In [83]: import pandas as pd import numpy as np import matplotlib.pyplot as plt from sklearn.preprocessing import MinMaxScaler columns = ["id", "diagnosis"] x columns = []for i in range (1, 31): columns.append("feature " + str(i)) x_columns.append("feature_" + str(i)) df = pd.read_csv("data/wdbc.data", names=columns) Data preprocessing Apply MinMaxScaler to the features to minimize the effect of features with large values In [84]: features = df[x_columns] features = MinMaxScaler().fit_transform(features) target = df["diagnosis"] target_name = df["diagnosis"].copy() target = target.map({"M": 1, "B": 0}) K-Means implementation In [85]: def k means(data, k num, max iters=100): cent = data[np.random.choice(data.shape[0], k_num, replace=False)] label = np.array([]) for _ in range(max_iters): distances = np.sqrt(((data - cent[:, np.newaxis]) ** 2).sum(axis=2)) label = np.argmin(distances, axis=0) new_centroids = np.array([data[label == j].mean(axis=0) for j in range(k_num)]) if np.allclose(cent, new_centroids): break cent = new centroids return cent, label In [86]: def calculate accuracy(1, t): acc = np.mean(l == t)**if** acc < 0.5: acc = 1 - accreturn acc **Elbow method** In [87]: def elbow method(data, max k=10, title="Elbow"): sum_squared_err = [] for current_k in range(1, max_k + 1): centroids, labels = k means(data, k num=current k) sum squared err.append(np.sum((data - centroids[labels]) ** 2)) plt.figure(figsize=(8, 6)) plt.plot(range(1, max_k + 1), sum_squared_err, marker='o') plt.xlabel('Number of clusters (k)') plt.ylabel('Sum of Squared Distances') plt.title(title) plt.grid() plt.show() Trying different k-means values and visualizing the results In [98]: $k_nums = [2, 3, 4, 5, 6, 7, 8, 9, 10]$ plots per row = 3num_rows = int(np.ceil(len(k_nums) / plots_per_row)) fig, axes = plt.subplots(num_rows, plots_per_row, figsize=(15, 5 * num_rows)) for i, k in enumerate(k_nums): centroids, labels = k means(features, k num=k) row = i // plots per row col = i % plots_per_row if num rows == 1: ax = axes[col]else: ax = axes[row, col]ax.scatter(features[:, 0], features[:, 1], c=labels, cmap='viridis') ax.scatter(centroids[:, 0], centroids[:, 1], c='red', s=200, alpha=0.5) ax.set_title(f'K = {k}') # Hide empty subplots if there are any for i in range(len(k_nums), num_rows * plots_per_row): row = i // plots_per_row col = i % plots_per_row if num rows == 1: ax = axes[col]else: ax = axes[row, col]ax.axis('off') plt.tight layout() plt.show() 1.0 1.0 0.8 0.8 0.8 0.6 0.6 0.4 0.2 0.0 0.0 1.0 0.6 0.8 1.0 0.6 1.0 1.0 1.0 0.8 0.8 0.8 0.6 0.2 1.0 1.0 0.8 K = 10 K = 81.0 0.8 0.8 0.6 0.2 1.0 1.0 elbow method(features, title="Elbow Exp 1 before PCA") In [88]: Elbow Exp 1 before PCA 350 300 Sum of Squared Distances 250 200 150 Number of clusters (k) **Observations** We can see that the elbow is at k=2, which is the number of classes in the dataset. In [89]: k = 2 # Best number of clusters from elbow method centroids, labels = k means(features, k num=k) # plt plt.figure(figsize=(8, 6)) plt.scatter(features[:, 0], features[:, 1], c=labels, cmap='viridis') plt.scatter(centroids[:, 0], centroids[:, 1], c='red', s=200, alpha=0.5) plt.show() # Calculate accuracy accuracy = calculate_accuracy(labels, target) print("Accuracy: " + str(round(accuracy * 100, 2)) + "%") sse_exp_1 = np.sum((features - centroids[labels]) ** 2) 1.0 0.8 0.6 0.4 0.2 0.0 0.2 0.4 0.6 0.8 0.0 1.0 Accuracy: 92.79% **PCA** implementation In [90]: def pca(data, num_components): # Step 1: Standardize the data data mean = np.mean(data, axis=0) standardized_data = data - data_mean # Step 2: Compute the covariance matrix covariance_matrix = np.cov(standardized_data, rowvar=False) # Step 3: Calculate eigenvalues and eigenvectors eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix) # Step 4: Select top k eigenvectors sorted_indices = np.argsort(eigenvalues)[::-1] top_k_indices = sorted_indices[:num_components] principal_components = eigenvectors[:, top_k_indices] # Step 5: Project the data onto the selected principal components transformed_data = np.dot(standardized_data, principal_components) return transformed data In [91]: t data = pca(features, num components=2) elbow method(t data, title="Elbow Ex 2 with PCA") Elbow Ex 2 with PCA 250 200 Sum of Squared Distances 150 100 50 Number of clusters (k) In [92]: k = 2 # Best number of clusters from elbow method centroids, labels = k means(t data, k) In [93]: plt.figure(figsize=(8, 6)) plt.scatter(t_data[:, 0], t_data[:, 1], c=labels, cmap='viridis') plt.scatter(centroids[:, 0], centroids[:, 1], c='red', s=200, alpha=0.5) plt.show() sse_exp_2 = np.sum((t_data - centroids[labels]) ** 2) accuracy = calculate accuracy(labels, target) print("Accuracy: " + str(round(accuracy * 100, 2)) + "%") 1.5 1.0 0.5 0.0 -0.5-1.0-2.0-1.5-1.0-0.50.0 0.5 1.0 Accuracy: 92.79% **Comparing Sum of Squared error Expectations Experiment 1 (Without PCA):** • SSE reflects distances in the original feature space without dimensionality reduction. • Higher SSE may indicate more scattered or dispersed clusters due to the higher dimensionality. **Experiment 2 (With PCA):** Lower SSE compared to Experiment 1 due to dimensionality reduction. PCA attempts to retain the most relevant information in fewer dimensions. • Lower SSE suggests that clustering in a reduced-dimensional space captures the inherent structure better. In [94]: print(f"SSE Experiment 1 (Without PCA): {round(sse exp 1, 2)}") print(f"SSE Experiment 2 (With PCA): {round(sse_exp_2, 2)}") SSE Experiment 1 (Without PCA): 215.84 SSE Experiment 2 (With PCA): 110.97 Trying different principale components and visualizing the results With 2 components In [95]: n components = 2 $_{-}$, axes = plt.subplots(1, 2, figsize=(12, 8)) t data = pca(features, n components) centroids, labels_pca = k_means(t_data, k_num=2) ax = axes[0]ax.scatter(t_data[:, 0], t_data[:, 1], c=labels_pca) ax.set_title(f'PCA with {n_components} Components') ax.set_xlabel('Principal Component 1') ax.set_ylabel('Principal Component 2') # Plot original labels $ax_{original} = axes[1]$ map_dict = {0: 'B', 1: 'M'} for i in [0, 1]: pca_points = t_data[np.where(target == i)] ax_original.scatter(pca_points[:, 0], pca_points[:, 1], label=map_dict[i]) ax_original.set_title('Original Labels') ax_original.set_xlabel('Feature 1') ax_original.set_ylabel('Feature 2') plt.tight_layout() plt.show() PCA with 2 Components Original Labels 1.5 1.5 1.0 1.0 Principal Component 2 0.5 0.5 Feature 2 0.0 0.0 -0.5-0.5-1.0-1.0-2.0 -1.5-1.0-0.5 0.0 0.5 1.0 -2.0 -i.5 -1.0-0.5 0.0 0.5 1.0 Principal Component 1 Feature 1 With 3 components

In [96]: from mpl_toolkits.mplot3d import Axes3D
n_components = 3 fig = plt.figure(figsize=(14, 8)) # Subplot for PCA with n_components ax_pca = fig.add_subplot(121, projection='3d')
t_data = pca(features, n_components) _, labels_pca = k_means(t_data, k_num=2)
ax_pca.scatter(t_data[:, 0], t_data[:, 1], t_data[:, 2], c=labels_pca)
ax_pca.set_title(f'PCA with {n_components} Components')
ax_pca.set_xlabel('Principal Component 1') ax_pca.set_ylabel('Principal Component 2') ax_pca.set_zlabel('Principal Component 3') # Subplot for Original Labels ax_original = fig.add_subplot(122, projection='3d') map_dict = {0: 'B', 1: 'M'} for i in [0, 1]: pca_points = t_data[np.where(target == i)] ax_original.scatter(pca_points[:, 0], pca_points[:, 1], pca_points[:, 2], label=map_dict[i]) ax_original.set_title('Original Labels') ax_original.set_xlabel('Feature 1') ax_original.set_ylabel('Feature 2')
ax_original.set_zlabel('Feature 3') plt.tight_layout() plt.show() PCA with 3 Components Original Labels + 0.8 0.8 0.6 0.6 0.4 0.4 0.2 0.2 0.0 0.0 -0.2 ⁵ -0.2 -0.4 -0.6 -0.6 0.5 principal component 2 1.5 1.5 0.5 0.0 _{keakure} 2 1.0 -2.0 -1.5 -2.0 -1.5 -1.0 $\begin{array}{ccc} 1.5 & -1.0 & \\ Principal & -0.5 & 0.0 \\ Component & 1 & 1 \end{array}$ Feature 1 -0.5 0.0 0.5 1.0 1.0 -1.0