Excersie 3

In [1]:
```python
import numpy

red = [1, 2, 3, 4, 5, 6]
blue = [1, 2, 3, 4, 5, 6]

counter=0       #All wanted outcomes
combinations=0  #All possible outcomes

for element_red in red:
    for element_blue in blue:
        if element_red+element_blue == 9:
            counter = counter + 1
            print(str(element_red) +" and "+ str(element_blue)) #Printing out the possible wanted Combinations
        combinations = combinations + 1

print("The Probability for rolling a 9 is " + str(counter) + "/" + str(combinations))
```

```
3 and 6
4 and 5
5 and 4
6 and 3
The Probability for rolling a 9 is 4/36
```

Nr. 3a)

$$\begin{aligned} P(W_{red} + W_{blue} = 9) &= P(W_{red} = 3) \cdot P(W_{blue} = 6) \\ &+ P(W_{red} = 4) \cdot P(W_{blue} = 5) \\ &+ P(W_{red} = 5) \cdot P(W_{blue} = 4) \\ &+ P(W_{red} = 6) \cdot P(W_{blue} = 3) \\ &= 4 \cdot \left(\frac{1}{6}\right)^2 = \frac{1}{9} \end{aligned}$$

In [2]:
```python
counter=0
combinations=0

for element_red in red:
    for element_blue in blue:
        if element_red+element_blue >= 9:
            counter = counter + 1
            print(str(element_red) +" and "+ str(element_blue))
        combinations = combinations + 1

print("The Probability for rolling higher than a 9 is " + str(counter) + "/" + str(combinations))
```

```
3 and 6
4 and 5
4 and 6
5 and 4
5 and 5
5 and 6
6 and 3
6 and 4
6 and 5
6 and 6
The Probability for rolling higher than a 9 is 10/36
```

Nr. 3b)

$$\begin{aligned} P(W_{red} + W_{blue} \geq 9) &= P(W_{red} = 3) \cdot P(W_{blue} = 6) \\ &+P(W_{red} = 4) \cdot P(W_{blue} = 5) \\ &+P(W_{red} = 4) \cdot P(W_{blue} = 6) \\ &+P(W_{red} = 5) \cdot P(W_{blue} = 4) \\ &+P(W_{red} = 5) \cdot P(W_{blue} = 5) \\ &+P(W_{red} = 5) \cdot P(W_{blue} = 6) \\ &+P(W_{red} = 6) \cdot P(W_{blue} = 3) \\ &+P(W_{red} = 6) \cdot P(W_{blue} = 4) \\ &+P(W_{red} = 6) \cdot P(W_{blue} = 5) \\ &+P(W_{red} = 6) \cdot P(W_{blue} = 6) \\ &= 10 \cdot \left(\frac{1}{6}\right)^2 = \frac{10}{36} = \frac{5}{18} \end{aligned}$$

In [3]:
```python
counter=0
combinations=0

for element_red in red:
    for element_blue in blue:
        if (element_red+element_blue == 9) & (3 < element_red < 6): #only 4 and 5 are wanted outcomes
            counter = counter + 1
            print(str(element_red) +" and "+ str(element_blue))
        combinations = combinations + 1

print("The Probability for rolling a 4 and a 5 is " + str(counter) + "/" + str(combinations))
```

```
4 and 5
5 and 4
The Probability for rolling a 4 and a 5 is 2/36
```

Nr. 3c)

$$\begin{aligned} P(W_1 = 4 \wedge W_2 = 5) &= P(W_{red} = 5) \cdot P(W_{blue} = 4) \\ &+ P(W_{red} = 4) \cdot P(W_{blue} = 5) \\ &= 2 \cdot \left(\frac{1}{6}\right)^2 = \frac{2}{36} = \frac{1}{18} \end{aligned}$$

In [4]:
```python
counter=0
combinations=0

for element_red in red:
    for element_blue in blue:
        if (element_blue == 5) & (element_red == 4): #Only one combination is wanted
            counter = counter + 1
            print(str(element_red) +" and "+ str(element_blue))
        combinations = combinations + 1

print("The Probability for the red dice to roll a 4 and the blue dice to roll a 5 is " + str(counter) + "/" + str(com
```

```
4 and 5
The Probability for the red dice to roll a 4 and the blue dice to roll a 5 is 1/36
```

Nr. 3d)

$$\begin{aligned} P(W_{red} = 4 \wedge W_{blue} = 5) &= P(W_{red} = 4) \cdot P(W_{blue} = 5) \\ &= 1 \cdot \left(\frac{1}{6}\right)^2 = \frac{1}{36} \end{aligned}$$

You roll the dice so that the blue die rolls behind an object so that you can't see it at first. The red die shows a 4. From this point forward:

$$P(W_{red} = 4) = 1$$

The condition for all throws is that the red die shows a 4.

In [5]:
```python
element_red = 4
```

In [6]:
```python
counter=0
combinations=0
for element_blue in blue:
    if element_red+element_blue == 9:
        counter = counter + 1
        print(str(element_red) +" and "+ str(element_blue))
    combinations = combinations + 1

print("The Probability for rolling a 9 is " + str(counter) + "/" + str(combinations))
```

```
4 and 5
The Probability for rolling a 9 is 1/6
```

Nr. 3e)

$$\begin{aligned} P(W_{red} + W_{blue} = 9 \mid W_{red} = 4) &= \frac{P(W_{red} + W_{blue} = 9) \wedge P(W_{red} = 4)}{P(W_{red} = 4)} \\ &= P(W_{blue} = 5) \qquad |\text{because} P(W_{red} = 4) = 1 \\ &= 1 \cdot \left(\frac{1}{6}\right) = \frac{1}{6} \end{aligned}$$

In [7]:
```python
counter=0
combinations=0
for element_blue in blue:
    if element_red+element_blue >= 9:
        counter = counter + 1
        print(str(element_red) +" and "+ str(element_blue))
    combinations = combinations + 1

print("The Probability for rolling higher than a 9 is " + str(counter) + "/" + str(combinations))
```

```
4 and 5
4 and 6
The Probability for rolling higher than a 9 is 2/6
```

Nr. 3f)

$$\begin{aligned} P(W_{red} + W_{blue} \geq 9 \mid W_{red} = 4) &= \frac{P(W_{red} + W_{blue} \geq 9) \wedge P(W_{red} = 4)}{P(W_{red} = 4)} \\ &= \frac{P(W_{blue} = 5) + P(W_{blue} = 6)}{P(W_{red} = 4)} \\ &= \frac{\frac{1}{6} + \frac{1}{6}}{1} \\ &= \frac{2}{6} = \frac{1}{3} \end{aligned}$$

In [8]:
```python
counter=0
combinations=0
for element_blue in blue:
    if (element_red==4) & (element_blue==5):
        counter = counter + 1
        print(str(element_red) +" and "+ str(element_blue))
    combinations = combinations + 1

print("The Probability for rolling the red dice to show a 4 and the blue dice a 5 is " + str(counter) + "/" + str(com
```

```
4 and 5
The Probability for rolling the red dice to show a 4 and the blue dice a 5 is 1/6
```

Nr. 3g)

$$\begin{aligned} P(W_{red} = 4 \wedge W_{blue} = 5 \mid W_{red} = 4) &= \frac{P(W_{red} = 4) \cdot P(W_{blue} = 5) \wedge P(W_{red} = 4)}{P(W_{red} = 4)} \\ &= \frac{P(W_{blue} = 5)}{P(W_{red} = 4)} \\ &= \frac{\frac{1}{6}}{1} \\ &= \frac{1}{6} \end{aligned}$$

Excersie 4

The function $f(v)$ is only definded for $v \in R_+$, since $f(v) = f(-v)$.

The normalisingfactor is

$$N = \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}}$$

(All calculations are at the end of the document) a)

The most probable value for the velocity can be calculated with the extrempoints. So that

$$f\prime(v) = 0\,.$$

With this the most probable value is

$$v_m = \sqrt{\frac{2k_BT}{m}}\,.$$

b)

The mean $\langle v \rangle$ is

$$\langle v \rangle = \int_0^{\inf} v f(v)\, dv = \frac{2v_m}{\sqrt{\pi}}$$

c)

The median v_0 can be calculated with

$$\int_0^{v_0} f(v)dv = \frac{1}{2}$$

since it is normed to 1. To calculate the median v_m, we substitute $v\prime = \frac{v}{v_m}$.

In [9]:
```python
import numpy as np
import scipy.optimize as op
from scipy.integrate import quad

#We subsituted v/v_m with v_strich

# Integral of f
def f(v_strich):
    return 4/np.sqrt(np.pi)*v_strich**2*np.exp(-v_strich**2)

'''
As we want to find the integral limit for which the integral equals 0.5,
we want to find the "root" (when it equal to zero) of the integral minus 0.5

'''

def f_root(v_ratio):
    return quad(f, 0, v_ratio)[0]-0.5

sol = op.root_scalar(f_root, bracket = [0, 5],method ='brentq') #guessing the border [0,5] for the method brentq

print(f'The ratio between v_0.5 and v_m is {sol.root}')
print(f'Thus v_0.5 = {sol.root}*v_m')
```

```
The ratio between v_0.5 and v_m is 1.0876520317581067
Thus v_0.5 = 1.0876520317581067*v_m
```

d)

$f(v_{FWHM})$ is given by $f(v_{FWHM}) = \frac{f(\frac{v_m}{v_m})}{2} = \frac{f(1)}{2}$ so that:

In [10]:
```python
def f_root_FWHM(v_ratio):
    return f(v_ratio)-f(1)/2


sol_FWHM_2 = op.root_scalar(f_root_FWHM, bracket = [1, 5],method ='brentq') #suche nach rechten Rand
sol_FWHM_1 = op.root_scalar(f_root_FWHM, bracket = [0, 1],method ='brentq') #Suche nach linkem Rand

print(f"The first value where we hit the half maximum is {sol_FWHM_1.root}*v_m")
print(f"The second value where we hit the half maximum is {sol_FWHM_2.root}*v_m")
print(f"Thus the FWHM is {sol_FWHM_2.root-sol_FWHM_1.root}*v_m")
```

```
The first value where we hit the half maximum is 0.4816232479714112*v_m
The second value where we hit the half maximum is 1.6365656082224946*v_m
Thus the FWHM is 1.1549423602510833*v_m
```

e)

In general:

$$\sigma_m^2 = \langle v^2 \rangle - \langle v \rangle^2\,.$$

$\langle v^2 \rangle$ can be expressed by

$$\langle v^2 \rangle = \int_0^{\inf} v^2 f(v)\, dv = \frac{3}{2} v_m^2\,.$$

From task b) we know that

$$\langle v \rangle = \frac{2v_m}{\sqrt{\pi}}$$

With this we can calculate the standard deviation to

$$\sigma_m = v_m \cdot \sqrt{\frac{9\pi - 16}{4\pi}}\,.$$

SMB Übung 2

Exercise 4

a) $f(v) = N \cdot \exp\left(-\frac{mv^2}{2k_BT}\right) \cdot 4\pi v^2$

$$\int_0^\infty f(v) \overset{!}{=} 1$$

$$\int_0^\infty \underbrace{4\pi N \cdot \exp\left(\frac{-mv^2}{2k_BT}\right) \cdot v}_{v'} \cdot \underset{u}{v} \, dv$$

$$= \underset{v}{\frac{-4\pi \cdot N \cdot k_BT}{m} \exp\left(\frac{-mv^2}{2k_BT}\right)} \cdot \underset{u}{v} \Big|_0^\infty - \int_0^\infty \underset{v}{\frac{-4\pi N \cdot k_BT}{m} \cdot \exp\left(\frac{-mv^2}{2k_BT}\right)} \underset{u'}{} dv$$

$$x = \sqrt{\frac{m}{2k_BT}} \, v \qquad = \int_0^\infty \frac{4\pi \, N \cdot k_BT}{m} \cdot \sqrt{\frac{2k_BT}{m}} \cdot \exp(-x^2) \, dx$$

$$dv = dx \cdot \sqrt{\frac{2k_BT}{m}}$$

$$= 4\pi N \sqrt{2} \left(\frac{k_BT}{m}\right)^{3/2} \cdot \int_0^\infty \exp(-x^2) \, dx = 4\pi N \cdot \sqrt{2} \cdot \frac{\sqrt{\pi}}{2} \cdot \left(\frac{k_BT}{m}\right)^{3/2}$$

$$= N \left(\frac{k_BT \, 2 \, \pi}{m}\right)^{3/2} \overset{!}{=} 1$$

$$\Rightarrow N = \left(\frac{m}{2\pi k_BT}\right)^{3/2}$$

$$\Rightarrow f(v) = \left(\frac{m}{2\pi k_BT}\right)^{3/2} \cdot 4\pi \exp\left(\frac{-mv^2}{2k_BT}\right) 4\pi v^2$$

Der Einfachheit halber setzen wir N hier nicht ein

$$f'(v_m) \overset{!}{=} 0$$

$$\frac{df(v)}{dv} = -\frac{N \cdot m \cancel{2} v_m}{\cancel{2} k_B T} \cdot 4\pi v_m^2 \cdot \cancel{\exp\left(\frac{-m v_m^2}{2 k_B T}\right)} + 8\pi v_m N \cancel{\exp\left(\frac{-m v_m^2}{2 k_B T}\right)} \overset{!}{=} 0$$

$$\Leftrightarrow -\frac{N \cdot m v_m}{k_B T} \cdot 4\pi v_m^2 + 8\pi v_m N = 0 \quad \Big| \cdot \frac{1}{N \cdot v_m \cdot 4\pi}$$

$$\Leftrightarrow \frac{-m v_m^2}{k_B T} + 2 = 0$$

$$\Leftrightarrow v_m^2 = \frac{2 \cdot k_B T}{m} \quad \Big| \sqrt{}$$

$$v_m = \sqrt{\frac{2 k_B T}{m}}$$

b) $$N = \left(\frac{m}{2\pi k_B T}\right)^{3/2}$$

$$\langle v \rangle = \int_0^\infty v \cdot f(v)\, dv$$

$$= \int_0^\infty 4\pi \cdot N \cdot v^3 \cdot \exp\left(\frac{-m v^2}{2 k_B T}\right) dv$$

$$x = \frac{m v^2}{2 k_B T} \Rightarrow v^2 = \frac{2 k_B T}{m} \cdot x$$

$$dx = \frac{m}{k_B T} v\, dv$$

$$dv = dx \cdot \frac{k_B T}{m v}$$

$$= \int_0^\infty 4\pi N \cdot \frac{v^2}{m} k_B T \cdot \exp(-x)\, dx$$

	D	I
+	x	$\exp(-x)$
−	1	$-\exp(-x)$
+	0	$\exp(-x)$

$$= \int_0^\infty 4\pi N \cdot \frac{2 k_B T \cdot x \cdot k_B T}{m^2} \cdot \exp(-x)\, dx$$

(2) $$= \int_0^\infty 4\pi N \cdot 2\left(\frac{k_B T}{m}\right)^2 \cdot x \cdot \exp(-x)\, dx$$

$$= 8\pi N \cdot \left(\frac{k_B T}{m}\right)^2 \cdot \int_0^\infty x \cdot \exp(-x)\, dx = 8\pi N \left(\frac{k_B T}{m}\right)^2 \Big[-x \exp(-x) - \exp(-x)\Big]_0^\infty$$

$$= 8\pi \cdot N \cdot \left(\frac{k_B T}{m}\right)^2 (1)$$

$$\langle v \rangle = 8\pi \cdot N \cdot \left(\frac{k_B T}{m}\right)^2 \qquad v_m = \sqrt{\frac{2k_B T}{m}}$$

$$= 8\pi \cdot \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \left(\frac{k_B T}{m}\right)^2$$

$$= 8\pi \cdot \left(\frac{1}{2\pi}\right)^{3/2} \cdot \left(\frac{m}{\cancel{2\pi} k_B T}\right)^{3/2} \cdot \left(\frac{m}{k_B T}\right)^{-2}$$

$$= 8\pi \cdot \left(\frac{1}{2\pi}\right)^{3/2} \cdot \left(\frac{k_B T}{m}\right)^{1/2}$$

$$= 8\pi \cdot (2\pi)^{-\frac{3}{2}} \cdot \left(\frac{k_B T}{m}\right)^{1/2}$$

$$= 2^3 \cdot \pi^1 \cdot 2^{-\frac{3}{2}} \cdot \pi^{-\frac{3}{2}} \cdot \left(\frac{k_B T}{m}\right)^{1/2}$$

$$= 2^{3/2} \cdot \pi^{-\frac{1}{2}} \cdot \left(\frac{k_B T}{m}\right)^{1/2}$$

$$= \sqrt{\frac{8 k_B T}{m\pi}} = \frac{2}{\sqrt{\pi}} \sqrt{\frac{2 k_B T}{m}}$$

$$= \frac{2}{\sqrt{\pi}} v_m$$

c)

$$v_m = \sqrt{\frac{2k_BT}{m}} \qquad N = \left(\frac{m}{2\pi k_B T}\right)^{3/2}$$

$$\Rightarrow f(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot 4\pi v^2 \cdot \exp\left(-\frac{mv^2}{2k_BT}\right)$$

$$= \frac{1}{v_m^3} \cdot \left(\frac{1}{\pi}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{v^2}{v_m^2}\right)$$

$$f(v) = \frac{1}{v_m^3} \cdot \frac{4}{\sqrt{\pi}} \cdot v^2 \cdot \exp\left(-\frac{v^2}{v_m^2}\right)$$

Idea:

$$0{,}5 = \int_0^{v_{0,5}} f(v)\,dv$$

$$\Rightarrow f = \int_0^{v_{0,5}} f(v)\,dv - 0{,}5 \overset{!}{=} 0 \rightarrow \text{~~down~~ find root}$$

(when zero with argument $v_{0,5}$)

$$\int_0^{v_{0,5}} \frac{1}{v_m^3} \frac{4}{\sqrt{\pi}} v^2 \exp\left(-\frac{v^2}{v_m^2}\right) dv - 0{,}5$$

$$\Rightarrow v = v' v_m$$

$$= \boxed{\int_0^{\frac{v_{0,5}}{v_m}} \frac{4}{\sqrt{\pi}} v'^2 \exp\left(-v'^2\right) dv' - 0{,}5} \qquad v' = \frac{v}{v_m}$$

$$\Rightarrow dv = dv' \cdot v_m$$

d)

$$f(v') = \frac{4}{\sqrt{\pi}} v' \exp\left(-v'^2\right)$$

$$v' = \frac{v}{v_m}$$

$$f(v') \overset{!}{=} 0{,}5 \cdot f\left(\frac{v_m}{v_m}\right) = 0{,}5\, f(1)$$

$$v_{FWHM} = v'_2 - v'_1$$

e) $\sigma_v^2 = \langle v^2 \rangle - \langle v \rangle^2$

$$\langle v^2 \rangle = \int_0^\infty v^2 f(v) dv \qquad v' = \frac{v}{v_m}$$

$$= \frac{4}{\sqrt{\pi}} \int_0^\infty \frac{v^2}{v_m^3} \cdot v^2 \cdot \exp\left(-\frac{v^2}{v_m^2}\right) dv$$

$$= \frac{4}{\sqrt{\pi}} \int_0^\infty v_m^2 \cdot v'^4 \cdot \exp(-v'^2) dv'$$

$$= \frac{4}{\sqrt{\pi}} v_m^2 \left(\int_0^\infty \underset{w}{v'^3} \; \underset{u'}{v' \exp(-v'^2)} dv \right)$$

$$= \frac{4}{\sqrt{\pi}} v_m^2 \left(\underbrace{v'^3 \left(-\frac{1}{2}\exp(-v'^2)\right)\Big|_0^\infty}_{0} + \int_0^\infty \frac{3}{2} \underset{v'}{\cancel{v}^2} \underset{u}{\exp(-\cancel{v}^2)} d\cancel{v}' \right.$$

a) $= \frac{4}{\sqrt{\pi}} v_m^2 \cdot \frac{3}{2} \cdot \frac{\sqrt{\pi}}{4} = \frac{3}{2} v_m^2$

$$\Rightarrow \sigma_v^2 = \left(\frac{3}{2} v_m\right)^2 - \left(\frac{2 \cdot v_m}{\sqrt{\pi}}\right)^2 = v_m^2 \cdot \left(\frac{9\pi - 16}{4\pi}\right)$$

$$\Rightarrow \sigma_v = v_m \cdot \sqrt{\frac{9\pi - 16}{4\pi}}$$