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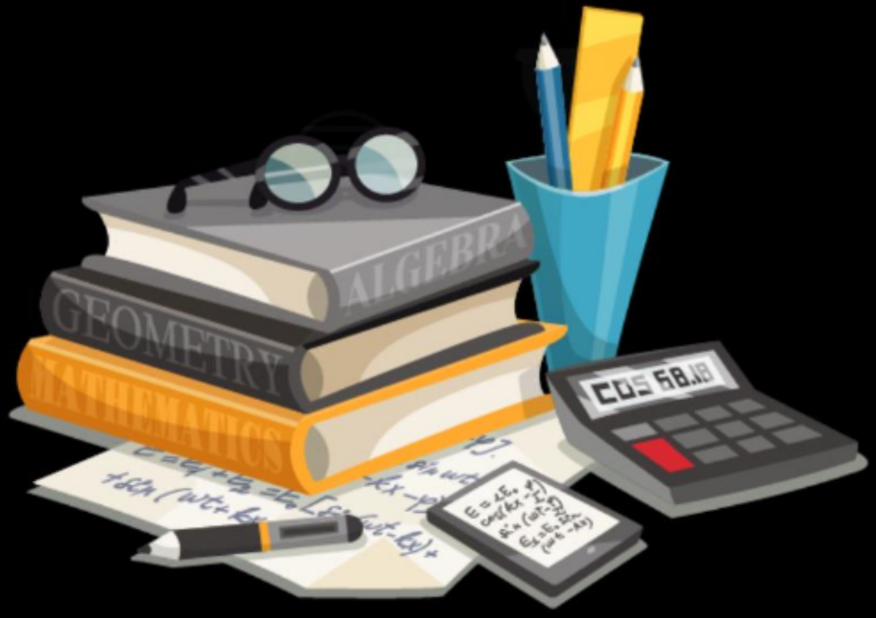
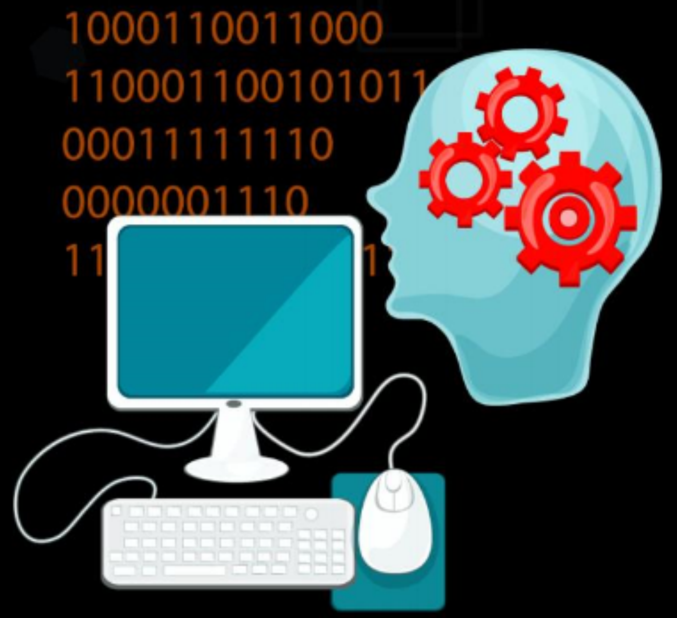
Higher Math 1st Paper

Chapter 10 : Integration

lec-2





$\circledast$ $\int \cos^4 x \, dx$

$\boxed{2\cos^2 x = 1+\cos 2x}$

$= \frac{1}{4}\int (2\cos^2 x)^2 \, dx$

$= \frac{1}{4}\int (1+\cos 2x)^2 \, dx$

$= \frac{1}{4}\int \left(1 + 2\cos 2x + \frac{1}{2}\cdot 2\cos^2 2x\right) dx$

$= \frac{1}{4}\int \left\{1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right\} dx$

$= \frac{1}{4}\int \left(1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$

$= \frac{1}{4}\left[x + 2\frac{\sin 2x}{2} + \frac{1}{2}x + \frac{1}{2}\cdot\frac{\sin 4x}{4}\right] + c$

$= \frac{1}{4}\left[\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right] + c$

⊛ $\int \sqrt{1-\sin 2x}\, dx$

$= \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x}\, dx$

$= \int \sqrt{(\sin x - \cos x)^2}\, dx$ / $= \int \sqrt{(\cos x - \sin x)^2}\, dx$

$= \int (\sin x - \cos x)\, dx$ / $= \int (\cos x - \sin x)\, dx$

$= \boxed{-\cos x - \sin x + c}$ / $= \boxed{\sin x + \cos x + c}$

$$\int \sqrt{1 \pm \sin x}\, dx$$

$$= \int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} \pm 2\sin\frac{x}{2}\cos\frac{x}{2}}\, dx$$

$$= \int \sqrt{\left(\sin\frac{x}{2} \pm \cos\frac{x}{2}\right)^2}\, dx \quad / \quad = \int \sqrt{\left(\cos\frac{x}{2} \pm \sin\frac{x}{2}\right)^2}\, dx$$

$$= \int \left(\sin\frac{x}{2} \pm \cos\frac{x}{2}\right) dx \quad / \quad = \int \left(\cos\frac{x}{2} \pm \sin\frac{x}{2}\right) dx$$

$$= \boxed{-2\cos\frac{x}{2} \pm 2\sin\frac{x}{2} + c} \quad / \quad = \boxed{2\sin\frac{x}{2} \pm 2\cos\frac{x}{2} + c}$$

(12) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

(12) $\int \frac{1}{2\sqrt{x}}\,dx = \sqrt{x}$

$\Rightarrow \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx = \sqrt{x}$

$\Rightarrow \boxed{\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c}$

(13) $\int \tan x\,dx = \ln|\sec x| + c$ ✓✓

(14) $\int \cot x\,dx = \ln|\sin x| + c$ ✓✓

□ $\int xe^{x^2}dx$ নির্ণয় কর।

$$\int xe^{x^2}dx$$

$$= \int e^z \frac{dz}{2}$$

$$= \frac{1}{2}\int e^z dz$$

$$= \frac{1}{2}e^z + c = \boxed{\frac{1}{2}e^{x^2} + c}$$

ধরি,

$$x^2 = z$$

$$\Rightarrow \frac{d}{dx}(x^2) = \frac{dz}{dx}$$

$$\Rightarrow 2x = \frac{dz}{dx}$$

$$\Rightarrow 2x\,dx = dz$$

$$\Rightarrow \boxed{x\,dx} = \frac{dz}{2}$$

❑ $\int xe^{x^2}dx$ নির্ণয় কর।

$$= \frac{1}{2}\int e^{x^2}\,\boxed{2x\,dx}$$

$$\boxed{\begin{aligned} &\frac{d}{dx}(x^2) = 2x \\ &\Rightarrow d(x^2) = 2x\,dx \end{aligned}}$$

$$= \frac{1}{2}\int e^{x^2}\,d(x^2)$$

$$= \boxed{\frac{1}{2}e^{x^2} + c}$$

$$\int e^x\,dx = e^x + c$$

$$\int e^{\text{চলক}}\,d(\text{চলক})$$

$$= e^{\text{চলক}} + c$$

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$\int f(x)\,dx$

❑ $\int x \sin x^2\,dx$ নির্ণয় কর।

$\circledast \int x \sin x^2\,dx$

$= \int \sin z \,\frac{dz}{2}$

$= \frac{1}{2}\int \sin z\,dz$

$= \boxed{-\frac{1}{2}\cos x^2 + c}$

let,

$x^2 = z$

$\Rightarrow \frac{d}{dx}(x^2) = \frac{dz}{dx}$

$\Rightarrow 2x = \frac{dz}{dx}$

$\Rightarrow \boxed{x\,dx} = \frac{dz}{2}$

- [ ] $\int x \sin x^2 \, dx$ নির্ণয় কর।

$$= \frac{1}{2}\int \sin x^2 \, \boxed{2x\,dx}$$

$$= \frac{1}{2}\int \sin x^2 \, d(x^2)$$

$$= \boxed{-\frac{1}{2}\cos x^2 + c}$$

(12) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

(12) $\int \frac{1}{2\sqrt{x}}\,dx = \sqrt{x}$

$\Rightarrow \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx = \sqrt{x}$

$\Rightarrow \boxed{\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c}$

(13) $\int \tan x\,dx = \ln|\sec x| + c$ ✓✓

(14) $\int \cot x\,dx = \ln|\sin x| + c$ ✓✓

$$\circledast \int \tan x \, dx$$

$$\int \frac{1}{x} dx = \ln x + c$$

let,

$$\cos x = z$$

$$\frac{d}{dx}(\cos x) = \frac{dz}{dx}$$

$$\Rightarrow -\sin x \, dx = dz$$

$$\Rightarrow \boxed{\sin x \, dx = -dz}$$

$$= \int \frac{\sin x}{\cos x} dx$$

$$= \int \frac{-dz}{z}$$

$$= -\int \frac{dz}{z} = -\ln|z| + c$$

$$= -\ln|\cos x| + c$$

$$= \ln\left|(\cos x)^{-1}\right| + c = \boxed{\ln|\sec x| + c}$$

$$(\cos x)^{-1} = \frac{1}{\cos x}$$

$$= \boxed{\sec x}$$

* $$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \frac{\cos x}{\sin x} dx$$

let, $\sin x = z$

$\Rightarrow \cos x = \frac{dz}{dx}$

$\Rightarrow \boxed{\cos x \, dx = dz}$

$$= \int \frac{dz}{z}$$

$$= \ln|z| + c$$

$$= \boxed{\ln|\sin x| + c}$$

☐ $\boxed{\int e^x \tan e^x \, dx}$ নির্ণয় কর।

$$= \int e^x \tan e^x \, dx$$

let,

$$e^x = z$$

$$\Rightarrow \boxed{e^x \, dx = dz}$$

$$= \int \tan z \, dz$$

$$= \ln|\sec z| + c$$

$$= \ln|\sec e^x| + c \checkmark$$

$\int e^x \tan e^x \, dx$ নির্ণয় কর।

$$= \int \tan e^x \; \boxed{e^x \, dx}$$

$$= \int \tan e^x \; d(e^x)$$

$$= \boxed{\ln|\sec e^x| + c}$$

$$\circledast \quad \int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$$

$$= \int \sin z \,(-dz)$$

$$= -\int \sin z \, dz$$

$$= \cos z + c$$

$$= \boxed{\cos \frac{1}{x} + c}$$

let,

$$\frac{1}{x} = z$$

$$\Rightarrow \frac{-1}{x^2} = \frac{dz}{dx}$$

$$\Rightarrow \boxed{\frac{1}{x^2} dx} = -dz$$

(*) $\int \sec^2 x \, e^{\tan x} \, dx$

let,

$\tan x = z$

$\Rightarrow \sec^2 x = \frac{dz}{dx}$

$\Rightarrow \boxed{\sec^2 x \, dx} = dz$

$= \int e^z \, dz$

$= e^z + c$

$= \boxed{e^{\tan x} + c}$

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□ $\int\left(e^x+\frac{1}{x}\right)(e^x+\ln x)dx$ নির্ণয় কর।

let,

$$e^x + \ln x = z$$

$$\Rightarrow e^x + \frac{1}{x} = \frac{dz}{dx}$$

$$\Rightarrow \boxed{\left(e^x + \frac{1}{x}\right)dx = dz}$$

$$= \int z\, dz$$

$$= \frac{z^2}{2} + c$$

$$= \boxed{\frac{1}{2}\left(e^x + \ln x\right)^2 + c}$$

❑ $\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}dx$ নির্ণয় কর।

$$= \int \tan(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}}dx$$

$$= \int \tan z \, dz$$

$$= \ln|\sec z| + c$$

$$= \boxed{\ln|\sec(\sin^{-1}x)| + c}$$

let, $\sin^{-1}x = z$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} = \frac{dz}{dx}$$

$$\Rightarrow \boxed{\frac{1}{\sqrt{1-x^2}}dx} = dz$$

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$\int \frac{(\sec^{-1} x)^4}{x\sqrt{x^2-1}} dx$ নির্ণয় কর।

$$= \int z^4 \, dz$$

$$= \frac{z^5}{5} + c$$

$$= \boxed{\frac{1}{5}(\sec^{-1} x)^5 + c}$$

let, $\sec^{-1} x = z$

$$\Rightarrow \frac{1}{x\sqrt{x^2-1}} = \frac{dz}{dx}$$

$$\Rightarrow \boxed{\frac{1}{x\sqrt{x^2-1}} dx} = dz$$

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□ $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ নির্ণয় কর।

[DU'15-16]

$$= \int \frac{1}{\cos^2 z} dz$$

$$= \int \sec^2 z \, dz$$

$$= \tan z + c$$

$$= \boxed{\tan xe^x + c}$$

let,

$$xe^x = z$$

$$\Rightarrow xe^x + e^x \cdot 1 = \frac{dz}{dx}$$

$$\Rightarrow \boxed{e^x(x+1)dx} = dz$$

- $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ নির্ণয় কর। [DU'15-16]

$$= \int \sec^2(xe^x)\ d(xe^x)$$

$$= \boxed{\tan(xe^x) + c}$$

$$\frac{d}{dx}(xe^x) = e^x(x+1)$$

$$\Rightarrow \boxed{d(xe^x) = e^x(x+1)dx}$$

$$\frac{d}{dx} f(x) = f'(x)$$

$$\Rightarrow d\, f(x) = \boxed{f'(x)\, dx}$$

→ কার অন্তরজ $f'(x)$?

উত্তর : $f(x)$ এর

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\Rightarrow d(\sin x) = \boxed{\cos x\, dx}$$

→ কার অন্তরজ $\cos x$?

→ $\sin x$ এর অন্তরজ $d(\sin x)$

$$\circledast \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\Rightarrow d(\tan x) = \boxed{\sec^2 x \, dx}$$

$$\downarrow$$

$$d(\tan x)$$

❑ $\int \sin^3 x\, dx$ নির্ণয় কর।

$$= \int \sin^2 x \boxed{\sin x\, dx}$$

$$= -\int \sin^2 x\, d(\cos x)$$

$$= -\int (1-\cos^2 x)\, d(\cos x)$$

$$= -\left[\cos x - \frac{\cos^3 x}{3}\right] + c$$

$$\frac{d}{dx}(-\cos x) = \sin x$$

$$\Rightarrow \boxed{d(-\cos x) = \sin x\, dx}$$

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❑ $\int \cos^3 x \, dx$ নির্ণয় কর।

$$= \int \cos^2 x \, \boxed{\cos x \, dx}$$

$$= \int (1 - \sin^2 x) \, d(\sin x)$$

$$= \left[\sin x - \frac{\sin^3 x}{3} \right] + c$$