Let G represent an undirected graph.

Prove that the problem of deciding whether G contains a simple path of length at least k from vertex a to vertex b is NP-complete.

Hamiltonian path in a graph goes through each vertex of the graph exactly once. We already know that the problem of deciding whether a graph contains a hamiltonian path is NP-complete. You can use this assumption for proof.

To show that any problem A is NP-Complete, we need to show four things:

(1) there is a non-deterministic polynomial-time algorithm that solves A, i.e., A $\in$ NP,

(2) any NP-Complete problem B can be reduced to A,

(3) the reduction of B to A works in polynomial time,

(4) the original problem A has a solution if and only if B has a solution.