Let $\Sigma = \{a, b\}$. Consider the following language over the Σ.

$L1$ = {w contains exactly two b}

$L2$ = {w contains aa or bb}

$L3$ = {w ends with even numbers of b}

$L4$ = {length of w is two more than multiple of four}

$L5$ = {each a in w is followed by at least two b}

$L6 = L1 \cap L5$

L7 = {ω starts with even numbers of a}

a) Give the regular expression for L1. (4 points)

$a^* b\, a^* b\, a^*$

b) Give the regular expression for $\overline{L2}$. (3 points)

$\overline{L2}$ = ω contains neither aa nor bb

Sol1: $(ab)^*(a+\varepsilon) + (ba)^*(b+\varepsilon)$ Sol2: $(a+\varepsilon)(ba)^*(b+\varepsilon)$

c) Write four four-letter strings that belongs to L3. (2 points)

2^3 choices aaaa, babb, aabb, bbbb (write any four)

d) Give the regular expression for L3. (3 points)

$(a+b)^* a (bb)^* + (bb)^*$

e) Give the regular expression for L4. (4 points)

Sol1: $((a+b)(a+b)(a+b)(a+b))^* (aa+ab+ba+bb)$

Sol2: $((a+b)(a+b)(a+b)(a+b))^* (a+b)(a+b)$

f) Give the regular expression for L5. (2 points)

$b^* (a\, bb\, b^*)^*$

g) Give the regular expression for L6. (2 points)

$bb + abb$

L7 = ω starts with even numbers of a

$(aa)^* b (a+b)^* + (aa)^*$

Let $\Sigma = \{a, b\}$. Consider the following language over the Σ. 'Precede' means come before in order or position.

$L1$ = {w contains at most two a}
$L2$ = {w contains ab or ba}
$L3$ = {w starts and ends with different symbols}
$L4$ = {length of w is not multiple of three}
$L5$ = {each b in w is preceded by even numbers of a}
$L6 = L1 \cap L3$

a) Give the regular expression for L1. (4 points)

Sol1 : $b^* + b^*ab^* + b^*ab^*ab^*$

Sol2 : $b^*(a+\varepsilon)b^*(a+\varepsilon)b^*$

b) Give the regular expression for $\overline{L2}$. (3 points)

$\overline{L2}$ = w contains neither ab nor ba

$a^* + b^*$

c) Give the regular expression for L3. (3 points)

$a(a+b)^*b + b(a+b)^*a$

d) Give the regular expression for L4. (4 points)

Sol1 : $((a+b)(a+b)(a+b))^*(a+b+ab+ba+aa+bb)$

Sol2 : $((a+b)(a+b)(a+b))^*(a+b+\varepsilon)(a+b)$

e) Write four four-letter strings that belongs to L5. (2 points)

aaaa, baaa, ~~baab~~, aaba, baab, bbaa, aabb, bbba, bbbb (write any four)

f) Give the regular expression for L5. (2 points)

$((aa)^*b)^*a^*$

g) Give the regular expression for L6. (2 points)

Sol1: $ab^*b + ab^*ab^*b + bb^*ab^*a + bb^*a$

Sol2: $ab^*(a+\varepsilon)b^*b + bb^*(a+\varepsilon)b^*a$

Let $\Sigma = \{a, b\}$. Consider the following language over the Σ. 'Precede' means come before in order or position.

$L1$ = {w contains at most two b}

$L2$ = {w contains ab or ba}

$L3$ = {w starts and ends with same symbols}

$L4$ = {length of w is not multiple of three}

$L5$ = {each a in w is preceded by odd numbers of b}

$L6 = L1 \cap L3$

a) Give the regular expression for L1. (4 points)

Sol1 : $a^* + a^*ba^* + a^*ba^*ba^*$

Sol2 : $a^*(b+\varepsilon)a^*(b+\varepsilon)a^*$

b) Give the regular expression for $\overline{L2}$. (3 points)

$\overline{L2}$ = ω contains neither ab nor ba

$a^* + b^*$

c) Give the regular expression for L3. (3 points)

$a(a+b)^*a + b(a+b)^*b + a + b$

d) Give the regular expression for L4. (4 points)

Sol1 : $((a+b)(a+b)(a+b))^*(a+b+aa+ab+ba+bb)$

Sol2 : $((a+b)(a+b)(a+b))^*(a+b)(a+b+\varepsilon)$

e) Write four four-letter strings that belongs to L5. (2 points)

bbbb, babb, bbba, baba

f) Give the regular expression for L5. (2 points)

$((bb)^*ba)^*b^*$

g) Give the regular expression for L6. (2 points)

Sol1 : $ba^*b + ~~ab~~ aa^*a + aa^*ba^*a + aa^*ba^*ba^*a$

Sol2 : $ba^*b + aa^*(b+\varepsilon)a^*(b+\varepsilon)a^*a$