

CSE221
Lab Assignment 03
Summer 2023

Submission Guidelines:

1. You can code all of them either in Python, CPP, or Java. But you should choose a specific language for all tasks.
2. For each task write separate python files like task1.py, task2.py, and so on.
3. For each problem, take input from files called "**inputX.txt**" and output at "**outputX.txt**", where X is the task number.
4. Add a hand written explanation of 3-4 lines for each of your solutions in a separate document. You may compile all of your explanations in a single file.
5. Finally zip all the files and rename this zip file as per this format: **LabSectionNo_ID_CSE221LabAssignmentNo_Summer2023.zip**
[Example: **LabSection01_21101XXX_CSE221LabAssignment03_Summer2023.zip**]
6. Don't copy from your friends.
7. You MUST follow all the guidelines, naming/file/zippping convention stated above.

Failure to follow instructions will result in a straight 50% mark deduction.

Task 01 [15 Points]:

Somewhere in the universe, the Biannual Regional Alien Competition is taking place.

There are **N** aliens standing in a line. You will be given a permutation of N, which denotes the height of each alien. A sequence of N numbers is called a permutation if it contains all integers from 1 to N exactly once. For example, the sequences [3,1,4,2], [1] and [2,1] are permutations, but [1,2,1], [0,1] and [1,3,4] – are not.

In the competition, for each alien, the judge wants to count how many aliens are standing on its right side with a strictly

smaller height. Then the judge wants to add up all the counts. To do this, the judge writes the following piece of code.

```
count = 0
for i in range(n):
    for j in range(i+1,n):
        if H[i] > H[j]:
            count+=1
```

However, their algorithm wasn't efficient at all. Hence, the alien calls you to write a better solution for the program.

More formally, you have to count how many pairs of aliens are standing in the line such that $H[i] > H[j]$ and $i < j$. Here, A is a permutation of the aliens' heights. And i, j denote the Aliens' positions.

Input

The first line contains a single integer $1 \leq N \leq 10^6$ - the number of total aliens.

The next line contains N integers $H_1, H_2, \dots, H_n (1 \leq H_i \leq N)$ - the height of the i -th alien. It is guaranteed that the given heights will be a permutation of N .

Output

Print a single integer, which denotes the total number of pairs (i, j) such that $i < j$ and $H_i > H_j$.

Sample Input/Output:

| | |
|----------------|-----------------|
| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|

| | |
|----------------------|-----------------|
| 5 1 2 3 4 5 | 0 |
| Sample Input 2 | Sample Output 2 |
| 5 5 4 3 2 1 | 10 |
| Sample Input 3 | Sample Output 3 |
| 8 2 7 4 1 5 6 8 3 | 11 |

Sample Input 3 Explanation:

In the sample input 3, the following pairs on alien's heights satisfy the condition: (2,1), (7,4), (7,1), (7,5), (7,6), (7,3), (4,1), (4,3), (5,3), (6,3), (8,3)

Task 02 [15 points]

You are given a list A of N integers. You have to choose two indices i and j such that $1 \leq i < j \leq N$ and $A[i] + A[j]^2$ is maximum possible. Here, we are considering 1-based indexing.

Write a code which will find the maximum value of $A[i] + A[j]^2$ in $O(N)$ or $O(N \log N)$.

Input

The first line contains a single integer $1 \leq N \leq 10^6$ - the length of the list.

The next line contains N integers A_1, A_2, \dots, A_n ($-10^8 \leq A_i \leq 10^8$) separated by a space.

Output

Print a single integer - which denotes the maximum possible value of $A[i] + A[j]^2$.

Sample Input/Output:

| | |
|--------------------------|-----------------|
| Sample Input 1 | Sample Output 1 |
| 5 9 6 5 8 2 | 73 |
| Sample Input 2 | Sample Output 2 |
| 8 5 10 4 -3 1 6 -10 2 | 110 |
| Sample Input 3 | Sample Output 3 |
| 7 -5 -2 -6 -7 -1 8 2 | 63 |

Task 03 [10 Points]

In this problem, you will be given a list of numbers. You have to sort the list using the Quick Sort algorithm in ascending order.

Pseudocode of Quick Sort Algorithm:

QUICKSORT(A, p, r)

```
1  if  $p < r$ 
2       $q = \text{PARTITION}(A, p, r)$ 
3      QUICKSORT( $A, p, q - 1$ )
4      QUICKSORT( $A, q + 1, r$ )
```

PARTITION(A, p, r)

```
1   $x = A[r]$ 
2   $i = p - 1$ 
3  for  $j = p$  to  $r - 1$ 
4      if  $A[j] \leq x$ 
5           $i = i + 1$ 
6          exchange  $A[i]$  with  $A[j]$ 
7  exchange  $A[i + 1]$  with  $A[r]$ 
8  return  $i + 1$ 
```

[The code snippet has been taken from the book: Introduction to Algorithms]

Input

The first line contains an integer N ($1 \leq N \leq 10^5$), denoting the length of Alice's list. In the next line, there will be N integers separated by space.

Output:

You have to sort the number using the Quick Sort algorithm in ascending order and show the sorted list.

Sample Input/Output:

| Sample Input 1 | Sample Output 1 |
|----------------------|-----------------|
| 8 9 5 4 6 1 3 2 9 | 1 2 3 4 5 6 9 9 |
| Sample Input 2 | Sample Output 2 |
| 1 10 | 10 |
| Sample Input 3 | Sample Output 3 |
| 6 8 1 4 2 1 3 | 1 1 2 3 4 8 |

| Sample Input 4 | Sample Output 4 |
|--------------------|-----------------|
| 7 7 6 5 4 3 2 1 | 1 2 3 4 5 6 7 |

Task 04 [10 Points]

In this problem, you will be given a list of numbers. You have to find the **k**-th smallest value from the list without sorting using the Partition function of Quick sort.

We will consider the 1-based indexing of the list.

Input

The first line contains an integer N ($1 \leq N \leq 10^6$), denoting the length of the list.

The next line contains N integers A_1, A_2, \dots, A_n ($1 \leq A_i \leq 10^6$) separated by a space.

The third line contains a single integer Q ($1 \leq Q \leq 100$) - which denotes the number of queries you have to answer.

Each of the next Q lines will contain a single integer K ($1 \leq K \leq N$).

Output:

For each query, you have to find the K -th smallest number from the given list.

Sample Input/Output:

| Sample Input 1 | Sample Output 1 |
|--|-----------------|
| 9 // Total Elements 10 11 10 6 7 9 8 15 2 4 // Total queries | 9 7 6 |

| | |
|------------------|----|
| 5 3 2 7 | 10 |
|------------------|----|