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HOPFIELD NETWORK IN SOLVING TRAVELLING SALESMAN PROBLEM IN NAVIGATION

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Abstract: *This paper considers the possibility of application Hopfield recurrent neural network in solving travelling salesman problem when nodes are given in sphere co-ordinates and when distances between nodes are not linear but sphere. Obtained numerical results in case of an arbitrary chosen example are presented.*

Key words: *Hopfield network, travelling salesman problem (TSP), navigation*

1. INTRODUCTION

Besides their role in associative recall, there is another class of problems that Hopfield network can be use to solve. This is class of combinatorial optimization problems. The best example is so-called travelling salesman problem (TSP). According to this problem a salesman (navigator) has to complete a round trip of a set of cities (nodes, ports) visiting each one only once in such way as to minimize the total distance traveled. This kind of problem is computationally very difficult and it is shown that the time to find a solution grows exponentially with number of nodes.

The paper is organized in a following way:

a) Section 2. considers the shortest path between two points on the Earth;
b) Section 3. considers TSP model appropriate for application within Hopfield neural network and
c) Section 4. presents the experimental numerical results of TSP in a case of four arbitrary chosen ports.

2. NAVIGATION BETWEEN TWO DISTANT POINTS ON THE EARTH

Navigation between two distant points on the Earth is possibly done in three ways: orthodrome navigation, loxodrome navigation and combined navigation. The shortest way from departure to arrival position is a shorter section of the great circle arc, i.e. orthodrome. Such navigation is difficult to achieve since the orthodrome intersects the meridians at different angles, so it would require constant, precisely defined change of course. In the case when course is constant, that is easily achievable in practice, navigator follows a curve asymptotically approaching nearer pole. Such a curve on the surface of the Earth is called a loxodrome. In the case of combined navigation, a standard orthodrome is replaced by two orthodromes tangent to the boundary parallel and a loxodrome between them. The dangerous areas that could be reached by following strictly the orthodrome navigation are avoided by applying the technique of combined navigation.

2.1. Orthodrome Navigation

Orthodrome is the shortest path between two distant points on the surface of the Earth (Figure 1). Therefore, the aim of orthodrome navigation is the shortest path and the least travelling time, resulting invariably in cost effectiveness. Since strict orthodrome navigation is difficult to achieve in practice, it is divided into a finite number of waypoints between which loxodrome navigation is applied. The difference between the sum of such loxodrome distances and orthodrome one represents saving in path length, which would otherwise, in case of strictly orthodrome navigation, be achieved. In case of orthodrome intersection with Equator or Greenwich, first of all points of intersection are to be determined. Then on each orthodrome segment, former described procedure of waypoints number optimization is to be done.

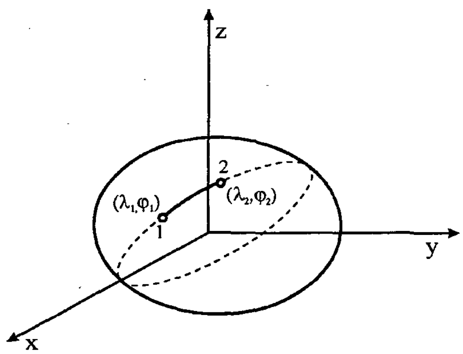

Figure 1. *Orthodrome on the Earth*

According to example presented in the paper, orthodrome distances are increased for value of certain deviation that could appear because necessity of keeping corridor or because coming across the obstacles. Namely, our task here is to find the shortest trip between four ports in accordance with TSP (Figure 2).

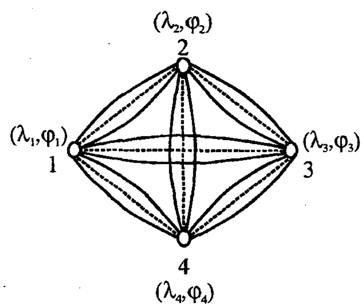

Figure 2. *TSP in a case of four ports*

3. NETWORK ENERGY REDUCTION AND TSP MODEL

3.1. Energy Function

In general, neural network optimization problems are based on minimization of energy function given by:

$$E(x) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}x_i x_j + \sum_{i=1}^{N}\theta_i x_i \qquad (3.1)$$

where

x_i - state of the i-th neuron;

θ_i - treshold of the i-th neuron and

w_{ij} - weight of the connection from the j-th neuron to i-th neuron ($w_{ij} = w_{ji}$).

In the neural algorithms for combinatorial optimization, a candidate of the solution is represented by one of the state vectors $x = (x_1, x_2, \ldots, x_N)$.

Energy function (3.1) consists of two parts, cost (3.2) and penalty (3.3) given by equations:

$$E_c(x) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}{}^{(c)}x_i x_j + \sum_{i=1}^{N}\theta_i^{(c)} x_i \qquad (3.2)$$

$$E_p(x) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}{}^{(p)}x_i x_j + \sum_{i=1}^{N}\theta_i^{(p)} x_i \qquad (3.3)$$

Both of them, in addition, are to be reduced as much as possible, according to the aim of *E(x)* minimization. The second one is to be reduced to zero in optimal solution.

3.2. Formulation of TSP

TSP could be formulated in following way: the shortest round tour, which covers all N nodes (cities, ports) salesman (navigator) is to visit, has to be found. The problem of N nodes may be coded into N by N network. Each row of the network corresponds to a node and the ordinal position of the node in the tour is given by the node at the place outputting a high value (1), while rest are all at very low values (0). The output $x_{ai} = 1$, means the node a is visited i-th in the tour and output $x_{ai} = 0$, means the node a is not visited i-th in the tour. The distance between nodes a and b is denoted as d_{ab} and energy function, i.e. its cost part, takes form:

$$E_c(x) = \frac{D}{2}\sum_{a}\sum_{b\neq a}\sum_{i} d_{ab}x_{ai}\left(x_{b,i+1} + x_{b,i-1}\right) \qquad (3.4)$$

where indexes are cyclic, i.e. $N+1=1$, $1-1=N$, and D is positive constant.

Constrains that must be satisfied could be defined as follows:

a) Only one node can be visited at the same time;
b) Each node is visited only once and
c) Every node must be visited in the tour.

These constrains could be expressed in following form:

$$E_p(x) = \frac{A}{2}\sum_a \sum_i \sum_{j \neq i} x_{ai}x_{aj} + \\ + \frac{B}{2}\sum_i \sum_a \sum_{b \neq a} x_{ai}x_{bi} + \\ + \frac{C}{2}\left(\sum_a \sum_i x_{ai} - N\right)^2 \quad (3.5)$$

where A, B and C are positive coefficients.

Total energy function of the network is given by:

$$E(x) = E_c(x) + E_p(x) \quad (3.6)$$

From the energy function weight matrix is:

$$w_{aibj} = -A\delta_{ab}(1-\delta_{ij}) - \\ - B\delta_{ij}(1-\delta_{ab}) - C - \\ - Dd_{ab}(\delta_{j,i+1} + \delta_{j,i-1}) \quad (3.7)$$

where

$$\delta_{i_1 i_2} = \begin{cases} 1, & i_1 = i_2 \\ 0, & i_1 \neq i_2 \end{cases}.$$

4. EXPERIMENTAL RESULTS

According to TSP, here four ports are taken into consideration: New York (New York), Lisbon (Portugal), San Juan (Puerto Rico) and Mogador (Marocco). Their geographical coordinates are given in Table1. As obvious, all of them are on northwest hemisphere.

no	Port	Longitude [W]	Latitude [N]
1.	New York (New York)	73° 00′	40° 42′
2.	Lisbon (Portugal)	08° 48′	38° 42′
3.	San Juan (Puerto Rico)	65° 30′	21° 30′
4.	Mogador (Marocco)	09° 54′	31° 30′

Table 1. *Ports geographical coordinates*

The orthodrome distances and increased distances for value of deviation - Δ_i for each pair of ports are given in Table 2, as well as corresponding values of courses. Deviations Δ_i are arbitrary chosen. The main reasons for existing deviation are necessity of keeping appropriate corridor, as well as possibility of coming across the obstacles.

Route		Direction (1) – (2)			
		d_{ort} [Nm]	k_{p1} [°]	Δ_1 [Nm]	Δd_1 [Nm]
1.	New York (1) - Lisbon (2)	2897.985993	70.232010	+40	2937.985993
2.	New York (1) - San Juan (2)	1213.505319	159.433814	+50	1263.505319
3.	San Juan (1) - Mogador (2)	3013.872531	66.247621	+15	3028.872531
4.	Mogador (1) - Lisbon (2)	435.350675	6.812714	+60	495.350675
5.	New York (1) - Mogador (2)	3042.885974	79.235098	+20	3062.885974
6.	San Juan (1) - Lisbon (2)	3066.667076	56.932170	+30	3096.667076
Route		Direction (2) – (1)			
		d_{ort} [Nm]	k_{p2} [°]	Δ_2 [Nm]	Δd_2 [Nm]
1.	New York (2) - Lisbon (1)	2897.985993	293.909821	+150	3047.985993
2.	New York (2) - San Juan (1)	1213.505319	343.366911	+35	1248.505319
3.	San Juan (2) - Mogador (1)	3013.872531	272.821798	+45	3058.872531
4.	Mogador (2) - Lisbon (1)	435.350675	187.446483	+15	450.350675
5.	New York (2) - Mogador (1)	3042.885974	299.130482	+25	3067.885974
6.	San Juan (2) - Lisbon (1)	3066.667076	267.543747	+10	3076.667076

* $\Delta d_i = d_{ort} + \Delta_i, i = 1,2$

where

Δd_i - real distance between two ports, d_{ort} - orthodrome distance and Δ_i - deviation

Table 2. *Orthodrome and real distances between ports*

Matrix representation of distances between ports is given bellow in Table 3. It is to be noticed that distances are given in nautical miles and that it is not possible to travel from a certain port to itself, i.e. $d_{11}=\infty$, $d_{22}=\infty$, $d_{33}=\infty$ and $d_{44}=\infty$.

	1.	2.	3.	4.
1.	∞	2,938	1,264	3,063
2.	3,048	∞	3,077	450
3.	1,249	3,097	∞	3,029
4.	3,068	495	3,059	∞

Table 3. *Matrix representation of distances between ports*

After taking into account all possible combinations of round tours (Table 4) the optimal solution with minimum distant value of 7,696 nautical miles has been found.

Route 1	Route 2	Route 3	Route 4	Route 5	Route 6
1-2	1-3	1-3	1-4	1-4	1-2
2-4	3-4	3-2	4-3	4-2	2-3
4-3	4-2	2-4	3-2	2-3	3-4
3-1	2-1	4-1	2-1	3-1	4-1
D_1: 7,696 [Nm]	D_2: 7,836 [Nm]	D_3: 7,879 [Nm]	D_4: 12,267 [Nm]	D_5: 7,884 [Nm]	D_6: 12,112 [Nm]

Table 4. *Potential routes and their lengths in nautical miles*

Numerical results for energy minimum and appropriate weights vector, are given in Table 5.

$E(x)$	1,924,000
w_{12}	-1,469,200
w_{24}	-225,200
w_{43}	-1,529,700
w_{31}	-624,700

Table 5. *Energy minimum and weights vector*

Coefficients being used take values: $A=500$, $B=500$, $C=200$ and $D=500$, according to Hopfield and Tank original adaptation.

CONCLUSIONS

The idea for implementation Hopfield neural network into TSP in navigation has been presented. The main differences between classical and here presented TSP are that nodes have sphere coordinates and that distances between nodes are not linear but sphere. Numerical results of optimal round tour, energy minimum and appropriate weight vector are given, although problem of greater dimension TSP still remains. Therefore further investigation should be surely focused on improving algorithms for solving this problem.

LITERATURE

[1] Bauk S., *Informaciono-komunikacione tehnologije u optimizaciji vođenja broda*, magistarski rad, Saobraćajni fakultet, Beograd, 2001.

[2] Demuth H., *Neural Network Toolbox for Use with MATLAB*, The Math Works Inc., 1994.

[3] Fuller R., *Introduction to Neuro-Fuzzy Systems*, Phusica-Verlag, Warsaw, 1999.

[4] Gimenez-Martinez V., A Modified Hopfield Auto-Associative Memory with Improved Capacity, *IEEE Transactions on Neural Networks*, vol. 11, no. 4, pp 867-878, 2000.

[5] Gorney K., *An Introduction to Neural Networks*, UCL Press, London, 1997.

[6] Hassoun M., *Fundamentals of Artifical Neural Networks*, MIT Press, London, 1995.

[7] Hopfield J.J., Tank D.W., Neural Computation of Decisions in Optimization Problems, *Biol. Cybern.*, vol. 52, pp 141-152, 1985.

[8] Juang J., Stability Analysis of Hopfield-Type Neural Networks, *IEEE Transactions on Neural Networks*, vol. 10, no. 6, pp 1366-1374, 1999.

[9] Kakeya H., Okabe Y., Fast Combinatorial Optimization with Parallel Digital Computers, *IEEE Transactions on Neural Networks*, vol. 11, no. 6, pp 1323-1331, 2000.

[10] Milenković S., *Veštačke neuronske mreže*, Zadužbina Andrejević, Beograd, 1997.

[11] Qiao H., Peng J., Xu Z., Nonlinear Measures: A New Approach to Exponential Stability Analysis for Hopfield-Type Neural Networks, *IEEE Transactions on Neural Networks*, vol. 12, no. 2, pp 360 -369, 2001.