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Error of Saturation Vapor Pressure Calculated by Different Formulas and Its Effect on Calculation of Reference Evapotranspiration in High Latitude Cold Region

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Abstract

Teten, Buck and Magnus formulas for saturation vapor pressure e_s calculation were evaluated under different temperature conditions, compared with Goff-Gratch formula. Then errors of vapor pressure deficit VPD and reference crop evapotranspiration ET_0 derived from e_s by Teten were examined in high latitude cold regions. Error of e_s by Teten increased linearly with reduction of temperature. When it was applied for ET_0 calculation by Penman-Monteith equation in high latitude cold region, errors of VPD and ET_0 were acceptable with relative error lower than 10% only when average daily temperature is above -10°C, but error increased with the decrease of temperature. Buck and Magnus may be feasible substitute for Teten because of well performance.

Keywords: Saturation vapor pressure; reference evapotranspiration; cold region; vapor pressure deficit

1. Introduction

Air drying force is the important parameter in the determination of evapotranspiration, which is essential in agricultural water management and hydrological practices. Saturation water vapor pressure (e_s) and the vapor pressure deficit (VPD) is necessary when reference crop evapotranspiration (ET_0) was calculated by Penman-Monteith equation (FAO-56 PM), the sole standard method proposed by Food and Agriculture Organization (FAO) [1]. In determination of ET_0 by FAO-56 PM, Teten formula [2] was adopted for calculation of e_s based on air temperature.

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$$\mathrm{ET}_0 = \frac{0.408\Delta(R_\mathrm{n} - G) + \gamma \dfrac{900}{t+273} u_2 (e_\mathrm{s} - e_\mathrm{a})}{\Delta + \lambda(1 + 0.34 u_2)} \tag{1}$$

It is assumed that e_s were influenced only by air temperature t[ºC] or T[K], many formulas have been presented, such as Goff-Gratch [3], Teten, Magnus[4,5], and Buck formula[6]. Goff-Gratch has been proposed as standard by World Meteorological Organization and collected into the Standard of Meteorological Observation in China [7,8]. Goff-Gratch is authoritative method [7,9-11], but Teten was frequently used for its calculation convenience[12,13]. It was reported that result of Teten is acceptable at normal temperature (20-40ºC), but error enlarged significantly under the condition of ultra-high or ultra-low temperature [14]. So those formulas (i.e. Goff-Gratch, Magnus and Buck) are likely with different equations or different parameters under low temperature condition compared to the normal temperature condition, to improve their performance in wide temperature range (i.e. form -40 ºC to 100 ºC).

But in FAO-56 PM, Teten is set as formula 4. If it is applicable in cold region, and to what degree will it influence on the calculation of ET_0, is still unknown. Maulé et al calculate e_s by Teten formula and hence ET_0 by the ASCE standard Penman-Montieth equation in high latitude cold region (latitude ranged from 49.1°N to 55.2°N) in Canada[15]. In fact, there will be long-time with temperature lower than 0ºC in high latitude cold region. Global climate change also puts high latitude region to be suffer with extreme low temperature [16]. So it is necessary to evaluate the influence of Teten formula on determination of saturation vapor pressure, vapor pressure deficits and hence on the evapotranspiration.

2. Saturation vapor pressure formulas

2.1. Goff-Gratch formula

For T>273.16K and T<273.16K, $\mathrm{e_s}$ was calculated by equation (2) and (3) [3].

$$\lg\left[e_\mathrm{s}(T)\right] = \mathrm{a}_1\left(\frac{373.16}{T} - 1\right) + \mathrm{b}_1 \lg\left(\frac{373.16}{T}\right) + \mathrm{c}_1\left(10^{\mathrm{d}_1\left(1-\frac{T}{373.16}\right)} - 1\right) + \mathrm{e}_1\left(10^{\mathrm{f}_1\left(\frac{373.16}{T}-1\right)} - 1\right) + \lg(\mathrm{g}_1) \tag{2}$$

$$\lg\left[e_\mathrm{s}(T)\right] = \mathrm{a}_2\left(\frac{273.16}{T} - 1\right) + \mathrm{b}_2 \lg\left(\frac{273.16}{T}\right) + \mathrm{c}_2\left(1 - \frac{T}{273.16}\right) + \lg(\mathrm{d}_2) \tag{3}$$

Wherein $\mathrm{a_1}$-$\mathrm{g_1}$, and $\mathrm{a_2}$-$\mathrm{d_2}$ are constants. $\mathrm{a_1}$=-7.90298, $\mathrm{b_1}$=5.02808， $\mathrm{c_1}=1.3816\times10^{-7}$, $\mathrm{d_1}$=11.344, $\mathrm{e_1}=8.1328\times10^{-3}$, $\mathrm{f_1}$=-3.49149, $\mathrm{g_1}$=1013.246; $\mathrm{a_2}$=-9.09718, $\mathrm{b_2}$=-3.56654, $\mathrm{c_2}$=0.876793, $\mathrm{d_2}$=6.1071.

2.2. Teten formula in FAO 56

$$e_\mathrm{s}(t) = 0.611 \times \exp\left[\frac{17.27t}{t + 237.3}\right] \tag{4}$$

2.3. Magnus formula

For t>0ºC and t<0ºC, $\mathrm{e_s}$ was calculated by equation (5) and (6) **[4]**.

$$e_\mathrm{s}(t) = 6.11 \times 10^{7.45t/(237.3+t)} \tag{5}$$

$$e_\mathrm{s}(t) = 6.11 \times 10^{9.5t/(265.5+t)} \tag{6}$$

2.4. Buck formula

For t>0ºC and t<0ºC, e_s was calculated by equation (7) and (8) [6].

$$e_s(t) = 6.1121 \times \exp\left(\frac{(18.678-\frac{t}{234.5})t}{257.14+t}\right) \tag{7}$$

$$e_s(t) = 6.1115 \times \exp\left(\frac{(23.306-\frac{t}{333.7})t}{279.82+t}\right) \tag{8}$$

Unit for e_s is hPa in Goff-Gratch, Magnus, and Buck formula, and kPa in Teten formula. Unit for T is ºC in Magnus, Teten and Buck formula, and K in Goff-Gratch formula.

3. Climate data and Statistical index

Four weather stations were selected from high latitude cold region in north China, Harbin, Heihe, Urumchi and Aletai. The characteristics of each station were listed in Table 1. In all the four stations, annual days with temperature below 0ºC exceeds 110d. In Heihe station, the station with the highest latitude in China, the days with temperature below 0ºC exceed five and a half months. Daily meteorology data set from 1954 to 2006 were collected in all the four station mentioned above.

Table1. Characteristics of weather stations in high latitude cold region in China

Station	Latitude	Longitude	Altitude /m	Annual Temperature/ºC			Days with temperature<0ºC /d.yr^{-1}
				Minimum	Maximum	Average	
Harbin	45.8	126.8	142.3	-32.5	34.1	4.2	112
Heihe	50.3	127.5	166.4	-36.3	33.8	0.4	165
Urumchi	43. 8	87.7	935.0	-27.2	37.8	6.8	130
Aletai	47.7	88.1	735.3	-35.1	34.8	4.4	142

For e_s, determined by different formula, relative error (RE) and average of relative error (ARAE) were determined with e_s by Goff-Gratch formula as standard. For VPD and ET_0 determined with e_s by Teten, average of absolute error (AAE) and ARAE were determined. Regression with zero interception was also applied, slope of the regression was used to illustrate the consistency of VPD and ET_0 determined with different e_s formulas to those determined with e_s by Goff-Gratch formula.

4. Results and discussion

4.1. Saturation vapor pressure by different formulas

Taking results by Goff-Gratch as standard, e_s estimated by Teten, Magnus and Buck were evaluated, with a wide range of temperature from -50ºC to 50ºC. REs were plotted in Fig.1. Buck and Magnus, those are characterized with different equation for t<0ºC and t>0ºC circumstance, result in high consistency with Goff-Gratch within all temperature range. When the temperature falls into the range of 0-40ºC, Teten performs quiet well, showing high consistency with Goff-Gratch, but for temperature below 0ºC, errors of e_s by Teten increased linearly with reduction of temperature, when the temperature dropped to -

40ºC, RE of e_s by Teten is about 40%. Error of e_s is remarkable by Teten formula which was proposed in determination of ET_0 by FAO-56 PM equation, under the condition of low temperature (t<0ºC), that caused the risks on the application of FAO-56 PM equation in high latitude cold region.

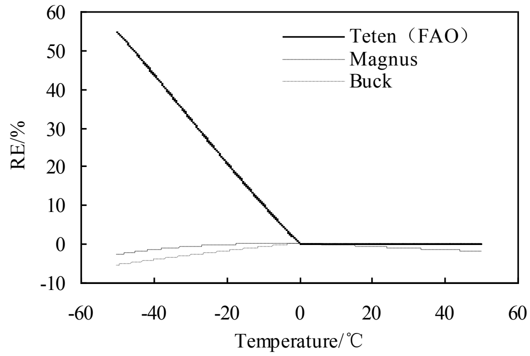

Fig.1 Relative errors of saturation vapor pressure derived from different formulas

Table 2 Errors of vapor pressure deficit (VPD) and reference evapotranspiration (ET0) with es derived from Telten formula

Station		Error	<-30ºC	-30~-20ºC	-20~-10ºC	-10~0ºC	>0ºC	Total
Harbin	VPD	AAE/kPa	0.0060	0.0085	0.0075	0.0049	0.0017	0.0036
		ARAE/%	67.68	39.05	15.47	2.86	0.41	5.96
		Slope of y=*a*x	0.6213	0.7384	0.9120	0.9989	1.0011	1.0080
	ET_0	AAE/kPa	0.0465	0.0503	0.0385	0.0219	0.0063	0.0172
		ARAE/%	66.54	40.08	14.78	2.45	0.31	5.77
		Slope of y=*a*x	0.6428	0.7508	0.9176	1.0006	1.0009	1.005
Heihe	VPD	AAE/kPa	0.0064	0.0091	0.0085	0.0053	0.0019	0.0045
		ARAE/%	74.45	43.50	16.69	2.90	0.48	10.58
		Slope of y=*a*x	0.5937	0.7289	0.9110	0.9992	1.0019	1.0012
	ET_0	AAE/kPa	0.0335	0.0413	0.0392	0.0239	0.0072	0.0200
		ARAE/%	74.37	41.66	15.21	2.49	0.34	9.95
		Slope of y=*a*x	0.5962	0.7393	0.9225	1.0007	1.0013	1.0007
Urumchi	VPD	AAE/kPa	0.0047	0.0071	0.0054	0.0031	0.0011	0.0023
		ARAE/%	71.01	40.85	14.38	3.25	0.38	3.78
		Slope of y=*a*x	0.5846	0.7312	0.9183	1.0093	1.0001	1.0001
	ET_0	AAE/kPa	0.0347	0.0480	0.0302	0.0143	0.0038	0.0105
		ARAE/%	71.86	39.69	13.39	2.65	0.28	3.44
		Slope of y=*a*x	0.5856	0.7410	0.9138	1.0118	1.0002	1.0002
Aletai	VPD	AAE/kPa	0.0044	0.0043	0.0042	0.0033	0.00013	0.0013
		ARAE/%	63.10	18.95	8.03	3.00	0.015	2.53
		Slope of y=*a*x	0.6613	0.8662	1.0036	1.0177	0.9999	1.0001
	ET_0	AAE/kPa	0.0334	0.0228	0.0104	0.0125	0.00029	0.0053
		ARAE/%	58.93	37.60	19.28	3.82	0.009	1.55
		Slope of y=*a*x	0.6871	0.8713	0.9985	1.0152	0.9999	1.0001

*All regressions of y=ax were significant at level of p=0.01.

4.2. Influence on determination of vapor pressure deficit and reference crop evapotranspiration

Based on e_s calculated by both Goff-Gratch and Teten, VPD and ET_0 were determined in the four stations by FAO-56 PM equation. For VPD and ET_0 determined based on different e_s formulas, as a whole, ARAE is less than 10%， in Harbin, Urumchi and Aletai, but in Heihe ARAE is a little bigger than 10%. Regression with zero interception indicates that the slope a is very close to 1, that means Teten formula results in acceptable result and shows high consistency with Goff-Gratch formula in the determination of VPD and ET_0, reviewing from the whole.

Errors of VPD and ET_0 in different temperature ranges (<-30ºC, -30~-20ºC, -20~-10ºC, -10~0ºC and >0ºC) conditions were listed in Table 2. AAE of VPD and ET_0 based on Teten increased with the decrease of temperature, reaching to the maximum when the temperature falls into the range of -30~-20ºC. ARAE of VPD and ET_0 also increased with the decrease of temperature. ARAE of VPD and ET_0 exceeds 10% when the temperature drops to lower than -10℃. Regression slope a decrease remarkably from appropriate 1.0 when temperature above -10ºC to less than 0.7 when the temperature lower than -30ºC. That indicates the lower temperature, the larger deviation of VPD and ET_0 between Teten and Goff-Gratch formula. So Teten formula recommended by FAO-56 will be debated when calculating ET_0 in high latitude in cold region.

5. Conclusion

Buck and Magnus, formulas characterized with different equations for t<0ºC and t>0ºC circumstance, result in acceptable e_s under all temperature conditions from -50ºC to 50ºC comparing with Goff-Gratch, but error of e_s by Teten increased rapidly with the drop of temperature when the temperature is below 0ºC, and relative error is about 40% when the temperature approaches -40ºC. Teten formula results in acceptable results of e_s and ET_0 compared with Goff-Gratch only when the temperature is above -10ºC. Large error in e_s calculation results in remarkable error in VPD and ET_0 when the temperature is lower than -10ºC, and the error increased with the decrease of temperature. That means when FAO-56 PM equation is used to calculate ET_0 in cold regions; the e_s calculation formula must be reconsidered. Since Buck and Magnus formulas result in acceptable result in e_s and they are simple comparing with Goff-Gratch, which may be feasible substitute for Teten formula.

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