

Assignment 01 Part B

General Feedback

Note: Finding out the corner cases is very important.

11. a) Draw a DFA of strings that have 1 as every 3rd symbol. $\Sigma = \{0,1\}$

- A common mistake is missing on accepting these strings: eps, 0, 1, 00, 0010, etc

Explanation: 0010 should be accepted since you can't show any 0 in the third position that violates the condition of L. Hence 0010 is a member of L, not the L'. Same goes for other strings.

13) Draw a DFA that accepts at least two "00" as a substring. $\Sigma = \{0,1\}$

- 000 should be accepted since there are two 00s in 000 as a substring, 00100 should be accepted.

14. a) Draw a DFA that accepts exactly two "00" as a substring. $\Sigma = \{0,1\}$

- 0000 should be **rejected** since there are three 00s in 0000 as a substring.

15. Construct a DFA defined as $L = \{\text{An even number of 0s follow the last 1 in } w\}$ $\Sigma = \{0,1\}$

- eps, 0, 00, 000, 1, 11 etc should be accepted.

17. Construct a DFA where the set of binary strings where numbers of 0s between two successive 1s will be even. $\Sigma = \{0,1\}$.

- eps, 0, 00, 000, 1, 11, 1000, 010010 etc should be accepted

- 0100100011 should be rejected.

Explanation: 00, 1000 should be accepted since there are no odd 0s between two successive 1s. As the strings are not violating the condition, the strings will be members of L, not the L'.

18. Construct a DFA of the Language, $L = \{w \in \{0,1\}^* : \text{no } 00 \text{ appears as a substring before the first } 11 \text{ in } w.\}$

- 00, 000, 001 etc should be accepted.

20. a) Construct a DFA of the Language, $L = \{w \in \{0,1\}^* : w \text{ contains } 01^m0 \text{ as a substring where } m \text{ is divisible by } 3\}$

- 00 should be accepted since 01^00 is 00 and 0 is divisible by 3. Another example of an accepted string is 01011101

21. a) Construct a DFA of the Language, $L = \{w \in \{0,1\}^* : w = 0^m1^n \text{ where } m \text{ and } n \text{ are both odd.}\}$

- The strings look like - 000....00001111....1111 where count 0 and 1 will be odd.

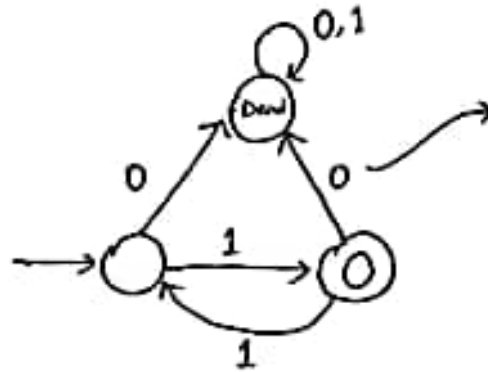
Section 05: MHB Sin

$$L_1 = \{ w : w = 1^m, \text{ where } m \text{ is odd} \}$$

$$L_2 = \{ w : w \text{ doesn't contain any } y \in L_1 \text{ as a substring} \}$$

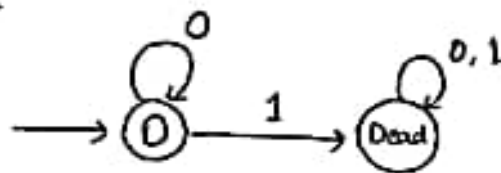
Solve:

b) DFA for L_1 :



Since $w = 1^m$, and it is not substring, getting 0 will violate the condition.

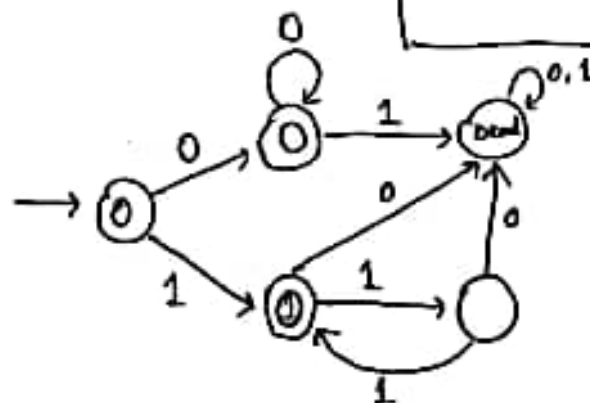
c) DFA for L_2 :



Since L_2 can't contain any substring of L_1 , now, if we want to have even numbers of 1s, for example, to get '11', we have to get '1' first. One getting a '1' actually violates the condition.

d) $L = L_1 \cap L_2$: and

if asked for $L_1 \cup L_2$:
'or'



for (d) you must use cross product, But it will take lots of time.

$$L1 = \{0, 10\}$$

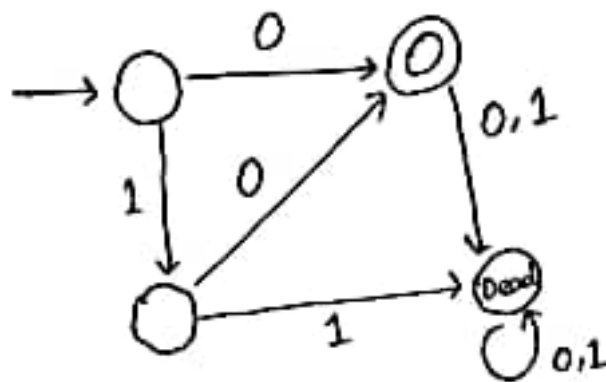
$$L2 = L1^*$$

$$L3 = \{\omega : \text{the length of } \omega \text{ is fun}\}$$

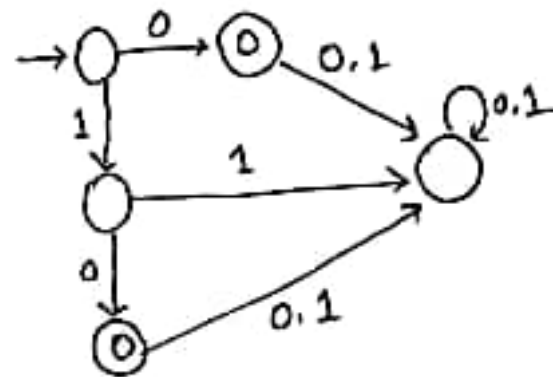
a) write down all strings in $L2 \cap L3$.

0000, 0010, 0100, 1000, 1010

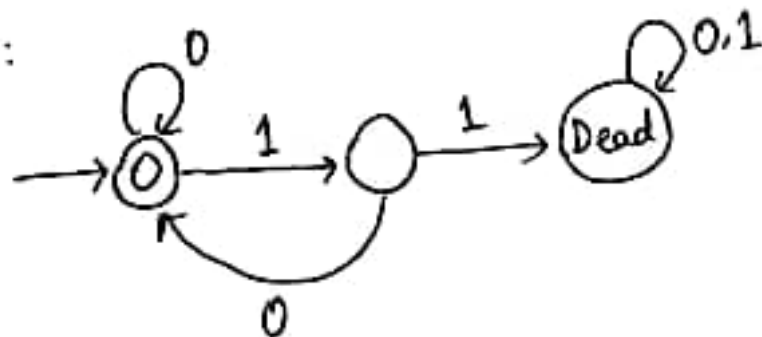
b) DFA for $L1$:



or, we can also do this:

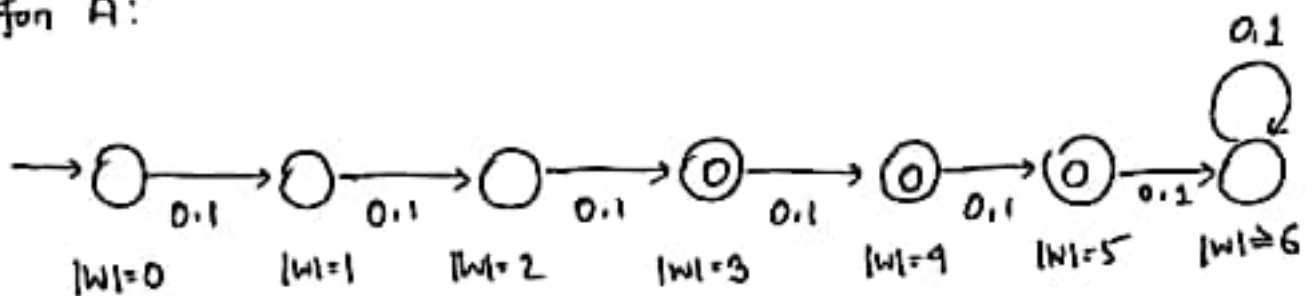


c) DFA for $L2$:

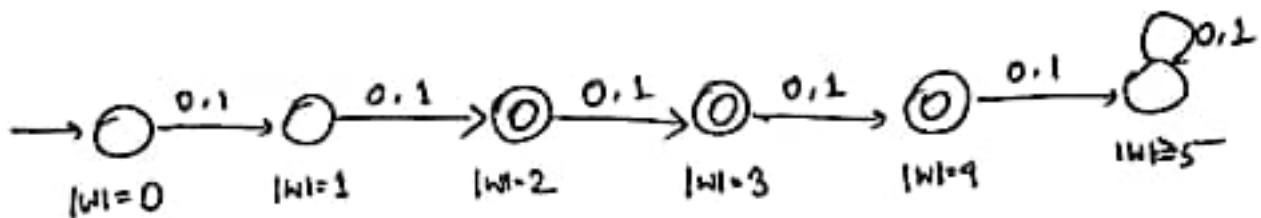


Section 6: MHB Sin

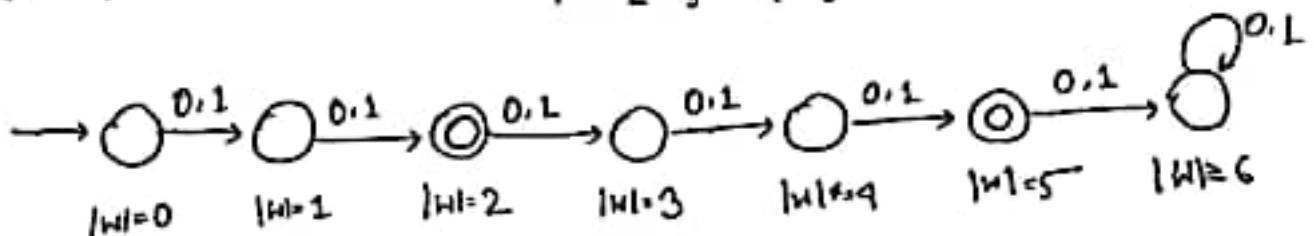
a) DFA for A:



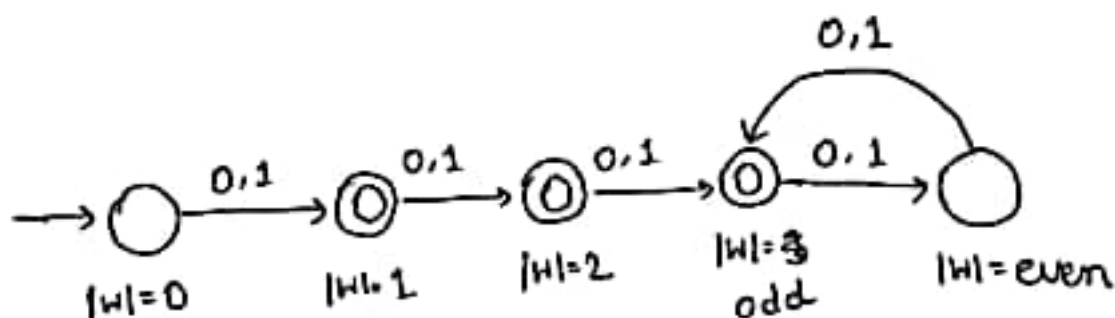
b) DFA for B:



c) DFA for $A \Delta B$:
 not both at the same time

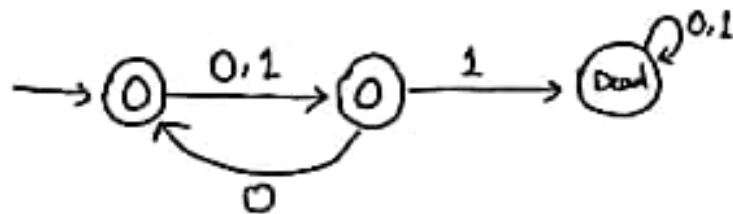


e) DFA for $(A \Delta B) \cup C$: (5 states)

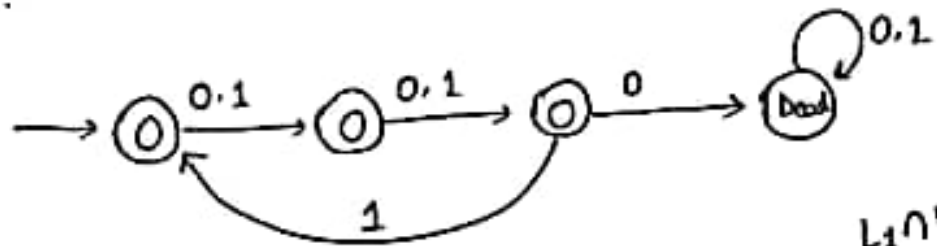


$L_1 = \{w : \text{every second letter of } w \text{ is } 0\}$
 $L_2 = \{w : \text{every third letter of } w \text{ is } 1\}$

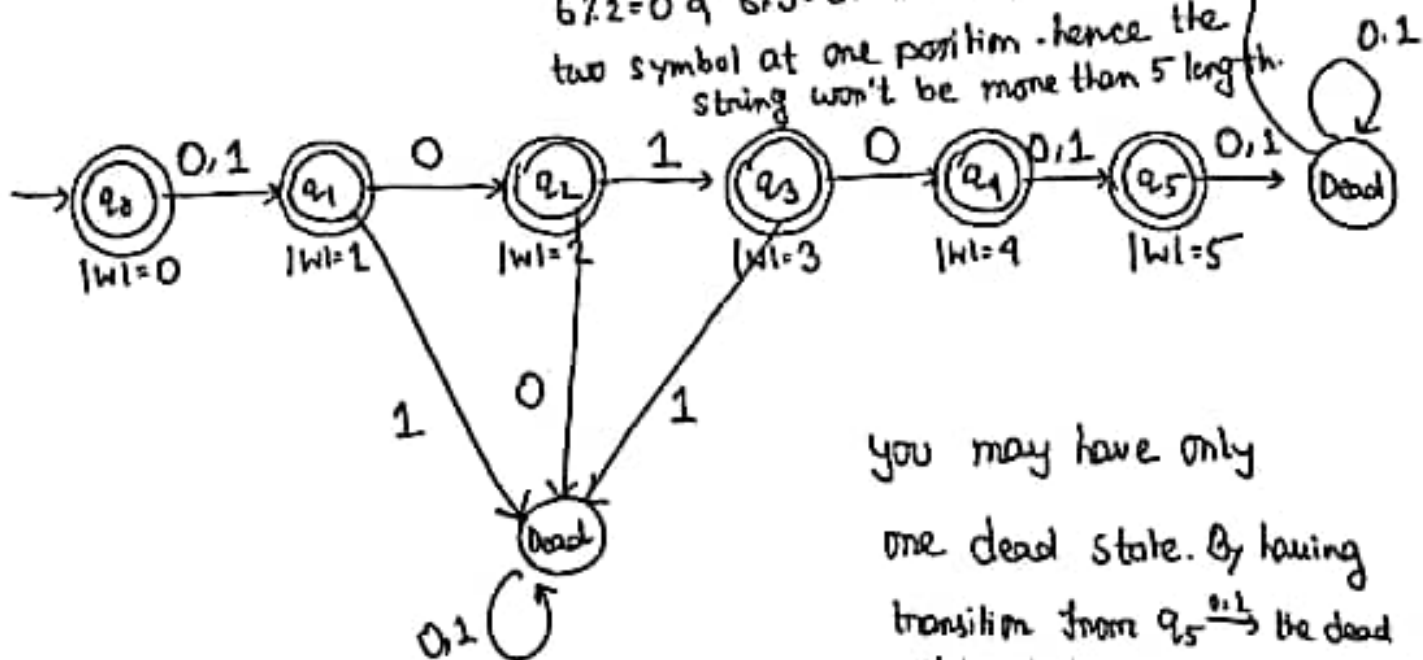
b) DFA for L_1 :



c) DFA for L_2 :



d) DFA for $L_1 \cap L_2$:

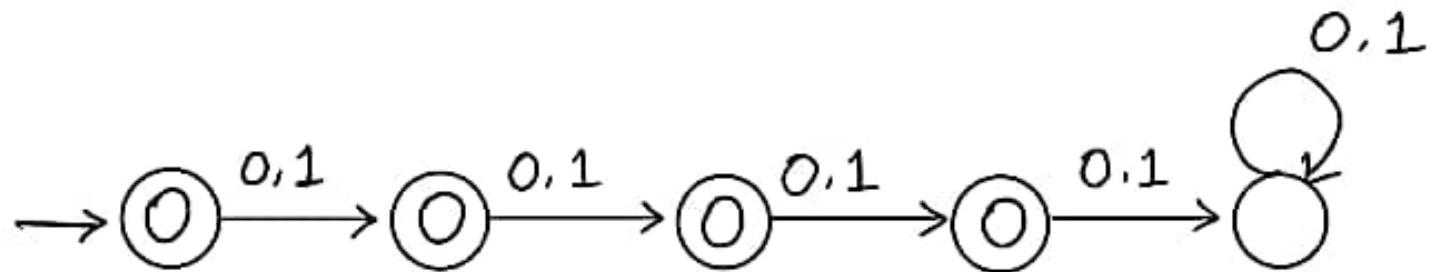


at the sixth position → we should have 0 and 1 both. Because $6 \div 2 = 0$ & $6 \div 3 = 0$. However, we can't have two symbol at one position. Hence the string won't be more than 5 length.

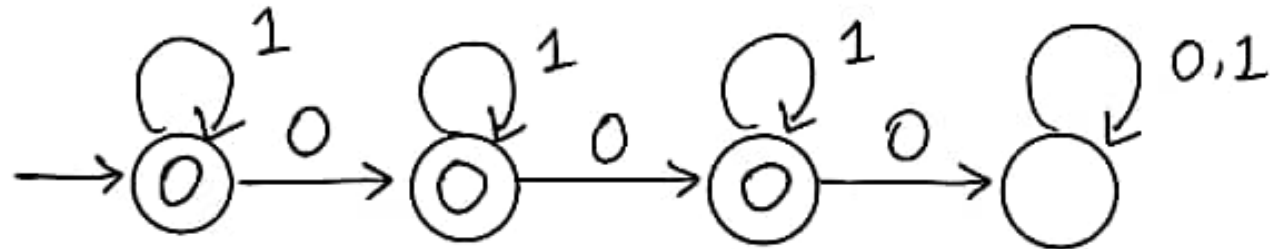
you may have only one dead state. By having transition from $q_5 \xrightarrow{0,1}$ the dead state at below.

1. Let $\Sigma = \{0, 1\}$. Consider the following language over the Σ .

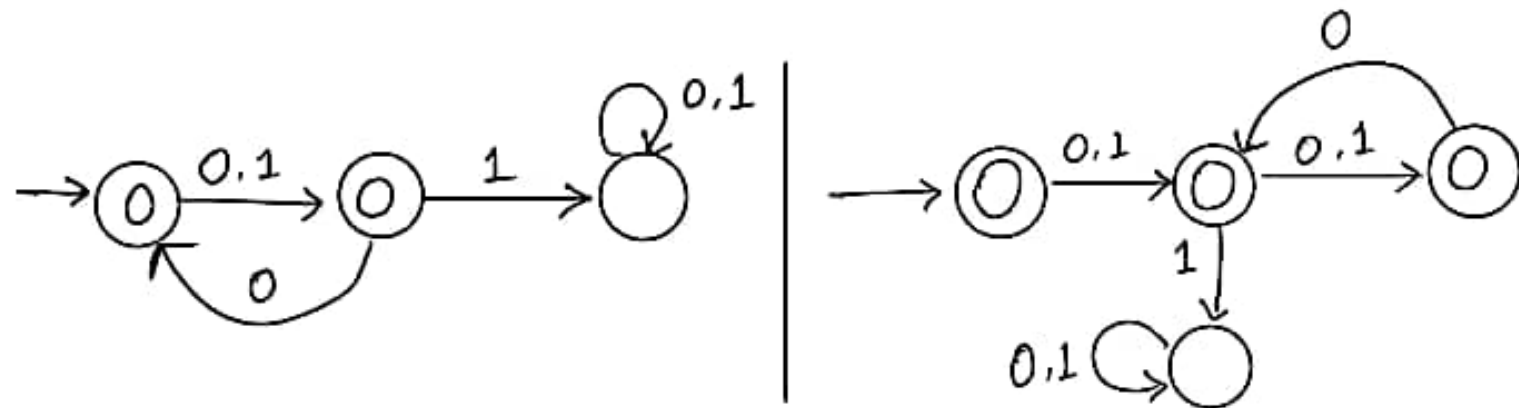
a) $L1 = \{\text{length of } w \text{ is at most three}\}$



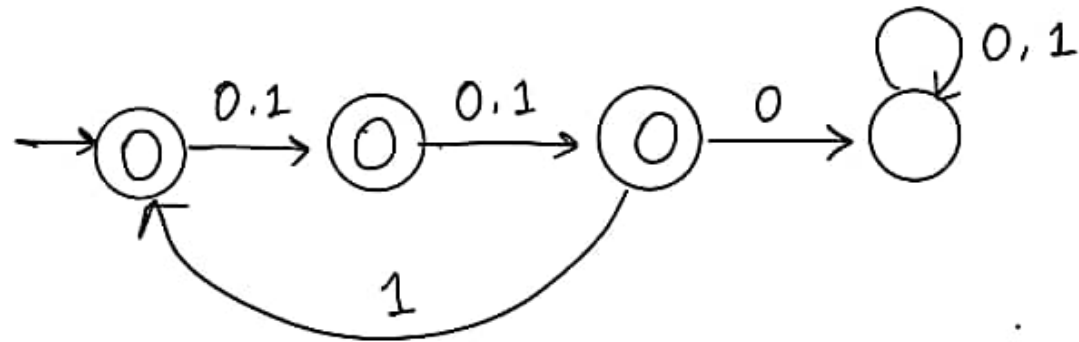
b) $L2 = \{w \text{ contains at most two 0s}\}$



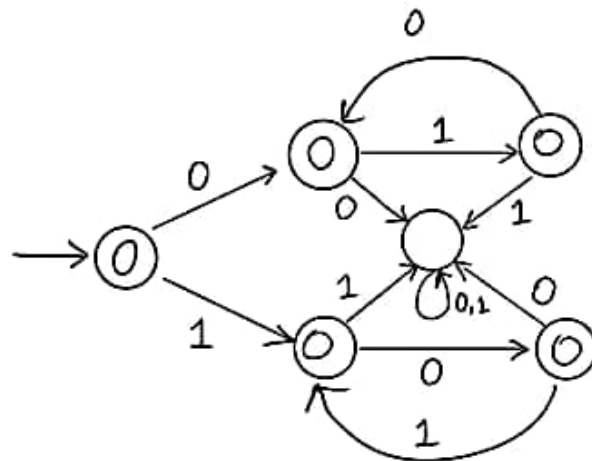
c) $L3 = \{\text{every second letter of } w \text{ is 0}\}$



d) $L4 = \{\text{every third letter of } w \text{ is } 1\}$

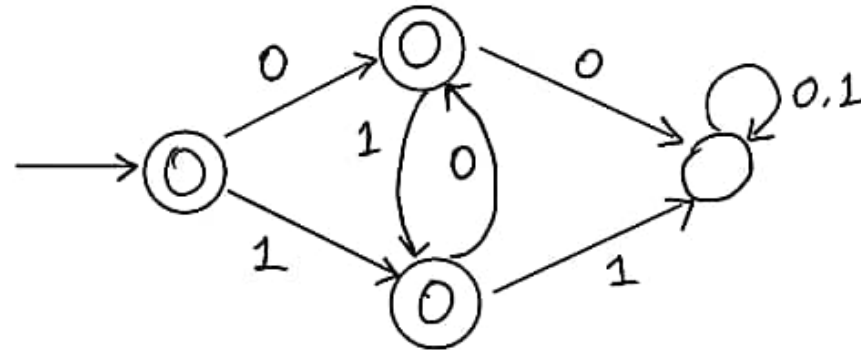


e) $L5 = \{0s \text{ and } 1s \text{ alternate in } w\}$

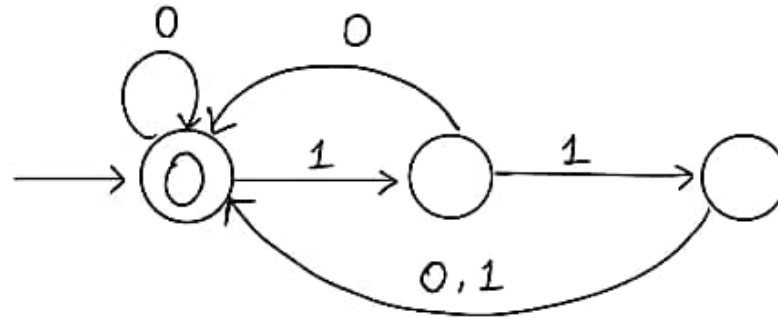


f) $L6 = \{w \text{ contains neither } 00 \text{ nor } 11\}$ [Same as e) L5]

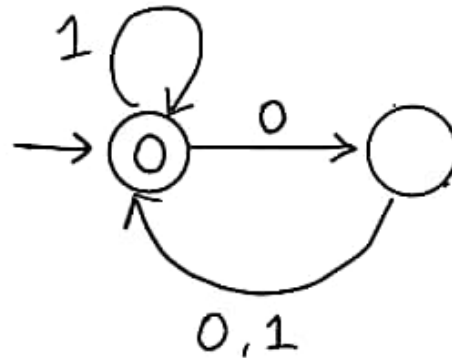
Another solution:



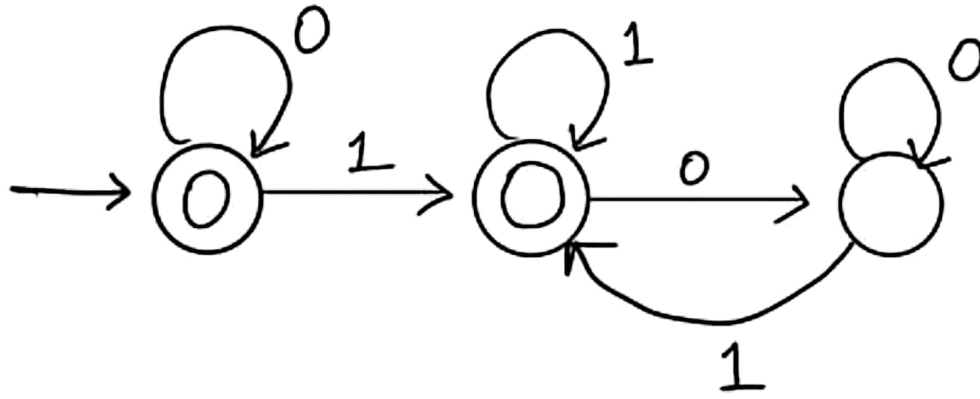
g) $L7 = \{w \text{ ends with } 1^m, \text{ where } m \text{ is multiple of three}\}$



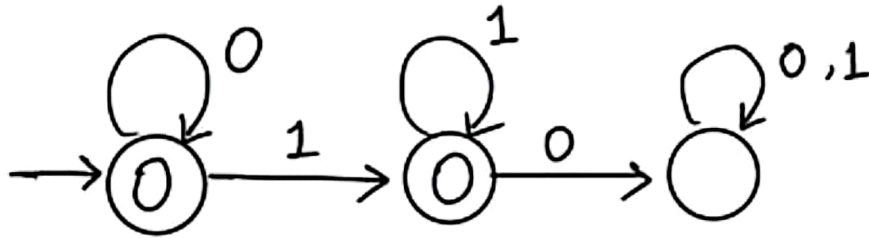
h) $L8 = \{w \text{ ends with even numbers of 0s}\}$



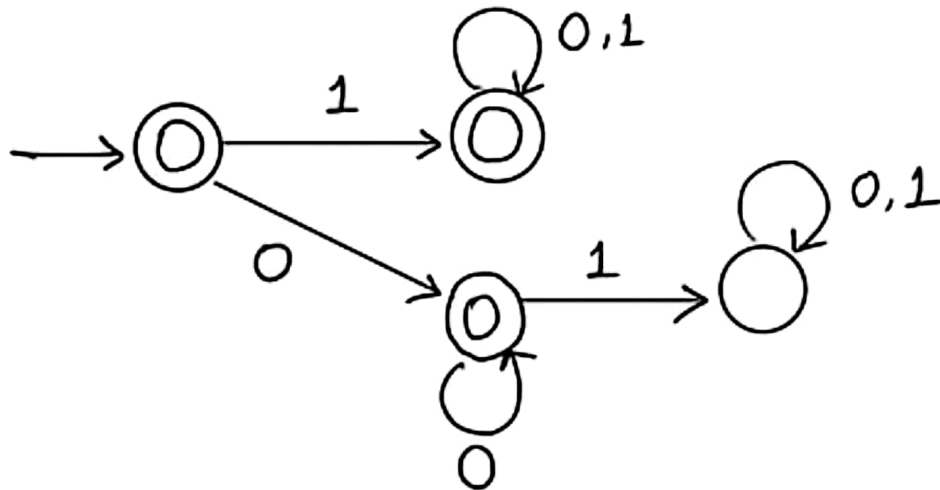
i) $L_9 = \{\text{no 0 appears after the last 1 in } w\}$



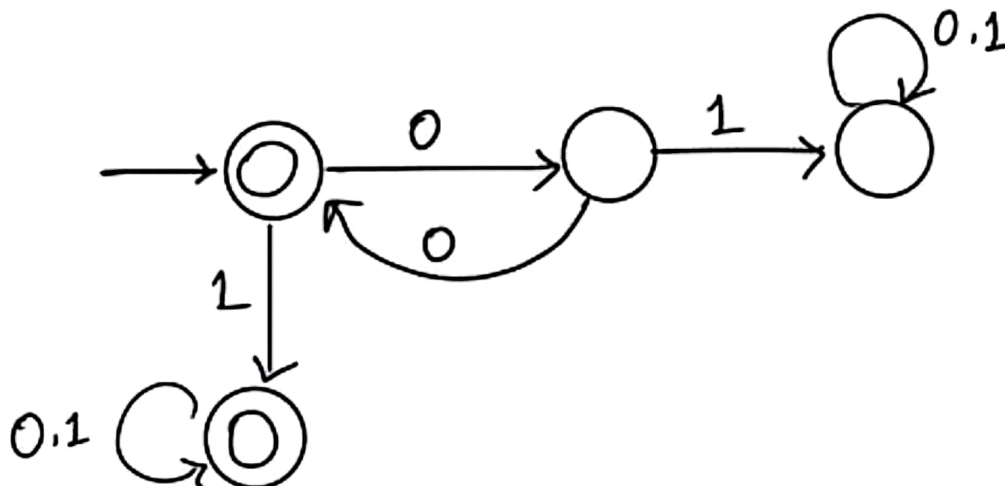
j) $L_{10} = \{\text{no 0 appears after the first 1 in } w\}$



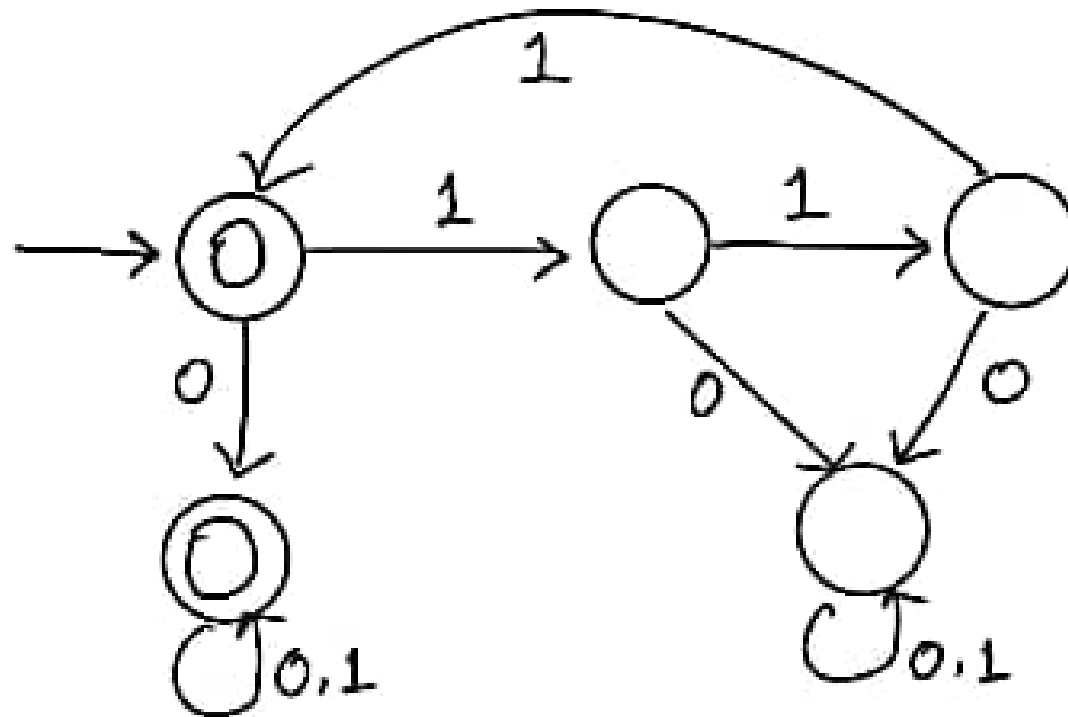
k) $L_{10} = \{\text{no 0 appears before the first 1 in } w\}$



l) $L_{11} = \{w \text{ starts with even numbers of 0s}\}$

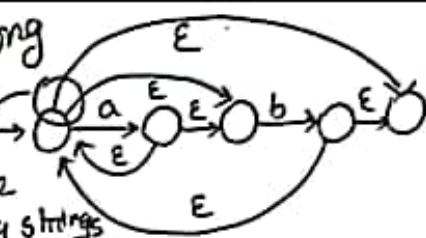


m) $L12 = \{w \text{ starts with } 1^m, \text{ where } m \text{ is multiple of three}\}$



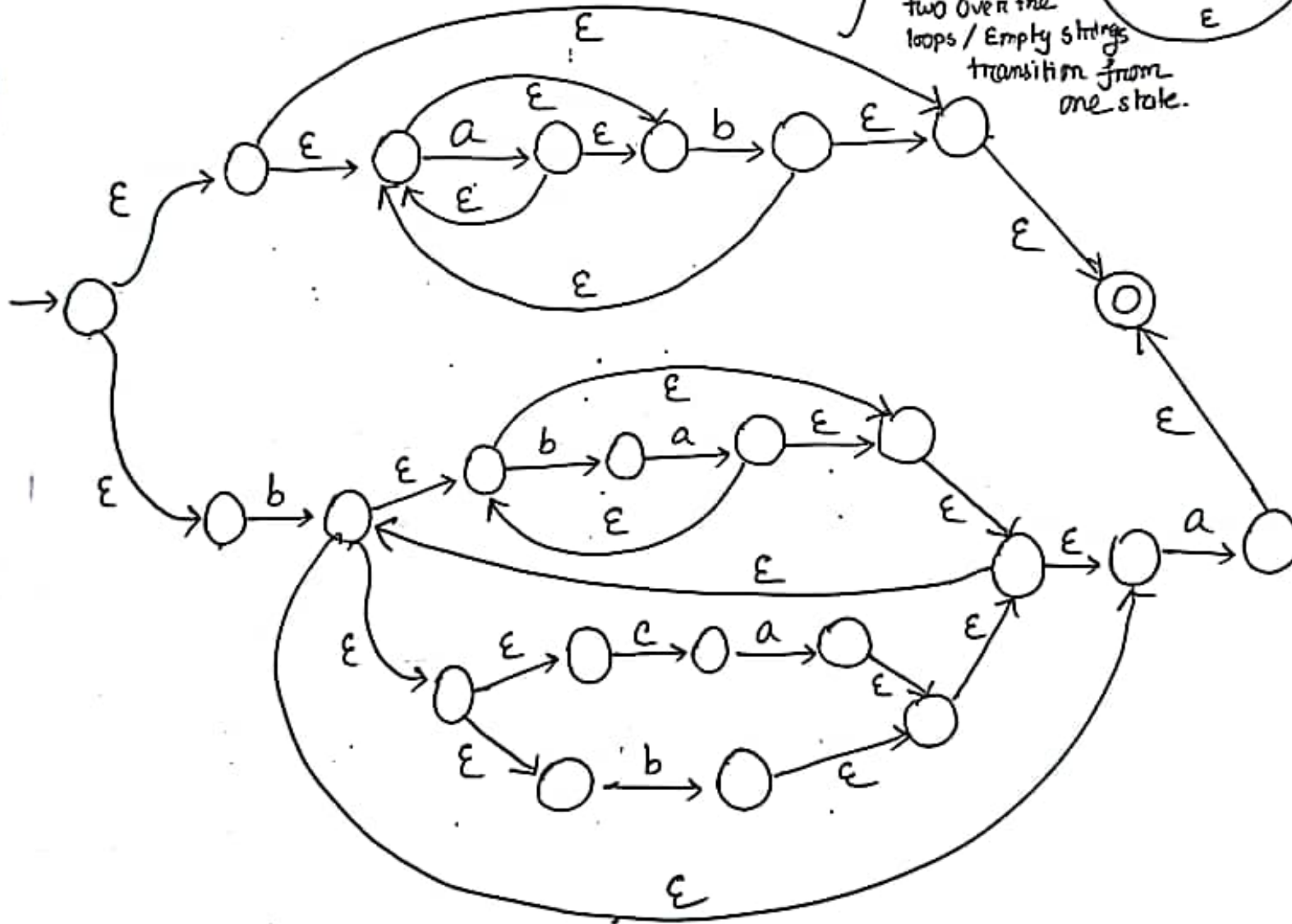
$$(a^*b)^* + b((ba)^* + (ca+ab)^*)a$$

will be wrong if two over the loops / Empty strings transition from one state.

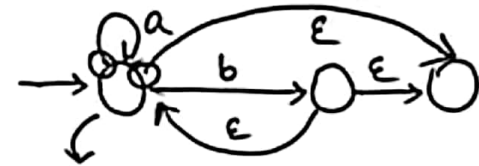


Problem 2: Regular Expression to NFA
Convert the following regular expression into an equivalent NFA.

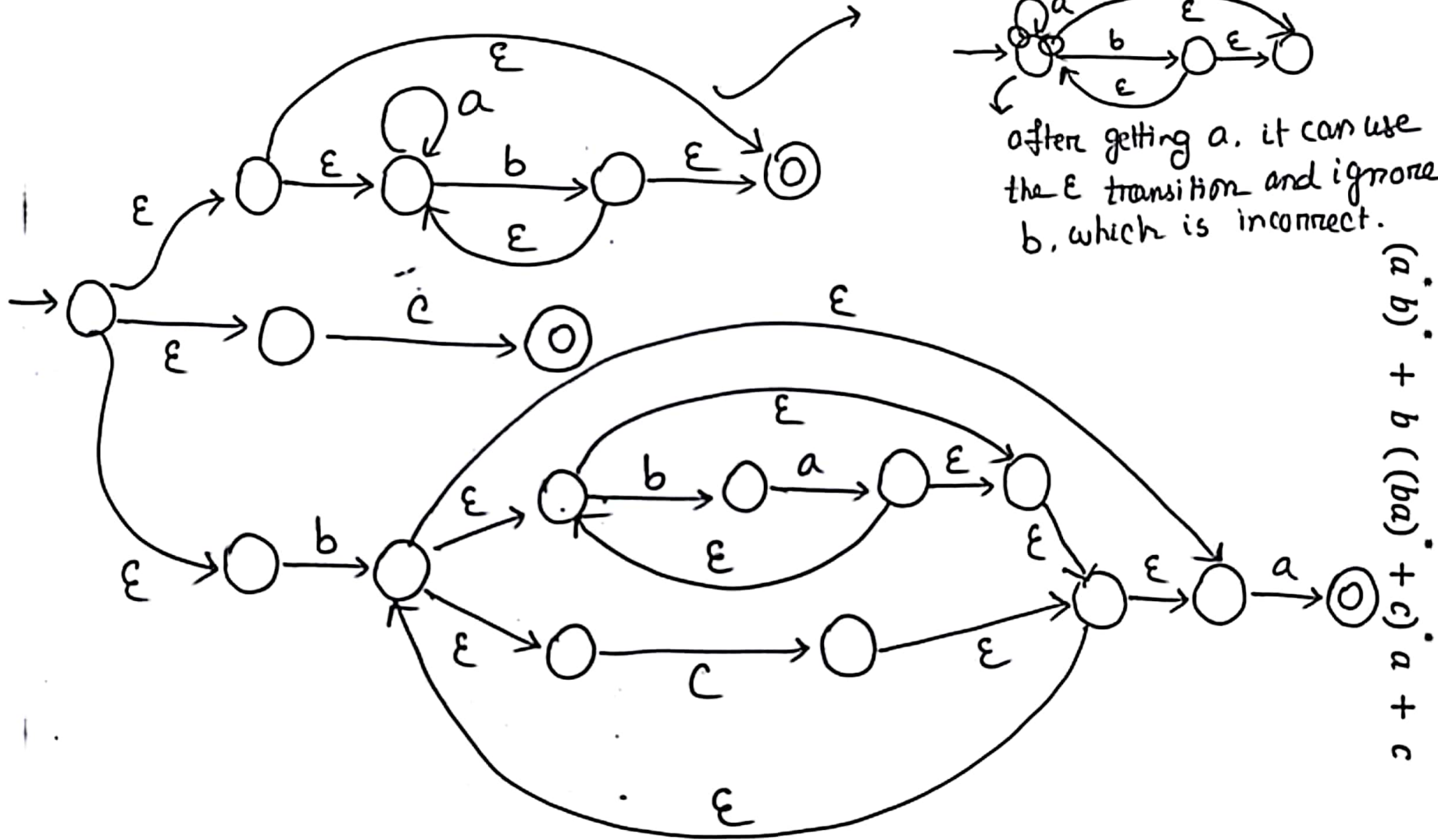
$$(a^*b)^* + b((ba)^* + (ca+ab)^*)a$$



$(a^*b)^* + b((ba)^* + c)^*a + c$ will be wrong if



after getting a, it can use the ϵ transition and ignore b, which is incorrect.



$(a^*b)^* + b((ba)^* + c)^*a + c$

Problem 2: Regular Expression to NFA

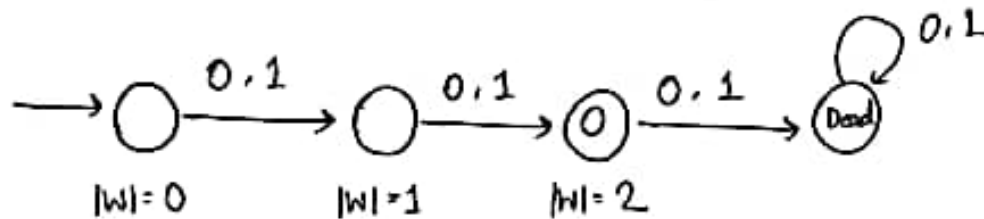
Convert the following regular expression into an equivalent NFA.

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ .

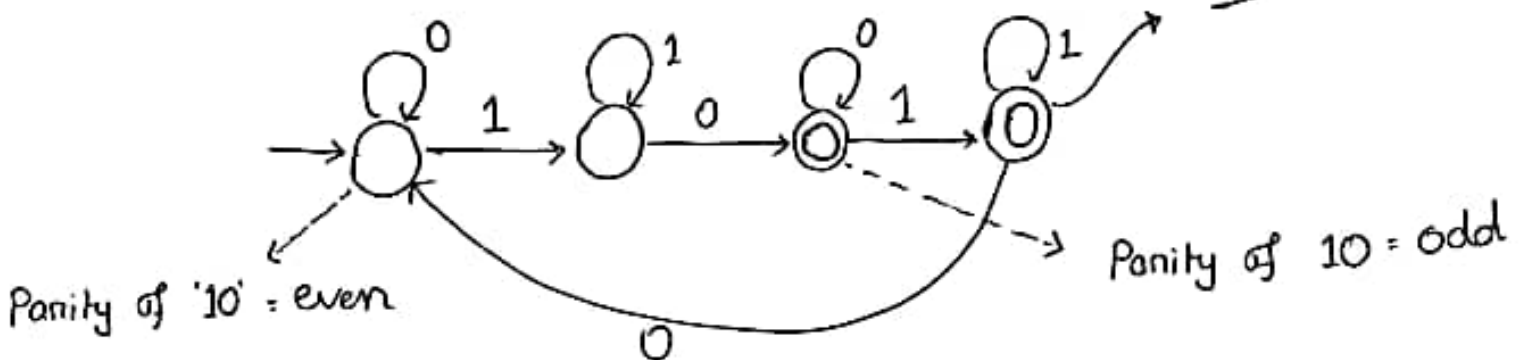
$L_1 = \{w : \text{the length of } w \text{ is two}\}$

$L_2 = \{w : \text{the number of times } 10 \text{ appears in } w \text{ is odd}\}$

(a) Give the state diagram for a DFA that recognizes L_1 . (2 points)



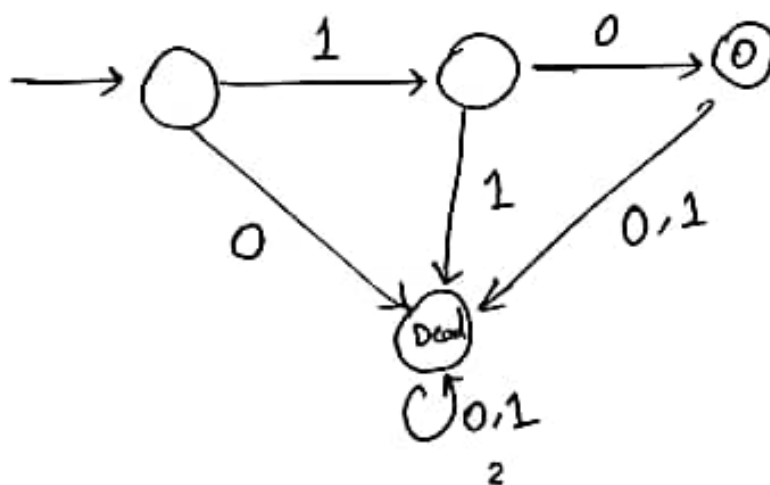
(b) Give the state diagram for a DFA that recognizes L_2 . (4 points)



(c) If were to construct a DFA for the language $L_1 \cap L_2$ using the construction shown in class, how many states would it have? (1 point) $4 \times 4 = 16$

(d) How many strings are in $L_1 \cap L_2$? (1 point) 1 (string 10)

(e) Give a 4-state DFA for the language $L_1 \cap L_2$. (2 points)



$L_1 = \{w : \text{the length of } w \text{ is at most three}\}$

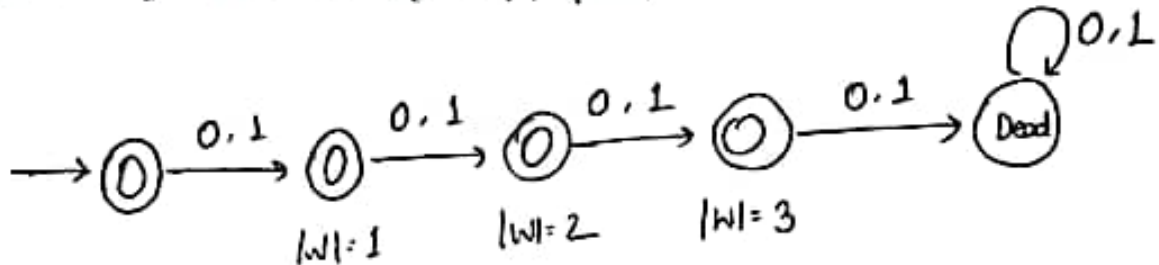
$L_2 = \{w : 00 \text{ appears at least twice as a substring in } w\}$

Now solve the following problems.

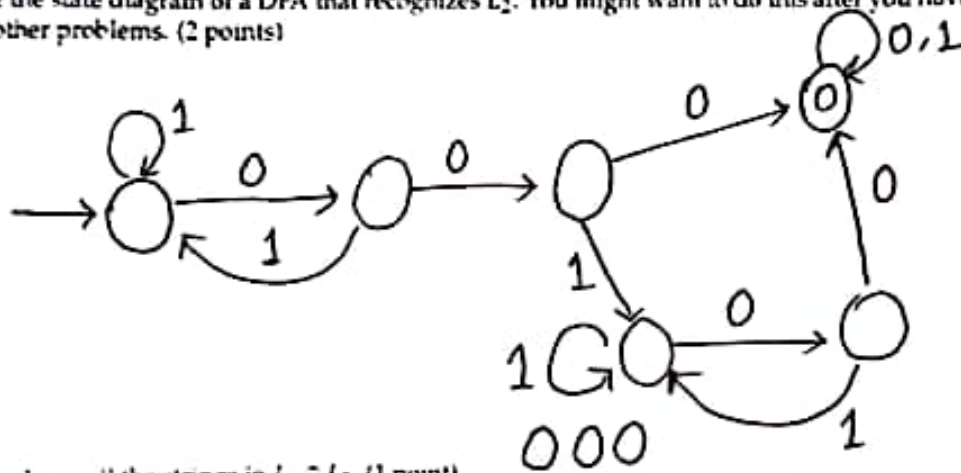
(a) Write down all the length-four strings in L_2 . (1.5 points)

0001, 1000, 0000

(b) Give the state diagram of a DFA that recognizes L_1 . (3.5 points)



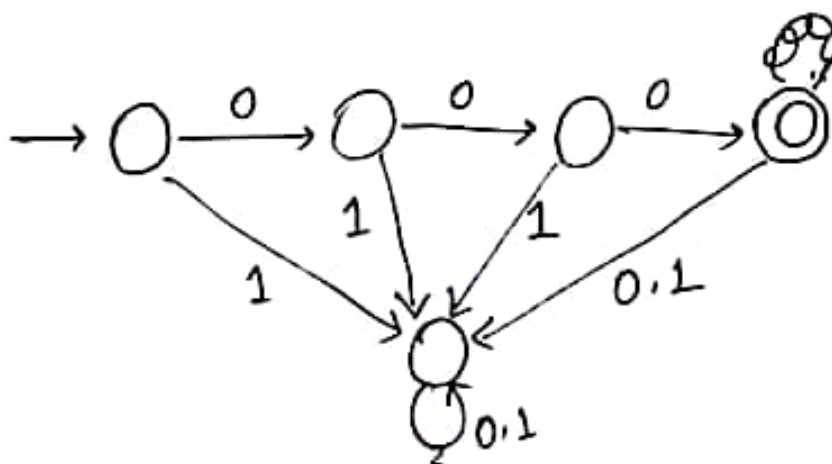
(c) Give the state diagram of a DFA that recognizes L_2 . You might want to do this after you have completed all the other problems. (2 points)



(d) Write down all the strings in $L_1 \cap L_2$. (1 point)

0000

(e) Give a five-state DFA that recognizes $L_1 \cap L_2$. Your answer to (d) should help you here. (2 points)



Problem 1 (CO1): DFA and Regular Languages (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ .

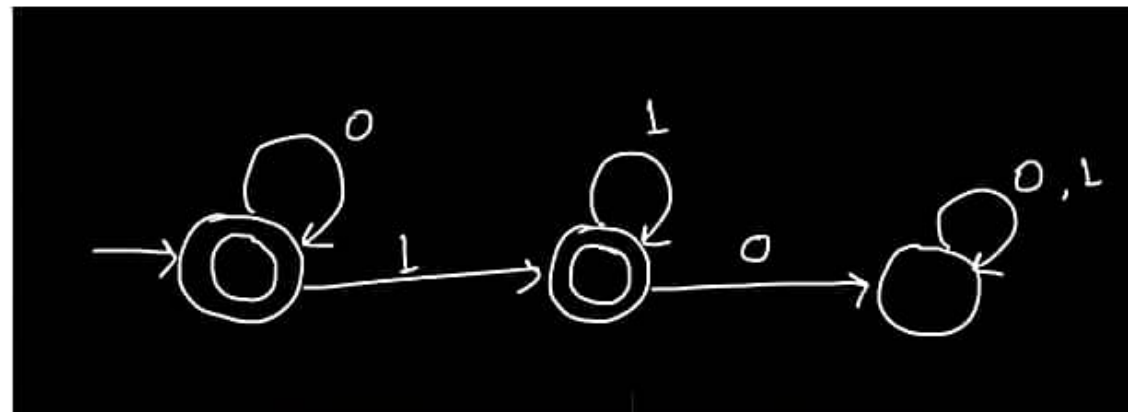
$$L_1 = \{w : w = 0^m 1^n, \text{ where } m, n \geq 0\}$$

$$L_2 = \{w : 1 \text{ does not appear at any even position in } w\}$$

Now solve the following problems.

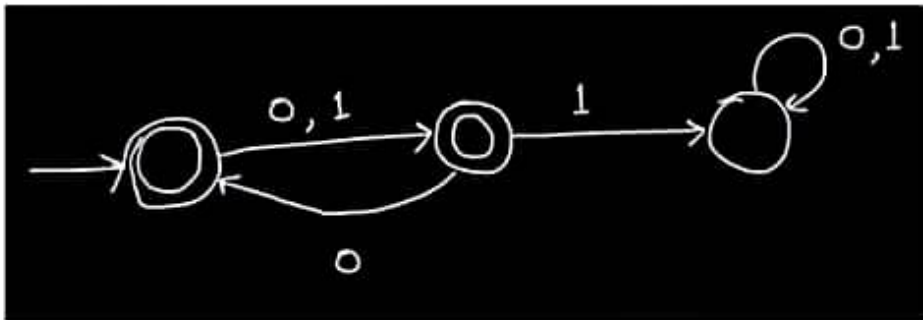
- (a) Give the state diagram for a DFA that recognizes L_1 . (3 points)
- (b) Give the state diagram for a DFA that recognizes L_2 . (3 points)
- (c) If you were to use the "cross product" construction shown in class to obtain a DFA for the language $L_1 \cap L_2$, how many states would it have? (1 point)
- (d) Find all five-letter strings in $L_1 \cap L_2$. (1 point)
- (e) Give the state diagram for a DFA that recognizes $L_1 \cap L_2$ using only four states. (2 points)

(a)



(b)

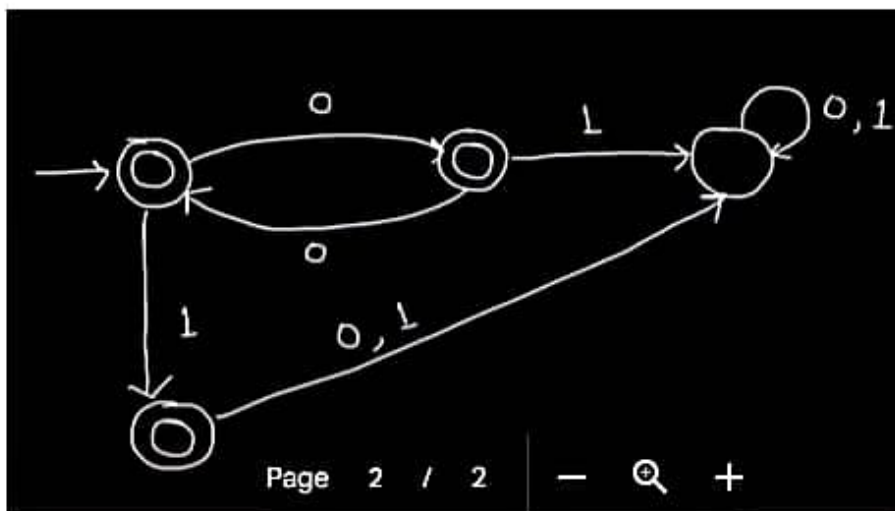
(b)



(c) The answer is $3 \times 3 = 9$.

(d) The strings are 00000 and 00001.

(e)



Problem 1 (CO1): DFA and Regular Languages (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ .

$$L_1 = \{w : w = 1^m 0^n, \text{ where } m, n \geq 0\}$$

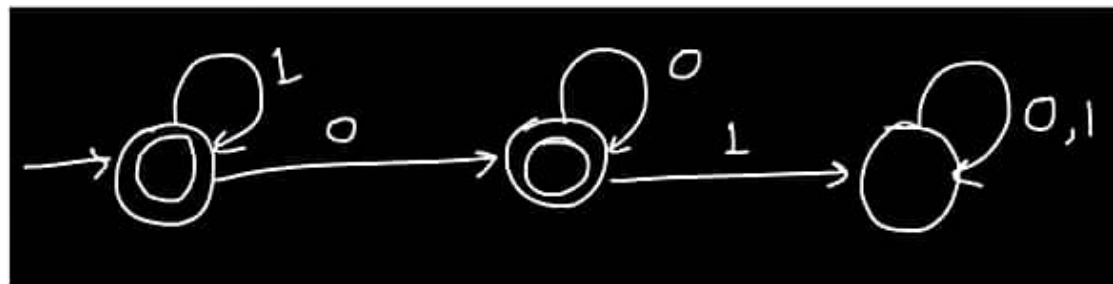
$$L_2 = \{w : 1 \text{ does not appear at any even position in } w\}$$

$$L_3 = L_1 \cap L_2$$

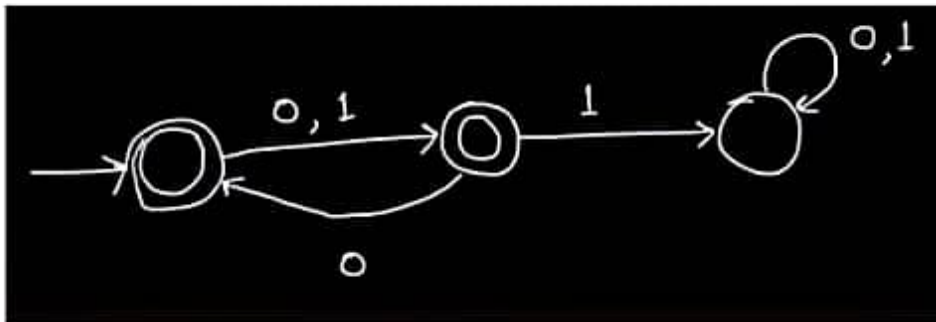
Now solve the following problems.

- (a) Give the state diagram for a DFA that recognizes L_1 . (3 points)
- (b) Give the state diagram for a DFA that recognizes L_2 . (3 points)
- (c) If you were to use the "cross product" construction shown in class to obtain a DFA for the language L_3 , how many states would it have? (1 point)
- (d) Find all four-letter strings in L_3 . (1 point)
- (e) Give the state diagram for a DFA that recognizes L_3 using only three states. (2 points)

(a)



(b)



(c) The answer is $3 \times 3 = 9$

(d) The strings are 0000, 1000.

(e)

