

Part A: Deterministic Finite Automata (DFA) [Each question contains 3 marks]

1. a) Draw a DFA for the set of strings that have three consecutive 0s. $\Sigma = \{0,1\}$

Or, b) Draw a DFA for the set of strings that don't contain 000. $\Sigma = \{0,1\}$

2. a) Construct a DFA that accept the language, $L = \{ w \in \{a,b\}^* : w \text{ starts and ends with different symbols.} \}$

Or, b) Construct a DFA that accepts the language, $L = \{ w \in \{a,b\}^* : w \text{ starts and ends with the same symbol.} \}$

3. a) Draw a DFA of strings that ends with "0101". $\Sigma = \{0,1\}$

Or, b) Design a DFA that accepts the language $L = \{ w \mid w \text{ ends with the substring "yxy"} \}$ over the alphabet $\{x,y\}$

4. a) Construct a DFA defined as $L = \{ w \in \{0,1\}^* : \text{the length of } w \text{ is two more than multiple of four} \}$

Or, b) Construct a DFA defined as $L = \{ w \in \{0,1\}^* : \text{numbers of 1s in } w \text{ is two more than multiple of four} \}$

5. Construct a DFA defined as $L = \{ w \in \{0,1\}^* : w, \text{ when interpreted as a binary number, is divisible by 5.} \}$

6. a) $L = \{ w \in \{0, 1, \#\}^* : w \text{ does not contain } \# \text{ and the number of 0s in } w \text{ is not a multiple of 3} \}$

Or, b) let's $\Sigma = \{0,1\}$

$L1 = \{w \text{ doesn't contain } \#\}$

$L2 = \{ \text{the number of 0s in } w \text{ is not a multiple of 3} \}$

$L = L1 \cap L2$

Prove L is a regular language by giving a state diagram for DFA.

7. Construct a DFA of the language L over the alphabet $\Sigma = \{a,b,c\}$ defined as follows-
 $L = \{ w \mid w \text{ does not contain "ba" and ends with "cb"} \}$

8. Draw a DFA of strings that contains at least three 0s or exactly two 1s. $\Sigma = \{0,1\}$

9. a) Draw a DFA of strings where the 2nd last symbol is a. $\Sigma = \{a,b\}$

Or, b) Draw a DFA of strings where the 3rd last symbol is 1. $\Sigma = \{0,1\}$ [You may draw the NFA for this problem if you find it difficult to solve using DFA]

10. $L = \{ w \in \{a, b\}^* : \text{the last letter of } w \text{ appears at least twice in } w. \}$

Part B: More Deterministic Finite Automata (DFA) [Each question contains 3 marks]

11. a) Draw a DFA of strings that have 1 as every 3rd symbol. $\Sigma = \{0,1\}$

Or, b) The set of binary numbers has 0 in all even positions. $\Sigma = \{0,1\}$.

12. a) Draw a DFA that accepts exactly one "ab". $\Sigma = \{a,b\}$

Or, b) Draw a DFA that accepts exactly two "ab". $\Sigma = \{a,b\}$

13. Draw a DFA that accepts at least two "00" as a substring. $\Sigma = \{0,1\}$

14. a) Draw a DFA that accepts exactly two "00" as a substring. $\Sigma = \{0,1\}$

Or, b) Draw a DFA that accepts at most two "00" as a substring. $\Sigma = \{0,1\}$

15. Construct a DFA defined as $L = \{ \text{An even number of 0s follow the last 1 in } w \}$ $\Sigma = \{0,1\}$

16. Construct a DFA defined as $L = \{ w \mid \text{each "b" is followed by at least one "a"} \}$ $\Sigma = \{a,b\}$

For example: baaa

17. Construct a DFA where the set of binary strings where numbers of 0s between two successive 1s will be even. $\Sigma = \{0,1\}$.

18. Construct a DFA of the Language, $L = \{ w \in \{0,1\}^* : \text{no 00 appears as a substring before the first 11 in } w. \}$

19. Construct a DFA of the Language, $L = \{ w \in \{0,1\}^* : \text{no 00 appears as a subsequence before the first 11 in } w. \}$

20. a) Construct a DFA of the Language, $L = \{ w \in \{0,1\}^* : w \text{ contains } 01^m0 \text{ as a substring where } m \text{ is divisible by 3} \}$

Or, b) Construct a DFA of the Language, $L = \{ w \in \{0,1\}^* : w \text{ contains } 01^m0 \text{ as a substring where } m \text{ leaves a remainder of 2 when divided by 3} \}$

Hints:

We denote by 1^m the string $\underbrace{111 \dots 111}_{m \text{ times}}$.

21. a) Construct a DFA of the Language, $L = \{ w \in \{0,1\}^* : w = 0^m 1^n \text{ where } m \text{ and } n \text{ are both odd.} \}$

Or, b) Construct a DFA of the Language, $L = \{ w \in \{0,1\}^* : w = 0^m 1^n \text{ where } m \text{ and } n \text{ are both even.} \}$

Or, c) The problem can also be designed as:

$$L1 = \{ w : w = 0^m, \text{ where } m \text{ is even} \}$$

$$L2 = \{ w : w = 1^n, \text{ where } n \text{ is even} \}$$

$$L = L1 \cdot L2$$

Prove L is a regular language by giving a state diagram for DFA.

Part C: Mursalin Sir's [MHB] Quiz Question from Previous semesters [Each question contains 10 marks.]

Question 1.

Let $\Sigma = \{0, 1\}$

$$L1 = \{ w : w = 1^m \text{ where } m \text{ is odd} \}$$

$$L2 = \{ w : w \text{ does not contain any } y \in L1 \text{ as a substring} \}$$

- (a) Write down a length 6 string that is in L2. (1 point) .
- (b) Give the state diagram for a DFA that recognizes L1. (5 points)
- (c) Give the state diagram for a DFA that recognizes L2. (3 points)
- (d) Give the state diagram for a DFA that recognizes $L1 \cap L2$. You can use the construction shown in class but there is a much simpler DFA. (2 points)

Question 4.

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ .

$$L1 = \{0, 10\}$$

$$L2 = L_1^*$$

$$L3 = \{w : \text{the length of } w \text{ is four}\}$$

- (a) Write down all the strings in $L2 \cap L3$. (2.5 points)
- (b) Give the state diagram for a DFA that recognizes $L1$. (4.5 points)
- (c) Give the state diagram for a DFA that recognizes $L2$. (3 points)

For Practice: [Don't have to submit]

Part D: Non-Deterministic finite automata (NFA)

1. Construct an NFA that recognizes the language $L = \{ w \in \{0,1\}^* : w \text{ contains both "000" and "111" as a substring} \}$
2. Construct a NFA which recognize the language $L = \{ w \in \{0,1\}^* : w \text{ contains at least two 0s or exactly two 1s} \}$
3. Construct an NFA for the languages $L = \{w \in \Sigma : w \text{ does not start with a Punctuation or contains only Alphabets}\}$ where $\Sigma = D \cup A \cup P$

Digit, $D = \{0,1,2,3,4,5,6,7,8,9\}$

Alphabet, $A = \{a, b, c, \dots, x, y, z\}$

Punctuation, $P = \{*, \#\}$

You can use the sets above to label the transitions of your NFA.