Critical Analysis of Hopfield's Neural Network Model for TSP and its Comparison with Heuristic Algorithm for Shortest Path Computation

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Abstract—**For shortest path computation, Travelling-Salesman problem is NP-complete and is among the intensively studied optimization problems. Hopfield and Tank's proposed neural network based approach, for solving TSP, is discussed. Since original Hopfield's model suffers from some limitations as the number of cities increase, some modifications are discussed for better performance. With the increase in the number of cities, the best solutions provided by original Hopfield's neural network were considered to be far away from those provided by Lin and Kernighan using Heuristic algorithm. Results of both approaches are compared for different number of cities and are analyzed properly.**

I. Introduction

The travelling salesman problem (TSP) was started in 1800s by the Irish mathematician W. R. Hamiltonian and by the British mathematician Thomas Kirkman, but its mathematical forms were initiated in 1930s. Since then researchers have been trying to compute best method for TSP, for its practical implementations. This method actually consists of visiting each city for only once following the shortest path available, also known as finding Hamiltonian tour. TSP is NP-complete and is among the most commonly tested optimization problems. Hopfield and Tank proposed a neural network based approach to solve such type of optimization problems with reduced programming complexity and faster convergence to stable states. Because of the high computational speed, neural networks are considered to provide optimal solution for shortest path computation in much less time. The major feature which neural networks offer is feedback connectivity, parallelism and collective analog mode, with each neuron summing up the inputs of hundreds and thousands of other neurons to provide graded output[6][13]. Neural networks have provided solution to many optimization problems previously. But original Hopfield's model offers some limitations as the number of cities increase and is discussed in later section.

On the other hand Heuristic algorithm is also considered to provide a better approach for TSP but with increased programming complexity. Lin and Kernighan gave 2-opt heuristic algorithm [12]. Their procedure is based on general approach to heuristics, with effective refinements, that is believed to have wide applicability in combinatorial optimization problems. Xu and Tsai have also presented some work in [18] and showed that heuristic algorithms give far better result for optimization than neural network. However Behrooz Shirazi and Sue Yih explained in [15] that the actual problem is not with the neural network but is with the mapping criterion of problem over this network. Many other reasons were also identified by researchers; however, Hopfield neural network has convergence property and is also discussed.

Section II explains the TSP problem, section III and section IV provides the Hopfield neural network and heuristic algorithm for TSP. Simulation results are presented in Section V.

II. Travelling Salesman Problem

Consider a problem of N cities for which a shortest path is needed to be found. Salesman should come back to starting city after completing its tour. Let N cities be defined as A, B, C, D... having pair wise distance as d_{AB}, d_{AC}, d_{AD},.... If shortest path consists of this sequence B,D,A,G,..., Q then total length of this tour will be

$$d = d_{BD} + d_{DA} + d_{AG} + \ldots.. + d_{QB}$$

with $d_{AB} = d_{BA}$

Let i = 1,2,3,…,N be the city location in a tour, then NxN matrix can be used to show the tour. It can be represented in two different ways but having same meanings, each row representing a particular city and column a particular position and vice versa. For example, for five-city tour DBEACD matrix is shown in Fig. 1.

In this permutation matrix, it can be seen that value of each entry is either 0 or 1. A '1' represents the visit of city 'x' at position 'i' and '0' otherwise, whereas x = A, B, C,..... Final output should be in this state so that a stable tour can be seen. For both of the approaches, finding shortest available path with a condition of visiting each city exactly once is the objective function. Neural network and heuristic algorithm provide totally different techniques for same type of optimization problems so computational complexity and practical implementations of both algorithms should be compared.

	1	2	3	4	5
A	0	0	0	1	0
B	0	1	0	0	0
C	0	0	0	0	1
D	1	0	0	0	0
E	0	0	1	0	0

Figure 1. DBEACD tour for 5-city problem

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III. HOPFIELD'S ORIGINAL MODEL

Neural network proposed by Hopfield and Tank has two-state neurons i.e. output of each neuron can take either of two states, V^0_i and V^1_i for representing 0 or 1 respectively [6][7][8]. Each neuron has sigmoid input-output relation. Although output V_i of each neuron can vary between 0 and 1 but it should finally settle down to either 0 or 1 after having comparison with a threshold value or it will may automatically converge to '0' or '1' eventually.

$$V_i = \begin{cases} 0, & V_i < threshold\ value \\ 1, & V_i \geq threshold\ value \end{cases} \tag{1}$$

Sigmoid function used to specify input-ouput relation is

$$V_i = g(u_i) = \frac{1}{2}\left(1 + tanh\left({}^{u_i}/_{u_0}\right)\right) \tag{2}$$

Here U_i is the respective input voltage for ith neuron. The input of each neuron came from two sources, external bias current I_i and outputs of other neurons.

$$U_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}V_j + I_i \tag{3}$$

Whereas

T_{ij} = Conductance between output of ith neuron and input of jth neuron

V_j = Output of jth neuron

I_i = External bias current of ith neuron, representing data provided by user

The electrical circuit corresponding to this neural network consists of operational amplifiers, capacitors and resistors as shown in Fig. 2. Operational amplifiers provide the input-output sigmoid relation; capacitors are used to show lagging nature of input u_i from instantaneous output V_j of other neurons and finite impedance is represented by T^{-1}_{ij} [8]. Thus this resistance-capacitance charging circuit determines the rate of change of input u_i. According to the Kirchhoff's current law,

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}V_j + I_i = C_i\left(\frac{du_i}{dt}\right) + \frac{u_i}{R_i} \tag{4}$$

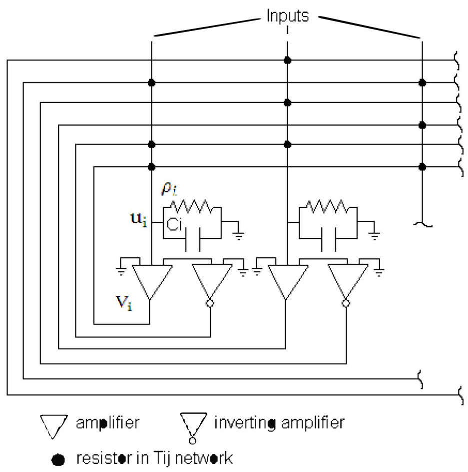

Figure 2. Hopfield's proposed analog neural network circuit

Rearranging (4)

$$C_i\left(\frac{du_i}{dt}\right) = \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}V_j + I_i - \frac{u_i}{R_i} \tag{5}$$

$$1/R_i = 1/\rho_i + \sum_j 1/R_{ij} \tag{6}$$

T_{ij} is thus the synapse efficacy and its magnitude is given by $1/R_{ij}$, where R_{ij} is the resistor connecting the output of j to the input line i. ρ_i, R_i, and Ci are input resistance of neuron i, total input resistance and capacitance of the ith neuron respectively.

Energy function for this network is

$$E = -\frac{1}{2}\sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}V_iV_j + \sum_i\left(1/R_i\right)\int_0^{V_i} g^{-1}(V)dV - \sum_{i=1}^{N} I_iV_i \tag{7}$$

Its time derivative for symmetric T is

$$\frac{dE}{dt} = -\sum_{i=1}^{N}\frac{dV_i}{dt}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}V_j - \frac{u_i}{R_i} + I_i\right) \tag{8}$$

From (5) and (8), (8) becomes

$$\frac{dE}{dt} = -\sum_{i=1}^{N} C_i\left(\frac{dV_i}{dt}\right)\left(\frac{du_i}{dt}\right) \tag{9}$$

$$= -\sum_{i=1}^{N} C_i\, g^{-1'}(V_i)\left(\frac{dV_i}{dt}\right)^2 \tag{10}$$

Since $g^{-1}(V_i)$ is a monotonic increasing function and C_i is positive, each term of (10) is nonnegative. Therefore

$$\frac{dE}{dt} \leq 0, \frac{dE}{dt} = 0 \rightarrow \frac{dV_i}{dt} = 0\ for\ all\ i$$

Above equation proves that energy function finally converges to a local minimum. The final solution may not be the most precise one but will always be a stable and valid solution because of this convergence property.

In terms of energy function, the dynamics of ith neuron becomes

$$\frac{dU_i}{dt} = -\frac{Ui}{\tau} + \frac{\partial E}{\partial V_i} \tag{11}$$

Hopfield proposed an energy function for TSP problem in [8].

$$E = \frac{A}{2}\sum_X\sum_i\sum_{j\neq i} V_{Xi}V_{Xj} + \frac{B}{2}\sum_i\sum_{Xx}\sum_{X\neq Y} V_{Xi}V_{Yi} + \frac{C}{2}\left(\sum_X\sum_i V_{Xi} - n\right)^2 + \frac{D}{2}\sum_x\sum_{y\neq x}\sum_i d_{XY}V_{Xi}\left(V_{Y,i+1} + V_{Y,i-1}\right) \tag{12}$$

In this energy function, first term will be zero iff each row has exactly one '1', which corresponds to the visit of one city for only once. Second term will be zero iff each column has exactly one '1', which corresponds to the presence of salesman in one city at given time. First two terms provide local inhibition and third term provides global inhibition of visit of only n cities. Fourth term provides total length of the tour, considering that $V_{Y,i+N}=V_{Y,i}$.

Mapping TSP system on to the Hopfield neural network, by equating energy functions of both systems; provide us following equation of motion

$$\frac{du_{Xi}}{dt} = -\frac{u_{Xi}}{\tau} - A\sum_{j\neq i} V_{xj} - B\sum_{Y\neq X} V_{Yi} - C\left(\sum_X \sum_j V_{xj} - N\right) - D\sum_Y d_{XY}\left(V_{Y,i+1} + V_{Y,i-1}\right) \quad (13)$$

These n(n-1) differential equations are solved using Euler's method to get the next state of neurons then output V_{xi}'s are updated accordingly. After 1000 iterations V_{xi}'s will be assigned a '1' or '0' as a final state to give a complete tour. The problem of self connections is solved with zero diagonal elements of the connection matrix T [9].

The limitation of this network is its dependence on the values of the parameters A, B, C and D. Wilson and Pawley, Bizzarri, Xu and many other researchers tried to solve TSP problem using the values of these parameters proposed by Hopfield. These values are A=B=500, C=200 and D=500. However with these parameter values system does not converge to stable tours and convergence rate of approximate 15% only was observed [1].

As parameters A and B enforce the system to have only one '1' in each row and each column respectively, so, increase in the value of these parameters can increases the system stability and convergence rate to a large extent, as will be shown in simulation results.

IV. Heuristic Algorithm

The abstract of heuristic algorithm used by Lin and Kernighan is [12] 'find from a set S a subset T that satisfies some criterion C and minimizes an objective function f'. In our case

Set S: all links, connecting cities with each other

Subset T: links that can make shortest path for traveling salesman

Criterion C: each city is visited only once and not more than one city can be visited at given time slot

Objective function f: total distance of the link should be minimum.

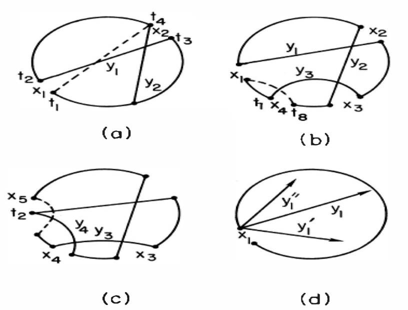

Figure 3. Description of Heuristic Algorithm. x, y_i and t_i are links in initial tour subset T, subset (S-T) and city or vertex at the end of each link. (a) i = 2, (b) i = 4 and (c) i = 5, possible exchanges are shown. (d) multiple possible choices of y_1 at Step 1.

The main steps involved in this algorithm are:

1. Generate a random initial tour T.
2. Set i=1.
3. Choose x_1 , x_2 , …… , x_n and y_1 , y_2 , …. , y_n in such a way that exchange of x_i with y_i maximize the improvement.
4. If it appears that no more gain can be made, according to an appropriate termination rule, go to step 5; otherwise, set i=i+1 and go back to step 3.
5. If the best improvement is found for i=k, exchange x_1 , x_2 , …… , x_k with y_1 , y_2 , …. y_k, to give new T, and go to step 2; if no improvement is found, go to step 6.
6. Repeat from step 1 if desired.

Lin and Kernighan refined this algorithm to get better results with improved usage of time for execution.

However Keld Helsgaun improved Lin and Kernighan's heuristic algorithm to solve TSP for greater number of cities [11].

V. Simulation Results

The n(n-1) differential equations, explaining the dynamics of the Hopfield's neural network, are implemented using Euler's method. Parameter values are listed in Table 1. It is observed that convergence rate of this network increases tremendously with the new suggested parameter values. System is initialized with random coordinate points for each city within a unit square and Euclidean distance is computed within each pair of cities. To give a start to system to move towards any stable state u_{xi}'s are selected randomly within the range of $-0.002\leq u_{xi}\leq 0.002$. It has been observed that convergence rate increases from 15% to 93.33% even for 50 cities problem. However, even for multiple runs for same distance matrix and with different initial values the obtained results provided by neural networks are much worse than those observed with the heuristic algorithm. Although new parameter values increased the convergence rate, however, this network is still not able to provide the optimum solution for greater number of cities. Changing the value of C or D parameter does not improve the performance of the system, but decrease the convergence rate. On the other hand, even with great programming complexity heuristic algorithm gives optimum and near-optimum solution in very less time.

Solutions are compared for N=5, 10, 20 and 50 in Table2.

Distance values for neural network are given after multiplying them with a constant value '100', to match the results with those of heuristic algorithm by Keld Helsgaun.

VI. Conclusion

Original model of Hopfield neural network and heuristic algorithm provided by Lin and Kernighan are implemented for N=5, 10, 20 and 50. Parameter values of Hopfield neural network are modified so as to improve the system capability to avoid unstable tours. Although the convergence rate increased from 15% to 93.33% with new parameter values but neural network is still not able to provide the optimum solution.

TABLE1.
PARAMETER VALUES

Parameters	Hopfield's parameter values	New Suggested Parameter Values
u_0	0.02	0.02
τ	0.0001	0.0001
A	500	1000
B	500	1000
C	200	200
D	500	500
n	15	(No. of cities)+5

TABLE2.
SIMULATION RESULTS

Size of Network (N)	Total Distance by HNN	Total Distance by Heuristic Algorithm
5	27	27
10	39	36
20	124	62
50	250	137

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