_2 for Δy

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$$

$$\Delta z = \frac{df}{dx} dx + \frac{df}{dy} dy + \varepsilon_1 dx + \varepsilon_2 dy$$

Important part = Differentiation of f

$$z = f(x, y) = x^2y - 3y \Rightarrow \Delta z = ? \quad dz = ?$$

$$\Delta x = -0,01 \quad \Delta y = 0,02$$

$$\Delta z = \left[(x + \Delta x)^2 (y + \Delta y) - 3(y + \Delta y) \right] - (x^2 y - 3y)$$

$$\left[\left(x^2 + 2x\Delta x + (\Delta x)^2 \right) (y + \Delta y) - 3(y + \Delta y) \right] - y(x^2 - 3)$$

$$y + \Delta y \left(x^2 + 2x\Delta x + (\Delta x)^2 - 3 \right)$$

$$\Delta z = 2 \times y \Delta x + (x^2 - 3) \Delta y$$

$$d_2 = 2xy \, dx + x^2 - 3 \, dy$$

As we seen here when Δx and Δy go to 0 then $d_2 \approx \Delta_2$

$$v = x^2 e^{yx} \Rightarrow dv = ? \quad dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy \dots$$

$$dx = \left(2x e^{yx} + \frac{y}{x^2} e^{yx} x^2 \right) dx$$

$$dy = (x e^{yx}) dy$$

$$dv = e^{yx} (2x + y) dx + e^{yx} dy$$

Chain Rule

$$z = f(x, y) \quad x = g(r, s) \quad y = h(r, s)$$

$$z = \begin{matrix} x \\ y \\ \downarrow s \end{matrix} \quad \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$z = e^{x \sin y}, \quad x = \ln t, \quad y = t^2$$

$$\frac{\partial z}{\partial x} = e^{x \sin y}$$

$$\frac{\partial z}{\partial t} \quad \boxed{?}$$

$$\frac{\partial z}{\partial y} = e^x \cos y$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial x}{\partial t} = \frac{1}{t}$$

$$\frac{\partial y}{\partial t} = 2t$$

$$e^{x \sin y} \cdot \frac{1}{t} + e^x \cos y \cdot 2t$$

$$e^x \left(\frac{\sin y}{t} + \cos y 2t \right)$$

$$e^{\ln t} \left(\frac{\sin(t)}{t} + \cos(t) 2t \right)$$

$$\sin(t^2) + 2t^2 \cos(t^2)$$

$$T = \left\{ \begin{array}{l} x - xy + y^3 \\ x = r \cos \theta \quad y = r \sin \theta \end{array} \right.$$

$$\frac{\partial T}{\partial r} = ?$$

$$\overline{T}$$

$$\frac{\partial T}{\partial \theta} = ?$$

$$\begin{array}{c} / \backslash \\ x \quad y \end{array}$$

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$\begin{array}{c} / \backslash \\ r \quad \theta \end{array}$$

$$\frac{\partial T}{\partial r} = (x^2 - y) \cos \theta + (3y^2 - x) \sin \theta$$

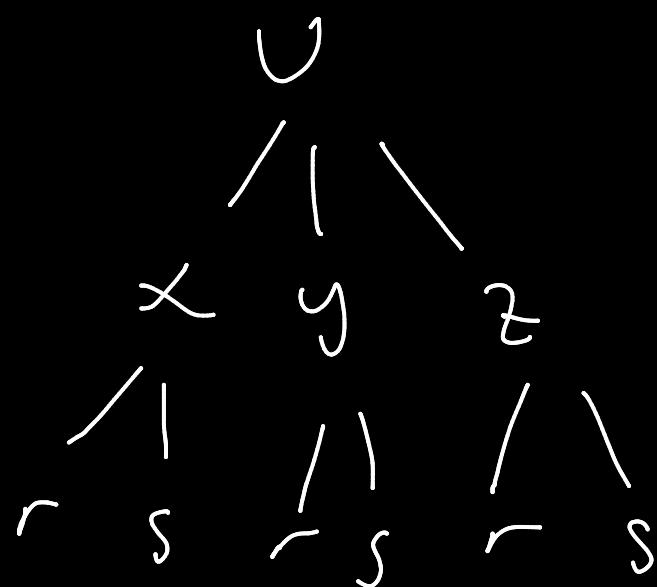
∂r

$$\frac{\partial T}{\partial \theta} = (x^2 - y) \cdot -r \sin \theta + (3y^2 - x) r \cos \theta$$

$\partial \theta$

$$v = \sin\left(\frac{s}{r}\right), x = 3r^2 + s, y = 4r - 2s^3, z = 2r^2 - 3s^2$$

$$\frac{dv}{ds} = ?, \quad \frac{dv}{dr} = ?$$



$$\frac{dv}{ds} = \frac{dv}{dx} \cdot \frac{dx}{ds} + \frac{dv}{dy} \cdot \frac{dy}{ds} + \frac{dv}{dz} \cdot \frac{dz}{ds}$$

$$\frac{dv}{dr} = \frac{dv}{dx} \cdot \frac{dx}{dr} + \frac{dv}{dy} \cdot \frac{dy}{dr} + \frac{dv}{dz} \cdot \frac{dz}{dr}$$

$z = f(v)$ $v = xy$ prove that $\frac{\partial z}{\partial x} = 2y \frac{\partial z}{\partial y}$

$$\begin{array}{c} z \\ | \\ v \\ / \backslash \\ x \quad y \end{array} \quad \frac{\partial z}{\partial x} = \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$\frac{\partial z}{\partial v} = \frac{1}{xy} \frac{\partial z}{\partial x} \quad \left[\frac{1}{xy} \frac{\partial z}{\partial x} = \frac{1}{x^2} \frac{\partial z}{\partial y} \right]$$

$$\frac{\partial z}{\partial v} = \frac{1}{x^2} \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} = 2y \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} = 2y \frac{\partial z}{\partial y}$$

$$\begin{aligned} & \left\{ \begin{array}{l} x+y=t \\ x^2+y^3=t^2 \end{array} \right. \quad v = x^3 y \quad \frac{dv}{dt} = ? \\ & \begin{array}{c} v \\ / \quad \backslash \\ x \quad y \\ | \quad | \\ t \quad t \end{array} \end{aligned}$$

$$\frac{dv}{dt} = \underbrace{\frac{dv}{dx}}_{3x^2} \cdot \frac{dx}{dt} + \underbrace{\frac{dv}{dy}}_{x^3} \cdot \frac{dy}{dt}$$

$$\frac{d}{dt} (x+y=t) = s_x^4 \frac{dx}{dt} + \frac{dy}{dt} = 1$$

$$\begin{bmatrix} s_x^4 & 1 \\ 2x & 3y^2 \end{bmatrix} \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix} \begin{bmatrix} 1 \\ 2t \end{bmatrix}$$

$$\frac{dx}{dt} = \frac{\begin{bmatrix} 1 & 1 \\ 2t & 3y^2 \end{bmatrix}}{\begin{bmatrix} s_x^4 & 1 \\ 2x & 3y^2 \end{bmatrix}} = \frac{3y^2 - 2t}{15x^4 y^2 - 2x}$$

$$\frac{dv}{dt} = 3x^2 y \left(\frac{3y^2 - 2t}{15x^4 y^2 - 2x} \right) + x^3 \left(\frac{10xt - 2x}{15x^4 y^2 - 2x} \right)$$

$$\frac{dy}{dt} = \frac{\begin{bmatrix} s_x^4 & 1 \\ 2x & 2t \end{bmatrix}}{\begin{bmatrix} s_x^4 & 1 \\ 2x & 3y^2 \end{bmatrix}} = \frac{10xt - 2x}{15x^4 y^2 - 2x}$$

Jacob (obj) & m

$$\frac{\partial(F, G)}{\partial(v, w)} = \begin{vmatrix} F_v & F_w \\ G_v & G_w \end{vmatrix}$$

$$F(v, w) = 0 \quad G(v, w) = 0 \quad H(v, w) = 0$$

$$\frac{\partial(F, G, H)}{\partial(v, w)} = \begin{vmatrix} F_v & F_w & F_h \\ G_v & G_w & G_h \\ H_v & H_w & H_h \end{vmatrix}$$

$$F(x, y, v, w) = 0 \quad G(x, y, v, w) = 0$$

$$\frac{\partial v}{\partial x} = - \frac{\begin{vmatrix} F_x & F_w \\ G_x & G_w \end{vmatrix}}{\begin{vmatrix} F_v & F_w \\ G_v & G_w \end{vmatrix}} \quad \frac{\partial v}{\partial x} = - \frac{\begin{vmatrix} F_v & F_x \\ G_v & G_x \end{vmatrix}}{\begin{vmatrix} F_v & F_w \\ G_v & G_w \end{vmatrix}}$$

$$\frac{\partial v}{\partial y} = - \frac{\begin{vmatrix} F_y & F_w \\ G_y & G_w \end{vmatrix}}{\begin{vmatrix} F_v & F_w \\ G_v & G_w \end{vmatrix}} \quad \frac{\partial v}{\partial y} = - \frac{\begin{vmatrix} F_v & F_y \\ G_v & G_y \end{vmatrix}}{\begin{vmatrix} F_v & F_w \\ G_v & G_w \end{vmatrix}}$$

$$\begin{aligned} v^2 - u &= 3x + y \\ v - 2u^2 &= x - 2y \end{aligned} \quad \left. \frac{\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}}{} \right\} ,$$

$$f(v, u, x, y) = 0 \Rightarrow v^2 - u - 3x - y = 0$$

$$g(v, u, x, y) = 0 \Rightarrow v - 2u^2 - x + 2y = 0$$

$$\frac{\frac{\partial v}{\partial x}}{\frac{\partial u}{\partial x}} = - \frac{\begin{vmatrix} F_x & F_u \\ 6x & 6u \end{vmatrix}}{\begin{vmatrix} F_v & F_u \\ 6v & 6u \end{vmatrix}} = - \frac{\begin{vmatrix} -3 & -1 \\ -1 & -4u \end{vmatrix}}{\begin{vmatrix} 2v & -1 \\ 1 & -4 \end{vmatrix}} = - \frac{12u - 1}{-8v + 1}$$

$$\frac{\frac{\partial v}{\partial y}}{\frac{\partial u}{\partial y}} = - \frac{\begin{vmatrix} F_y & F_u \\ 6y & 6u \end{vmatrix}}{\begin{vmatrix} F_v & F_u \\ 6v & 6u \end{vmatrix}} = - \frac{\begin{vmatrix} -1 & -1 \\ 2 & -4u \end{vmatrix}}{\begin{vmatrix} 2u & -1 \\ 1 & -4 \end{vmatrix}} = - \frac{4u + 2}{-8v + 1}$$

$$\frac{\frac{\partial v}{\partial x}}{\frac{\partial u}{\partial x}} = - \frac{\begin{vmatrix} F_u & F_x \\ 6u & 6x \end{vmatrix}}{\begin{vmatrix} F_v & F_u \\ 6v & 6u \end{vmatrix}} = - \frac{\begin{vmatrix} 2u & -3 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2u & -1 \\ 1 & -4 \end{vmatrix}} = - \frac{-2u + 3}{-8v + 1}$$

$$\frac{\frac{\partial v}{\partial y}}{\frac{\partial u}{\partial y}} = - \frac{\begin{vmatrix} F_u & F_y \\ 6u & 6y \end{vmatrix}}{\begin{vmatrix} F_v & F_u \\ 6v & 6u \end{vmatrix}} = - \frac{\begin{vmatrix} 2u & -1 \\ 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2u & -1 \\ 1 & -4 \end{vmatrix}} = - \frac{4u + 1}{-8v + 1}$$

$$z^3 - xz - y = 0$$

$$\frac{d^2 z}{dx dy} = ?$$

It is closed-form
but we have 2 independent but
1 dependent variable that's why
we can't use jacobian

$$\frac{\partial z}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right)$$

$$\frac{dz}{dy} = 3z^2 \frac{dz}{dy} - x \frac{dz}{dy} - 1 = 0$$

$$\frac{dz}{dy} = \frac{1}{3z^2 - x}$$

We still
don't know

$$\frac{\partial}{\partial x} \left(\frac{1}{3z^2 - x} \right) = \frac{\partial (3z^2 - x)^{-1}}{\partial x} = - (3z^2 - x)^{-2} \cdot \left(6z \frac{dz}{dx} - 1 \right) = \frac{6z \frac{dz}{dx} - 1}{-(3z^2 - x)^2}$$

$$\frac{\partial z}{\partial x} = 3z^2 \frac{dz}{dx} - z - x \frac{dz}{dx} = 0 \quad \Rightarrow \frac{\partial z}{\partial x} = \frac{z}{(3z^2 - x)}$$

$$\frac{6z}{3z^2 - x} - 1$$

$$-\frac{(3z^2 - x)^2}{(3z^2 - x)^2}$$

$$v = \sqrt{xy}$$

is there a connection
between them?
If it's then how?

$$\begin{array}{ccc} v & & u \\ / \backslash & & / \backslash \\ x & y & x & y \end{array}$$

$$\begin{vmatrix} v_x & v_y \\ u_x & u_y \end{vmatrix} - \begin{vmatrix} \frac{y}{2\sqrt{xy}} & \frac{x}{2\sqrt{xy}} \\ -ye^{-xy} + y & -xe^{-xy} + x \end{vmatrix} = 0$$

so there is a
functions) connection
between v and u

$$v^2 = xy$$

$$u = e^{-xy} + xy \rightarrow u - e^{-v^2} + v^2 = 0$$

$$x = r \cos \theta$$

$$0 = r \cos \theta$$

$$\theta = 90^\circ \text{ and } r = 0$$

$$0 = r \sin \theta$$

$$\theta = 0^\circ \text{ and } r = 0$$

Directional

Derivative

$$\frac{\partial F}{\partial s} = \nabla F \cdot T$$

$$F = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
$$x = e^{-v}$$
$$y = 2 \sin v + 1$$
$$z = v - \cos v$$
$$v = 0$$

Find directional derivative
through x, y, z

$$\nabla F = \begin{pmatrix} 2xy \\ x^2 \\ x^2y^3z^2 \end{pmatrix} \quad T = \frac{T_0}{|T_0|} \quad T_0 = \frac{dr}{dv}$$

$$\begin{matrix} v=0 \\ x=1 \\ y=1 \\ z=-1 \end{matrix} \quad \left. \nabla F \right|_{v=0} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$$

$$r = \begin{pmatrix} i \\ j \\ k \end{pmatrix}$$

$$r = \begin{pmatrix} e^{-v} \\ 2 \sin v + 1 \\ v - \cos v \end{pmatrix}$$

$$\frac{dr}{dv} = \begin{pmatrix} -e^{-v} \\ 2 \cos v \\ 1 + \sin v \end{pmatrix}$$

$$\left. T_0 \right|_{v=0} = \begin{pmatrix} i \\ j \\ k \end{pmatrix}$$

$$\left(-2 \hat{i} - \hat{j} + 3 \hat{k} \right) \cdot \left(\frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}} \right)$$

$$= (2 - 2 + 3) \frac{1}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2}$$

$$T = \frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}$$

$$F = 2x^3y - 3y^2z \quad P(1, 2, -1) \quad Q(3, -1, 5)$$

Find directional derivative of F for P through Q direction.

$$\nabla F \cdot T = 6x^2y \hat{i} + (2x - 6yz) \hat{j} - 3y^2 \hat{k}$$

$$|\nabla F|_P = 12 \hat{i} + 14 \hat{j} - 12 \hat{k}$$

$$T_0 = r_q - r_p = (3\hat{i} - \hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$T = \frac{T_0}{|T_0|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4+9+36}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

$$\nabla F \cdot T = (12\hat{i} + 14\hat{j} - 12\hat{k}) \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7}$$

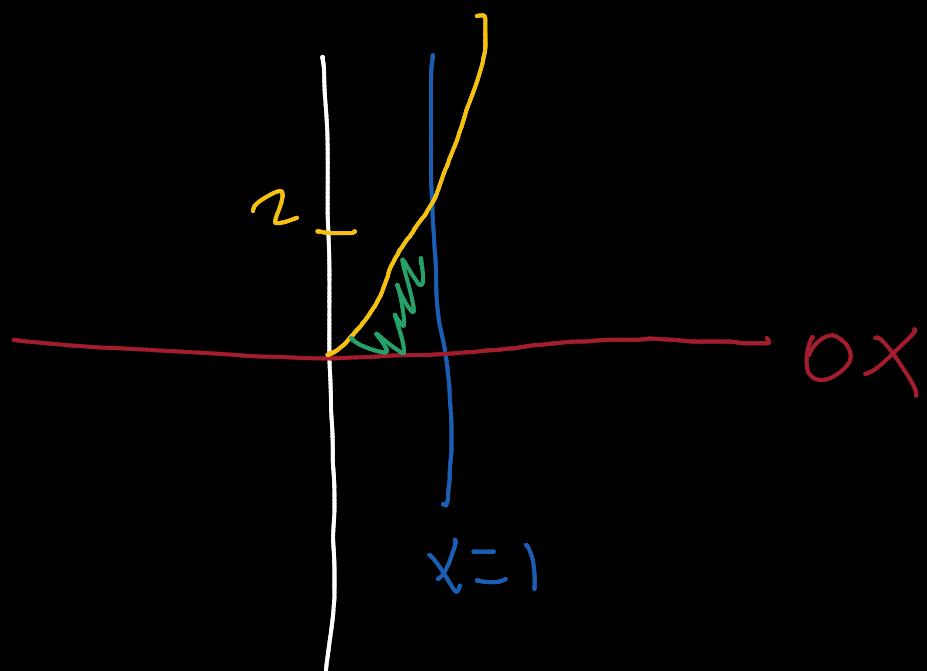
$$= \frac{24 - 42 - 72}{7} = \frac{-90}{7}$$

what is the max value of directional derivative?

$$|\nabla F|_P = \sqrt{12^2 + 14^2 + (-12)^2}$$

Double Integral

$$y = 2x^2 \quad x=1 \quad \text{and} \quad 0x \quad f(x,y) = x^2y - x + y$$



$$\iint_D x^2y - x + y \, dy \, dx$$

$$\left[\frac{x^2y}{2} - xy + \frac{y^2}{2} \right]_0^{2x}$$

$$x^2 \frac{(2x^2)^2}{2} - x(2x^2) + \frac{(2x^2)^2}{2}$$

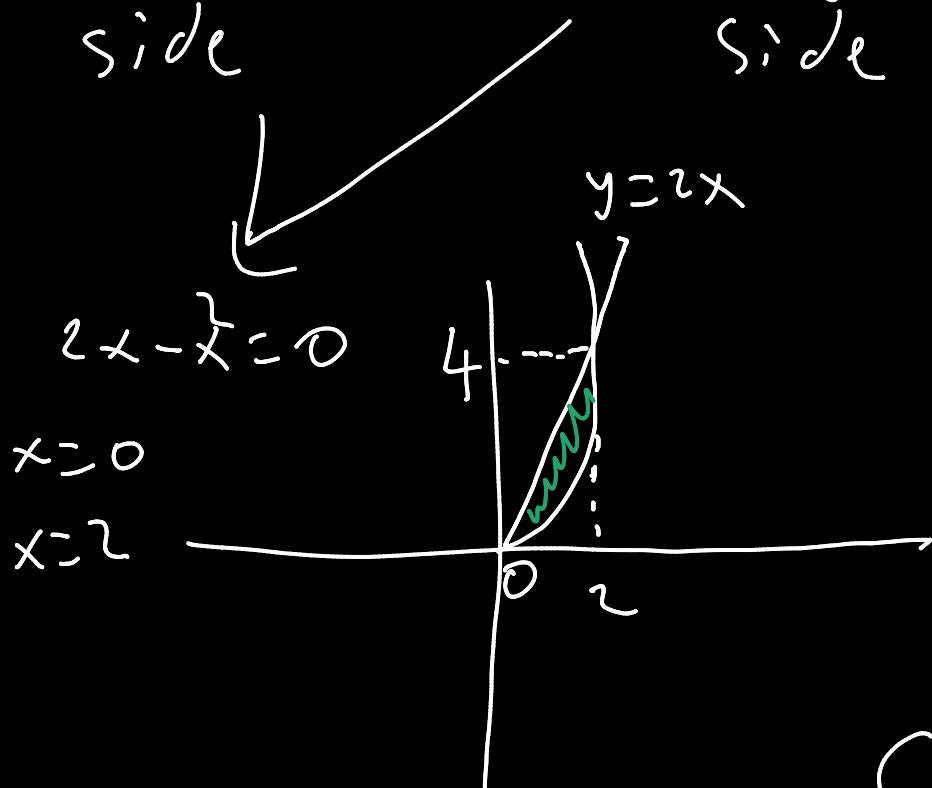
$$\int_0^1 2x^6 - 2x^3 + 2x^4 \, dx$$

$$2 \left(\frac{x^7}{7} - \frac{x^4}{4} + \frac{x^5}{5} \right) \Big|_0^1$$

$$2 \left(\frac{1}{7} - \frac{1}{4} + \frac{1}{5} \right) = \frac{20 - 35 + 28}{70} = \frac{13}{70}$$

$$y \leq 2x, x^2 - y \leq 0 \quad f(x,y) = x^3 + y$$

$y=2x$
but shrinking side
 $x=y$
but growing side



$$\int_0^2 \int_{x^2}^{2x} x^3 + y dy dx$$

$$\int_0^2 \left(x^3 y + \frac{y^2}{2} \right) \Big|_0^2 dx$$

$$\int_0^2 \left(2x^4 + 2x^2 \right) - \left(x^5 + \frac{x^4}{2} \right) dx$$

Area of the defined place

$$\int_0^2 \int_{x^2}^{2x} dy dx$$

$$\left. \frac{2x^5}{5} + 2x^3 - \frac{x^6}{6} - \frac{x^5}{10} \right|_0^2$$

$$\int_0^2 (2x - x^2) dx$$

$$\frac{64}{5} + \frac{16}{3} - \frac{64}{6} - \frac{32}{10}$$

$$\left. x^2 - \frac{x^3}{3} \right|_0^2$$

$$= \frac{394 + 160 - 320 - 96}{30} = \frac{128}{30}$$

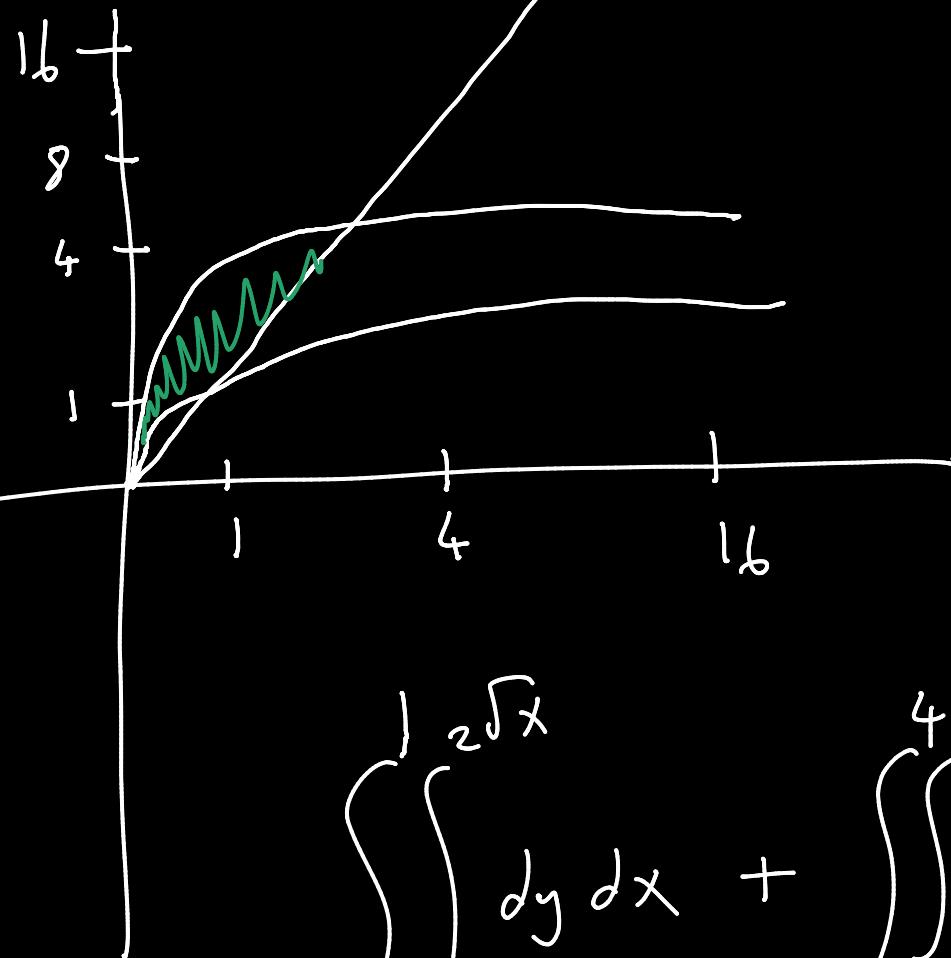
$$= 4 - \frac{8}{3}$$

$$= \frac{4}{3} \text{ or } \sim$$

$$= \frac{64}{15}$$

$$\begin{aligned}y &= 2\sqrt{x} \\y &= \sqrt{x} \\y &= x\end{aligned}$$

Area of the this
conjunction



$$\int_{0}^{\sqrt{x}} dy dx + \int_{\sqrt{x}}^{2\sqrt{x}} dy dx$$

$$\int_0^1 (2\sqrt{x} - \sqrt{x}) dx + \int_1^4 (2\sqrt{x} - x) dx$$

$$\left[\frac{2x^{\frac{3}{2}}}{3} \right]_0^1 + \left[\frac{4x^{\frac{3}{2}}}{3} - \frac{x^2}{2} \right]_1^4$$

$$= \frac{2}{3} + \left(\frac{32}{3} - 8 \right) - \left(\frac{4}{3} - \frac{1}{2} \right)$$

$$= \frac{2}{3} + \frac{8}{3} - \frac{5}{6} - \frac{4}{6} + \frac{16}{6} - \frac{5}{6} = \frac{15}{6} = \frac{5}{2}$$

$x^2 + y^2 \leq 4x$ area of this?

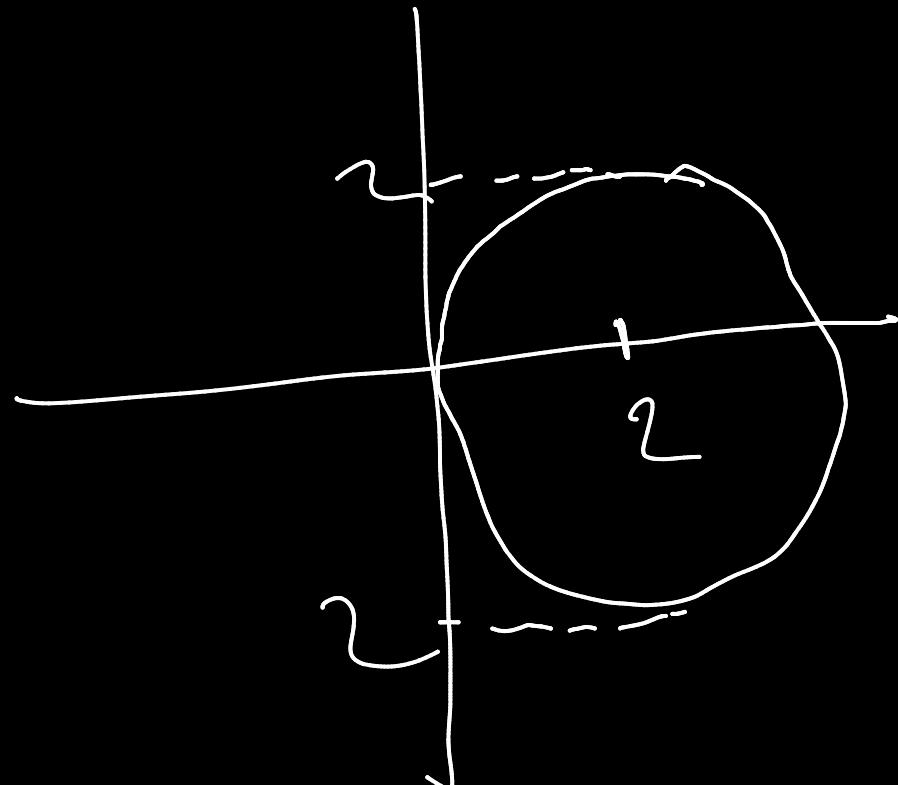
$$x^2 + y^2 - 4x = 0$$

$$x^2 + y^2 - 4x + 4 - 4 = 0$$

$$x^2 - 4x + 4 + y^2 = 4$$

$$(x-2)^2 + y^2 = 4$$

$$(x-a)^2 + (y-b)^2 = r^2$$



$$y = \sqrt{4x - 4}$$

$$y = \sqrt{4 - (x-2)^2}$$

choose this for easier integration
dx = dv

$$\frac{A}{2} = \int_0^2 \int_0^{\sqrt{4-(x-2)^2}} dy dx = \int_{-2}^2 \int_{-\sqrt{4-v^2}}^{\sqrt{4-v^2}} dv = \int_{-2}^2 \int_{-\sqrt{4-v^2}}^{\sqrt{4-v^2}} dv$$

$$\sqrt{a^2 - x^2} \rightarrow x = a \sin t$$

$$v = 2 \sin t$$

$$d_v = 2 \cos t dt$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{4 - (2 \sin t)^2} 2 \cos t dt$$

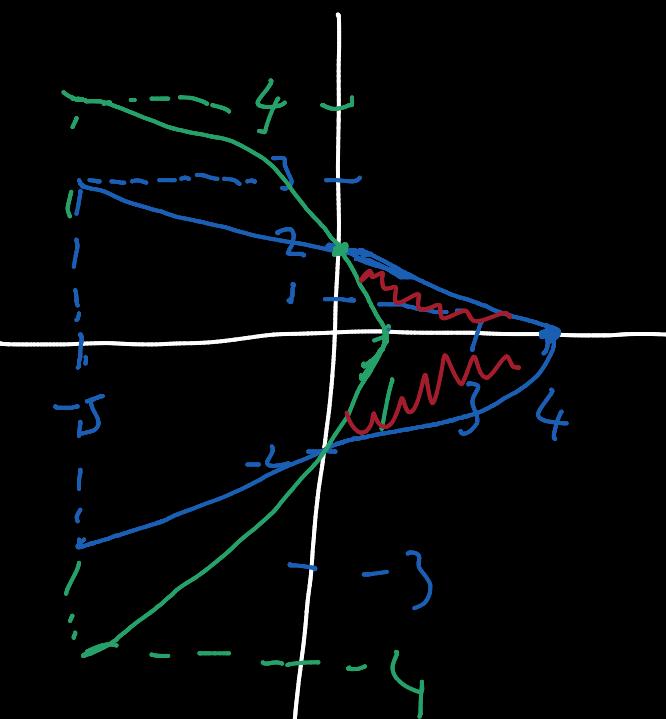
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{4(1 - \sin^2 t)} 2 \cos t dt$$

$\cos x = \frac{1 + \cos 2x}{2}$

$$4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + \cos 2t dt$$

$$2 \left(t + \frac{\sin 2t}{2} \right) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2 \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) = 2\pi = \frac{A}{2}$$

$A = 4\pi$

$y^2 = 4 - x$ $x = \frac{4-y^2}{4}$ $x = 4 - y^2$
 $y^2 = 4 - 4x$ Area ?


$$\int_{-2}^2 \int_{\frac{4-y^2}{4}}^{4-y^2} dx dy$$

$$= \int_{-2}^2 4-y^2 - \left(\frac{4-y^2}{4} \right) dy$$

$$= \frac{1}{4} \int_{-2}^2 16 - 4y^2 - 4 + y^2 dy$$

$$= \frac{1}{4} \int_{-2}^2 12 - 3y^2 dy$$

$$= \frac{1}{4} \left(12y - y^3 \right) \Big|_{-2}^2$$

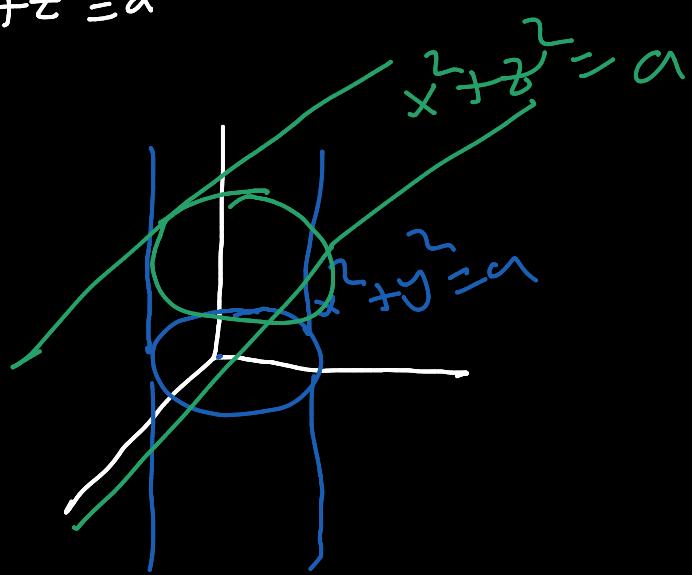
$$= \frac{1}{4} (24 - 8) - (-24 + 8)$$

$$= 8 \text{ Area}$$

$$x^2 + y^2 = a^2$$

Volume }
 $z = \sqrt{a^2 - x^2}$

$$y = \sqrt{a^2 - z^2}$$



$$V = \iiint_{-a}^a \sqrt{a^2 - x^2} dy dx$$

$$\left[\sqrt{a^2 - x^2} y \right]_{-\sqrt{a^2 - x^2}}^{\sqrt{a^2 - x^2}} = 2 \int_{-a}^a \sqrt{a^2 - x^2} dx$$

$$= 2 \left(\left(\frac{2}{3}x - \frac{x^3}{3} \right) \Big|_{-a}^a \right) = 2 \left(\left(\frac{2a^3}{3} - \frac{a^3}{3} \right) - \left(-\frac{2a^3}{3} + \frac{a^3}{3} \right) \right)$$

$$= 2 \left(\frac{3a^3}{2} \right) = 4a^3 - \frac{4a^3}{3} = \frac{8a^3}{3}$$

$$\int_0^4 (8y - x) \sqrt{4 - (x-2)^2} dx = \int_0^4 8\sqrt{4 - (x-2)^2} - x\sqrt{4 - (x-2)^2} - \left(8\sqrt{4 - (x-2)^2} + x\sqrt{4 - (x-2)^2} \right) dx$$

$$= \int_0^4 \left(16\sqrt{4 - (x-2)^2} - 2x\sqrt{4 - (x-2)^2} \right) dx = 2 \int_0^4 (8-x)(\sqrt{4-(x-2)}) dx$$

$\sqrt{a^2 - x^2}$
 $x = a \sin t$
 $dx = a \cos t dt$

Transformation with Double Integrals

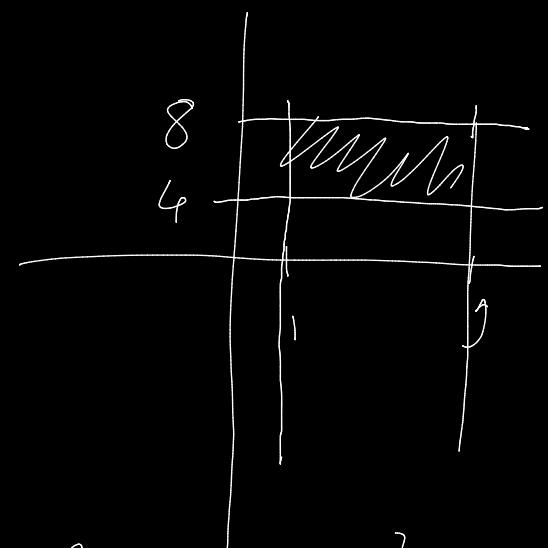
$$\left| \begin{array}{l} x^2 + y^2 = 1 \\ x^2 + y^2 = 9 \\ xy = 2 \\ xy = 4 \\ \iint (x^2 + y^2) dx dy = ? \end{array} \right|$$

$$\left| \begin{array}{l} x^2 - y^2 = u \\ u = 1 \\ u = 9 \\ x^2 y^2 = v \\ v = 4 \\ v = 8 \end{array} \right|$$

$$x^2 y^2 = u$$

$$v = 4$$

$$v = 8$$



$$\begin{aligned} u^2 + v^2 &= (x^2)^2 - 2x^2 y^2 + (y^2)^2 + 4x^2 y^2 \\ &= (x^2 + y^2)^2 \end{aligned}$$

$$x^2 + y^2 = \sqrt{u^2 + v^2}$$

$$J = \frac{\lambda(u, v)}{\lambda(x, y)} = \begin{vmatrix} 2x & -2y \\ 2y & 2x \end{vmatrix} = 4x^2 + 4y^2$$

$$J = \frac{1}{4x^2 + 4y^2} = \frac{1}{4\sqrt{u^2 + v^2}}$$

$$\iint_{\text{region}} \sqrt{u^2 + v^2} \frac{1}{4\sqrt{u^2 + v^2}} du dv = \frac{1}{4} \int_1^9 \int_4^8 4 du = 8$$

