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# NP-Completeness Practice

We need to prove - "*The problem of deciding whether G contains a simple path of length at least k from vertex a to vertex b is NP-complete.*" To prove that this problem (A) is NP-complete we need to show four things :

***(1) There is a non-deterministic polynomial-time algorithm that solves A, i.e., A ∈ NP***

- This can be proved by using the "Hamiltonian Path Existence" algorithm. We first check whether the graph has a Hamiltonian path starting from a and ending at b. If it does, then there must exist a linear path between a and b where length is n (number of nodes). If it doesn't then we reduce the vertices and check this again, until we have k vertices left. If at any point we find a Hamiltonian path then we can say "*there is a path of length at least k between a and b"*.

So since the problem A can be reduced to the "Hamiltonian Path Existence" problem, it is a problem in NP.

***(2) Any NP-Complete problem B can be reduced to A***

- We saw that the problem A ("k-length Simple Path Existence") can be reduced to the "Hamiltonian Path Existence" problem. Also to check whether we have a Hamiltonian path we can simply use problem A i.e. check whether we have a simple path between any two vertices a to b where k=number of nodes - 1.

So the "Hamiltonian Path Existence" can be reduced to A and we already know it is an NP-Complete problem so all other NP-Complete problems can be reduced to "Hamiltonian Path Existence" problem and subsequently can be reduced to A.

***(3) The reduction of B to A works in polynomial time***

- Any problem B can be reduced to the "Hamiltonian Path Existence" problem in polynomial time. Now that problem can be reduced to A very easily with k=number of nodes -1. So we can say B can be reduced to A in polynomial time.

***(4) the original problem A has a solution if and only if B has a solution.***

- A will have a solution if a problem B ("Hamiltonian Path Existence" and subsequently all NP-Complete) problems have a solution. Now if we can check the existence of a hamiltonian path, we can solve this problem because we already established an algorithm in (1) using this. Again if A can be solved then so can B because we can reduce B to A by using k = number of nodes -1. So this proof works bi-directionally.