Checking for Collisions: Validating Optimality/Feasibility

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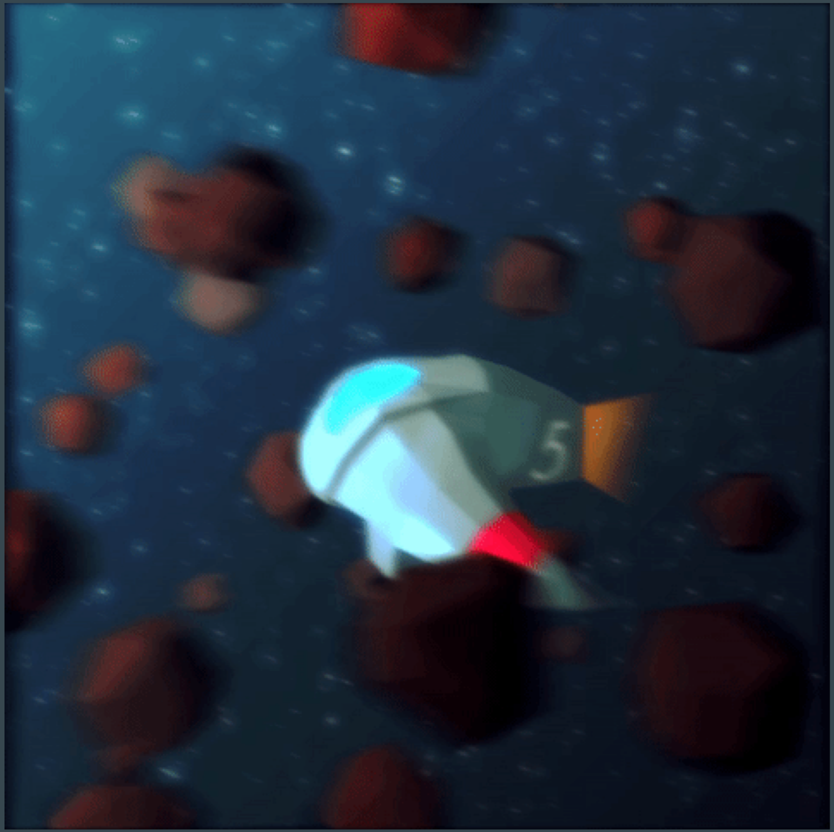
Group 14

Motivation: Safety-critical systems

- Detect collisions BEFORE they happen



Approximate vs Rigorous Algorithms

- Rigorous:
  - Pros: Finds correct exact solution
  - Cons: Requires a lot of time
- Approximate:
  - Pros: Faster computation
  - Cons: Closeness or even existence of real solution not certain
- Combining the two?
  - Rigorously verifying whether an exact solution exists close to an approximation, is faster than finding the solution

Research question

...

Can we validate a given approximation by proving the existence of a real solution 'nearby'?

Concepts and Approach

Approach:

- Combining approximate and rigorous algorithms

Tools:

- Ariadne
- Intlab

Concepts and Approach

- Interval arithmetic
- Newton method
- Interval Newton method
- Karush–Kuhn–Tucker (KKT) conditions

Milestone:

- Solving one-dimensional (single variable) equations

Interval Arithmetic

- Technique for doing a rigorous calculations
- Number is represented by an interval - lower and upper bound
- Classical operations of real arithmetic can be used on intervals

Example

Difference between the two intervals: $[x] - [y] = [\underline{x} - \overline{y}; \overline{x} - \underline{y}]$

Product of two intervals: $[x] \times [y] = [\min\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\}; \max\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\}]$

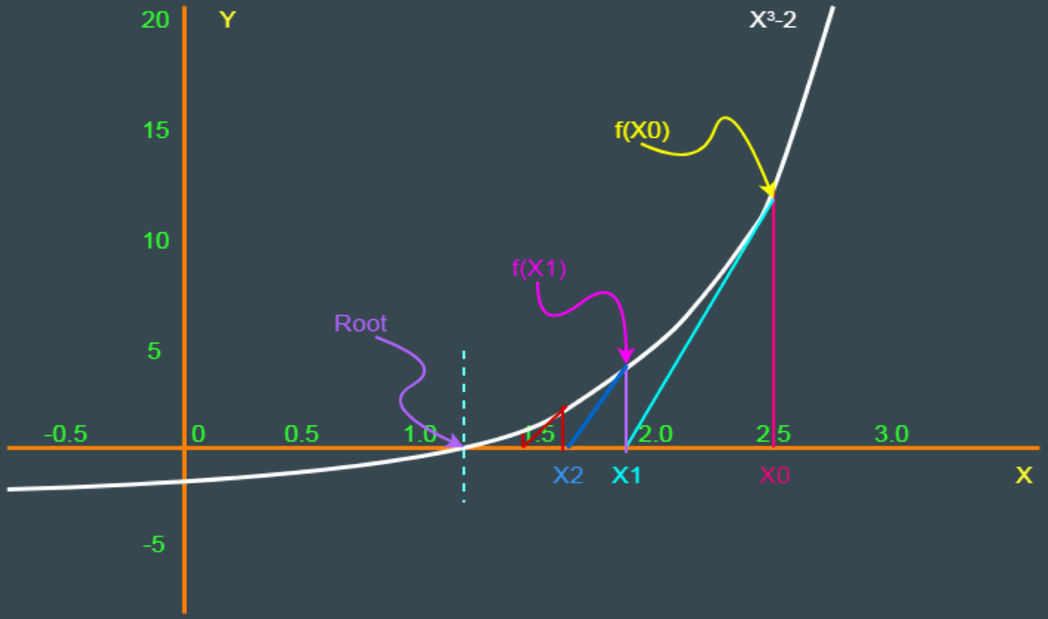
$$x \in [\frac{1}{2}, \frac{4}{3}]$$
$$(1 - x) \in [-\frac{1}{3}, \frac{1}{2}]$$

$$x(1 - x) \in [-\frac{4}{9}, \frac{2}{3}]$$

Newton Method

$$X_{n+1}=X_n-\frac{f(X_n)}{f'(X_n)}$$

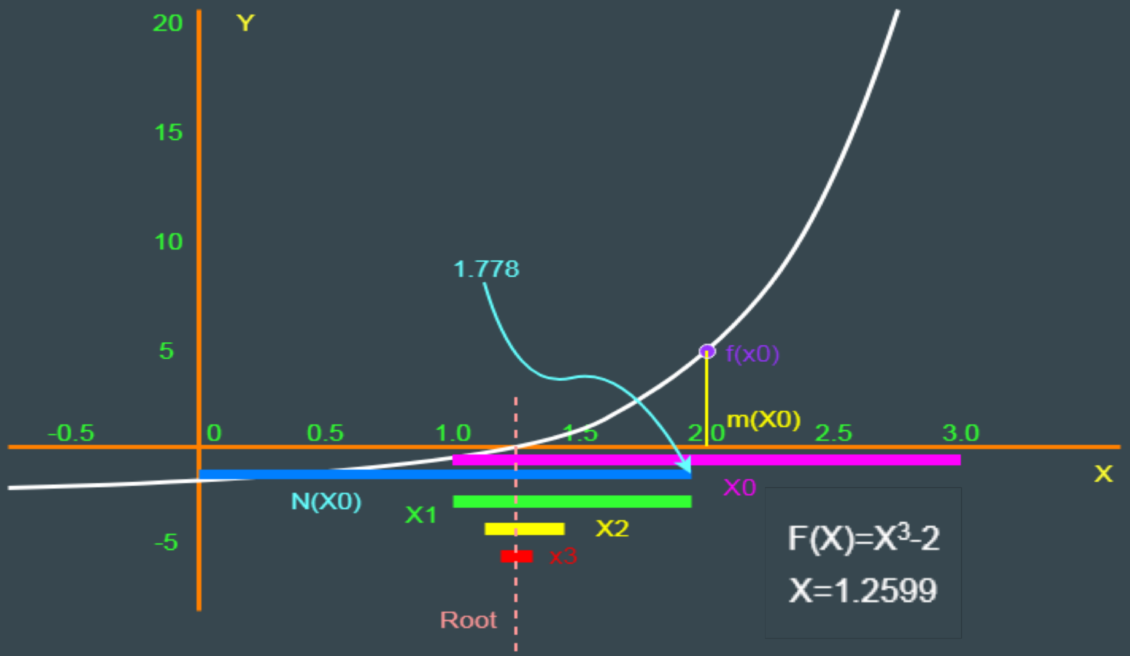
X0=2.5
Root=1.259



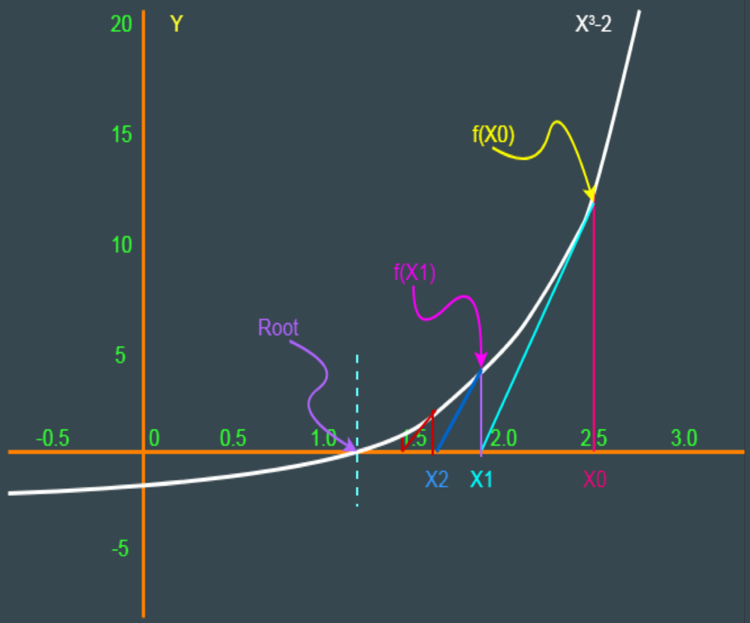
Interval Newton Method

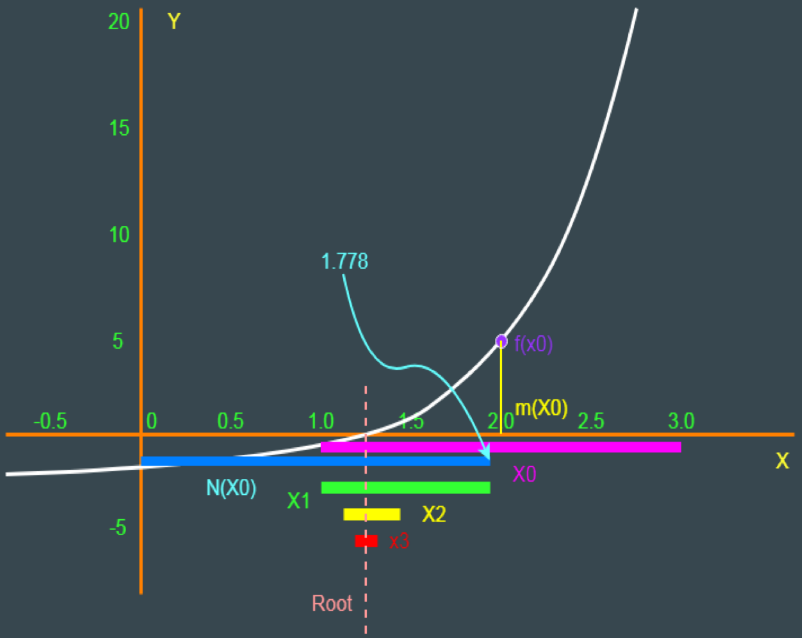
$$N(X_i) = m(X_i) - \frac{F(m(X_i))}{F'(X_i)}$$

$$X_{i+1} = N(X_i) \cap X_i$$



Newton Method and Interval Newton Method



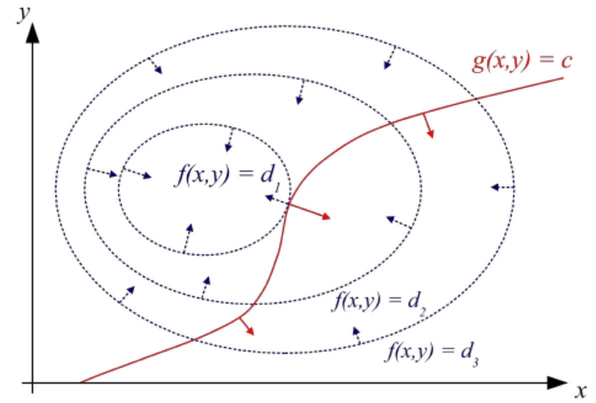


Optimality/feasibility Conditions

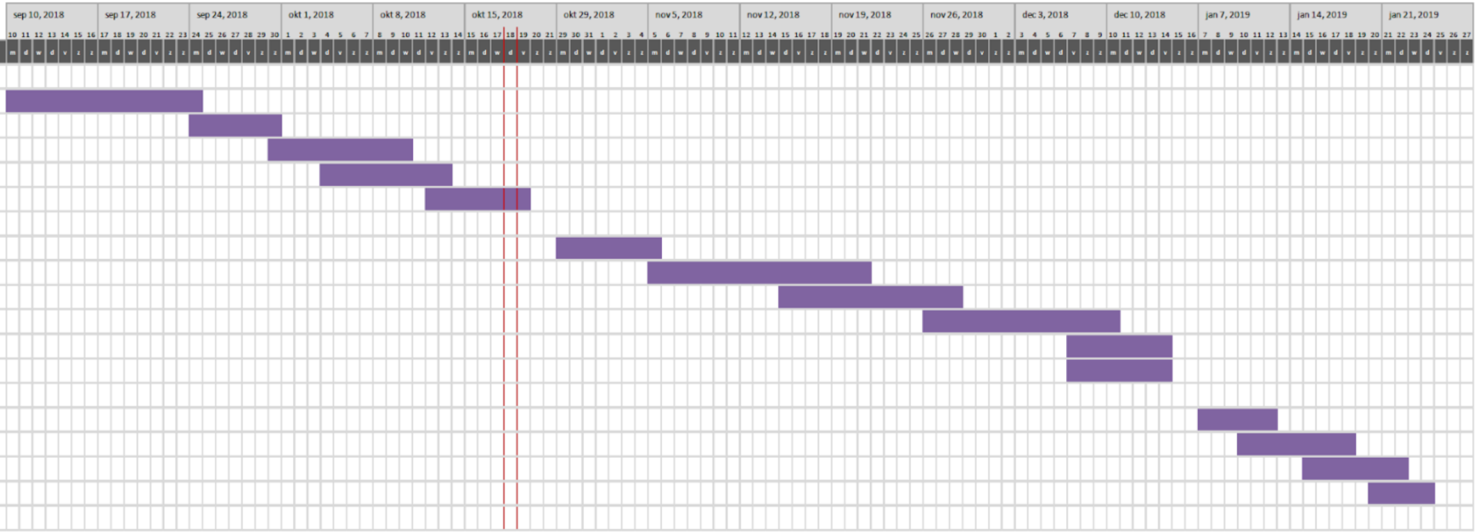
Karush-Kuhn-Tucker conditions check for optimality and feasibility

- Based on Lagrange multipliers
- Optimality is reached when,

$$
\begin{gathered}
\nabla f(x^*) - \lambda^* \cdot \nabla g(x^*) = 0; \\
g_j(x^*) = 0 \ for \ j \in \mathcal{E}; \\
g_j(x^*)\lambda_j^* = 0 \ for \ j \in \mathcal{I}, \\
g_j(x^*), \lambda_j^* \geq 0 \ for \ j \in \mathcal{I}.
\end{gathered}
$$



Time Management



Checking for feasibility

Master Project
Pieter Collins

Project Start: ma, 9-10-2018
Display Week: 1

TASK	ASSIGNED TO	PROGRESS	START	END
Phase 1 Project	10-09-18			
Literature Review			9-10-18	9-24-18
Ariadne installation			9-24-18	9-30-18
Ariadne exploration			9-30-18	10-10-18
Project Plan			10-4-18	10-13-18
Presentation			10-12-18	10-19-18
Phase 2 Project	29-10-18			
Implementation Approximation Newton's method			10-29-18	11-5-18
Define alternative approximation algorithms			11-5-18	11-21-18
Explore rigerious algorithms			11-15-18	11-28-18
Create program to check approximation			11-26-18	12-10-18
Website deliverable			12-7-18	12-14-18
Presentation			12-7-18	12-14-18
Phase 3 Project	07-01-19			
Experiment with software environment created			1-7-19	1-12-19
Analyzing results			1-10-19	1-18-19
Writing Report			1-15-19	1-22-19
Presentation			1-20-19	1-24-19

Risk Analysis

- General Risks Involved:
  - Approximation too far from real solution
  - Infeasibility (no real solution)
- Minimizing those: start off with 1-dimensional problems
- Additional risk: no efficient algorithms beyond 1 dimension

Social Impact & Conclusion

- Rigorous vs Approximate
- More confidence in safety-critical systems

Questions?