Assignment - 5

Hypothesis - Testing

1) During the 1980s, the general consensus is that about 5% of the nation's children had autism. Some claimed that increases certain chemicals in the environment has led to an increase in autism.

a) Write a appropriate hypothesis test for this situation.

b) Give an appropriate test for this hypothesis, stating what are the necessary conditions for performing the test.

c) A recent study examined 384 children and found that 46 showed signs of autism. Perform a test of the hypothesis and state the p-value.

d) What are your conclusions? State how you use the p-value.

Solution :-

Step 1 : The null hypothesis of the general consensus is 5%.
The alternative hypothesis is to be claimed that increase of certain chemicals in the environment causes increase of autism.

$H_0 : p = 0.05$

$H_A : p > 0.05$

It is a ~~Rig~~ one-tailed test. Because it is higher side so we have check for Right-tailed test.

Step 2: The statistical test for which it is qualified is z-test ($\because$ Because number of the samples are greater than 30 and proportion are given).

Step 3:- As we required "α" value so, we set it for ~~1~~% as it is not mentioned.

$\alpha = 5\%$

$\alpha = 0.05$

step 4: Decision Rule

$$Z \text{ critical score} = \frac{\hat{p} - p}{\sqrt{\frac{p \times q}{n}}}$$

$p = 0.05$, $n = 384$

$$\hat{p} = \frac{46}{384} = 0.12 \ (0.119)$$

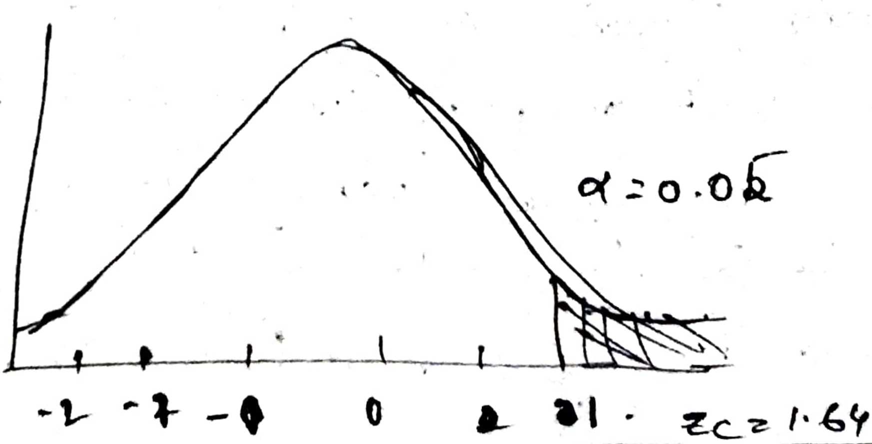
$q = 1 - p = 1 - 0.05$

$q = 0.95$

$$Z \text{ score} = \frac{\hat{p} - p}{\sqrt{\frac{p \times q}{n}}}$$

$$= \frac{0.12 - 0.05}{\sqrt{\frac{0.05 \times 0.95}{384}}} = 6.2938 \simeq 6.29$$

$$\boxed{Z \text{ score} = 6.29}$$



Step 7 : Take statistical action using Z-table, we have calculated,

$$Z_{critical} = 1.64$$

$$Z\text{-score} = 6.29$$

on the basis of decision rule

$$Z_{critical} < Z_{score}$$

$$1.64 < 6.29$$

Step 8 :- ~~Business Im~~

we will reject Null hypothesis so, that the increase in chemicals in the environment led to the increase in autism.

(2) A company with a fleet of 150 cars found that the emission system of 7 out of the 22 cars tested failed to meet pollution guidelines.

a) Write a hypothesis to test if more than 20% of the entire fleet might be out of compliance.

b) Test the hypothesis based on the binomial distribution and report a p-value.

c) Is the test significant at the 10%, 5%, 1% level?

Step 1: Null hypothesis; 20% of the cars are out of compliance

Alternative hypothesis: more than 20%. of the entire fleet might be out of compliance

$H_0 : P = 0.20$

$H_1 : P > 0.20$

It is a one-tailed test (Right tailed test) as it is greater than null hypothesis.

Step 2: The statistical test for which it is qualified is z-test. Because number of the samples are higher than 30 and proportion is given).

Step 3: (i) For 10% of level significance

$$\alpha = 10\% \text{ (or) } \alpha = 0.10$$

Step 4:
Decision Rule
For critical value
$z_{critical} < z_{score}$ (Test Score)

we will reject the null hypothesis

For P-value.

P. value < Significance level (α)

we will reject our null hypothesis

Step 5: Data collection

A company with a fleet of 150 cars found that the emission system of 7 out of the 22 cars tested failed to meet population guidelines.

Step 6: Analyse of data :- statistical decision action using z table.

$$Z \text{ score} = \frac{\hat{p} - p}{\sqrt{\frac{p \times q}{n}}}$$

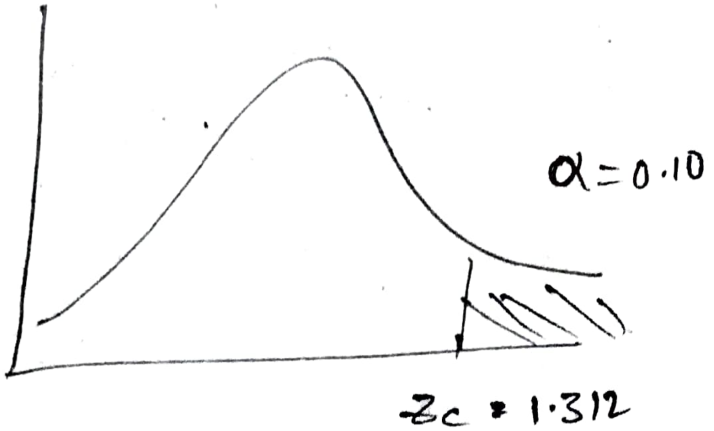
We know that $\boxed{p = 0.20}$, $\boxed{n = 22}$

$$q = 1 - p = 1 - 0.20 = 0.80$$

$$\hat{p} = 7/22 = 0.31$$

$$Z \text{ score} = \frac{0.31 - 0.20}{\sqrt{\frac{0.20 \times 0.80}{22}}} = 1.28$$

$$\boxed{Z \text{ Score} = 1.28}$$



By z-table,

$\boxed{z_c = 1.31}$

P-Value = 1.003

Step 7 : Statistical action

Zscore < Zcritical

Failed to reject null hypothesis.

Pvalues > α

1.0003 > 0.10

Failed to reject null hypothesis

Hence therefore,

20% of the entire fleet of the cars are failed to meet the population guidelines.

Step 8 : Business Implication.

So, the 20% of the cars from a company had failed to meet the population guidelines. So we should improve cars emmission system.

II) For $\alpha = 5\%$ significance level.

$$\boxed{\alpha = 0.05}$$

we have calculated.

$$\boxed{Z_{score} = 1.28}$$

Using z-table.

$$\boxed{Z\text{-critical} = 1.64}$$

on the basis of decision rule.

$$Z_{critical} > Z_{score}$$

$$\boxed{1.64 > 1.28}$$

Failed to reject the null hypothesis.

Therefore,

20% of the fleet of the entire cars failed to meet the population guidelines.

Business Implication :

The car company should improve the emmission system of the cars.

III) For 1% of significance level.

$\alpha = 1\%$ or $\boxed{\alpha = 0.01}$

$\boxed{Z_{score} = 1.28}$

Using Z table

$\boxed{Z_{critical} = 2.33}$

So,

decision rule.

$Z_{critical} > Z_{score}$

$\boxed{2.33 > 1.28}$

Failed to reject the null hypothesis.

Hence 20% of the fleet of the entire cars failed to meet the population guidelines.

Step 8: Business Implications.

So, the 20% of the cars from company had failed to ~~the~~ meet the population guidelines. So we should improve cars emission system.

5) The body temperature in degrees Fahrenheit of 52 randomly chosen healthy adults is measured with the following summary of the data.

$n = 52$, $\bar{x} = 98.2846$, $S = 0.6824$.

(a) Are the necessary conditions for constructing a valid t-interval satisfied? Explain.

(b) Find a 98% confidence interval for the mean body temperature and explain its meaning.

(c) Give a two-side hypothesis test for a mean body temperature of 98.6° Fahrenheit and use the information above to evaluate a test with significance level α = 0.01 fo

(d) Find the power of the test at the parameter value $\mu = 98.2$ and indicate this value using the cutoff value for the test and drawing the sample distribution for the null and alternative hypothesis.

<u>step 1</u>: Null hypothesis: Hypothesis test for a mean body temperature of 98.6° Fahrenheit.

Alternative hypothesis: A two-side hypothesis test for a mean body temperature of 98.6°F

$H_0 : \mu = 98.6°F$

$H_1 : \mu \neq 98.6°F$

As it is a two-tailed test we have to check for left and right tailed test.

$\alpha = 0.02$, $\alpha/2 = 0.01$

step 2: Decision Rule The statistical test to determine is t-test.

step 3: Significance level value

$$\boxed{\alpha = 0.02}$$

step 4: Decision rule

for critical value,

t critical < tscore

we reject the null hypothesis.

for p-value,

Pvalue < α

we reject the null hypothesis.

step 5: collect data.

The body temperature in degrees Fahrenheit of 52 randomly chosen healthy adults is measured with the following data $n = 52$, $\bar{x} = 98.2846$, $s = 0.6824$.

Step 6 : Analysis of the data

$$t_{score} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$n = 52$

degree of freedom $df = 52 - 1$

$= 51$

$s = 0.6824$

$\bar{x} = 98.2846$

$\mu = 98.6$

$$t_{score} = \frac{98.28 - 98.6}{\frac{0.682}{\sqrt{52}}}$$

$\boxed{t_{score} = 3.33}$

Since it is two-tailed test.

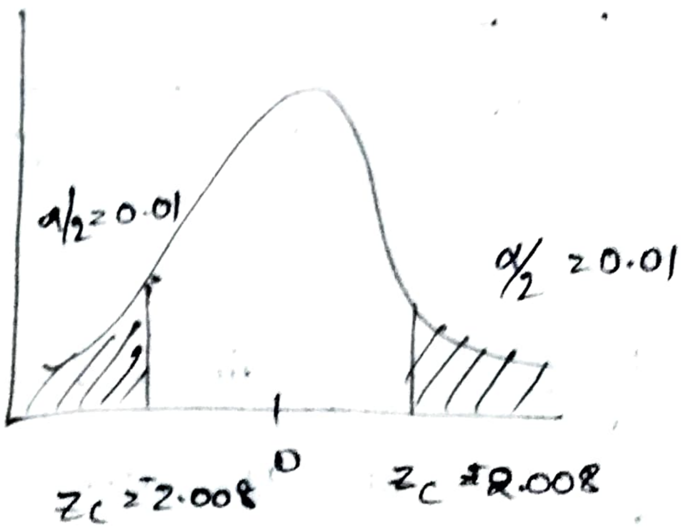
$$\alpha = \frac{\alpha}{2} = \frac{0.02}{2}$$

$\boxed{\alpha = 0.01}$, $\boxed{DF = 51}$

Using t-table

$\boxed{t_{critical} = 2.008}$

$\boxed{P\text{-Values} = 0.0016}$



step 7: statistical action

$$t_{critical} < t_{score}$$

$$\boxed{2.008 < 3.33}$$

we reject the null hypothesis.

step 8: Business Implication.

The body temperature in degrees fahrenheit of 52 randomly chosen healthy adults is not equal to the mean of the all adults.

6) Drivers of cars calling for regular gas sometimes premium in the hopes that it will improve gas mileage. Here a rental car company takes 10 randomly chosen cars in its fleet and runs a tank of gas according to a coin toss, runs a tank of gas of each type.

car	1	2	3	4	5	6	7	8	9	10
Regular	16	20	21	22	23	22	27	25	27	28
Premium	19	22	24	24	25	25	26	26	28	3[illegible]

Step 1: Null hypothesis: Sometimes regular gas and premium has no difference in mileage.

Alternative hypothesis: Some times regular gas and premium are different.

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

It is observed as two-tailed test and we need to check for both left-handed and right handed test.

Step 2: Number of samples are less than 30 so we go for t-test.

Step 3: As the significance level value is not given, so we assume as.

$\alpha = 0.05$

Since it is a two-tailed test

$$\frac{\alpha}{2} = 0.025$$

step 4: Decision Rule

for critical value

$$t_{critical} < t_{score}$$

we reject the null hypothesis

for p-values

$$p\text{-value} < \alpha$$

we reject the null hypothesis

step 5: collecting data.

Here a rental car company taken 10 randomly chosen cars in its fleet and runs a tank of gas according to a coin toss, runs a tank of gas of each type.

step 6: Analysis of data.

Since it is t-test for two samples.

$$DF = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

$n_1 = 10$, $n_2 = 10$

$\mu_A = \bar{x}_1 = 23.1$, $s_1 = 3.72$

$\mu_B = \bar{x}_2 = 25.1$, $s_2 = 3.44$

$$DF = \frac{[1.3838 + 1.183]^2}{\frac{(1.38)^2}{10-1} + \frac{(1.183)^2}{10-1}}$$

$$= \frac{(2.56)^2}{0.2116 + 0.1554}$$

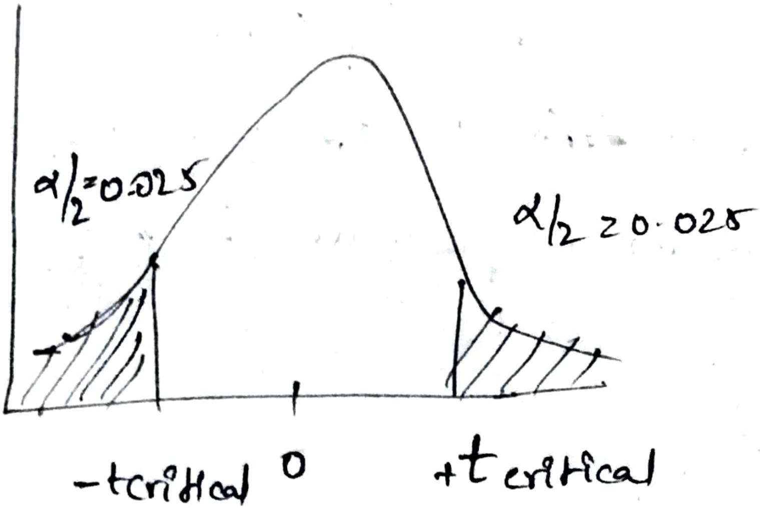
$$\boxed{DF = 17.857}$$

$$t\text{-score} / t\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{23.1 - 25.1}{\sqrt{\frac{3.72^2}{10} + \frac{3.44^2}{10}}}$$

$$= \frac{-2}{1.602}$$

$$\boxed{|t_{test}| = \pm 1.248}$$



By t-table.

$$\boxed{t_{critical} = 2.10}$$

$$\boxed{P = 0.22}$$

Step 7: Statistical Action

on the above decision rule.

for t-critical

if $t_{critical} < t_{score}$ (2.10 > 1.24)

Failed to reject null hypothesis

for P-value

$$P_{value} > \alpha$$

$$0.22 > 0.05$$

We reject the null hypothesis

So, on the basis of decision rule we can say that there is no difference between mileage of regular & premium tank of car.

3) National Data in 1960 showed that about 44% of the adult population had never smoked

a) State a null and alternative to test that the fraction of the 1995 of adults that had never smoked had increased.

b) A national random sample of 891 adults were interviewed and 463 stated that they had never smoked. Perform a z-test of the hypothesis and given an appropriate P-value

c) Create a 98% confidence interval for the proportion of the adults who had never been smoked.

Null hypothesis - 44% adult population had never smoked.

Alternate hypothesis - more than 44% of adult population had never smoked.

$H_0 : P = 0.44$

$H_a : P > 0.44$

$\therefore$ One tailed test, since only one ~~ed~~ end is being tested

Z - test :

Set the value of significance level

Confidence level is 98%.

$\alpha = 2\%$, $\alpha = 0.20$

$$\boxed{\therefore \alpha = 0.02}$$

Establish the decision rule, for z-critical

if z critical < z-test (test scale)

$\therefore$ Null hypothesis will be rejected

For P-values

if P-value < significance value

$\therefore$ Null hypothesis will be rejected

Given, a national random sample of 891 adults were interviewed and 463 stated that they had never smoked

For Z-score,

$$Z\text{-score} = \frac{\hat{P} - P}{\sqrt{\frac{P \times q}{n}}}$$

We know that,

$\boxed{P = 0.44}$ $\boxed{n = 891}$

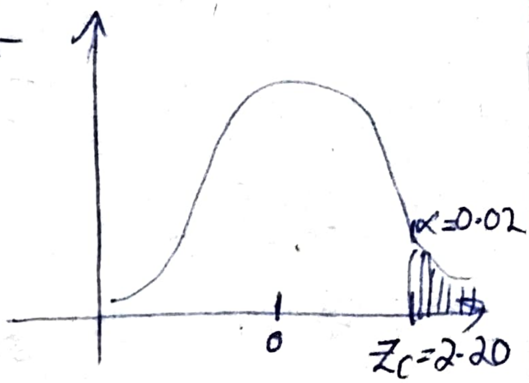
$q = 1 - p$
$= 1 - 0.44$
$= 0.56$
$\boxed{q = 0.56}$

$\hat{P} = \frac{463}{891}$

$\boxed{\hat{p} = 0.519}$

$$Z\text{-score} = \frac{0.519 - 0.44}{\sqrt{\frac{0.44 \times 0.56}{891}}}$$

$\boxed{Z\text{-score} = 4.75}$



By using Z-table

$\boxed{Z\text{-critical} = 2.20}$

On the basis of Decision Rule

$Z\text{-critical} < Z\text{-score}$

$2.20 < 4.75$

∴ Null hypothesis is rejected

Hence, more than 44% of adult population had never smoked.

4) One of the lenses of your supply is suspected to have a focal length of 9.1cm rather than 9cm claimed by the manufac-turer. Here S_1 is the distance from the lens to the real image of the object. The distances S_1 & S_2 are each independently measured 25 times. The sample mean of the measurement is $\bar{S}_1 = 26.6$ cm & $\bar{S}_2 = 13.8$ cm respectively, The std deviation of the measurement is 0.1cm for S_1 & 0.5cm for S_2.

a) Write the appropriate test hypothesis test for this situation.

b) Use this to devise a Z-test for the hypothesis and report a p-value for the test.

A)

Null hypothesis :- The distance from the lens to the object and distance from the lens to real image is same

Alternate hypothesis :- The distance from the lens to the object and distance from the lens to real image is not same

$H_0 - \mu_A = \mu_B$

$H_A - \mu_A \neq \mu_B$

z-test :- set significance value

Default significance value, $\alpha = 0.05$

Its a two tailed test, we are going to check left and right ends of the experiment

So,

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

Decision rule :- for critical value

if

z-critical $\leq$ Z-score

∴ Null hypothesis gets rejected

For P-values

Pvalues < significance value

∴ Null hypothesis gets rejected

for p-values

if Pvalues < significance value

We will reject null hypothesis.

Since we are going to perform the z-test for two samples

$\alpha = 0.05$

$$\boxed{\frac{\alpha}{2} = 0.025}$$

For distance from the lens to object s_1,

$\bar{s}_1 = 26.6cm$

$\sigma_1 = 0.1cm$

$n_1 = 25$

For distance from lens to real image s_2,

$\bar{s}_2 = 13.8$

$\sigma_2 = 0.5cm$

$n_2 = 25$

~~For~~ z-score

$$z\text{-score} = \frac{\bar{s}_1 - \bar{s}_2}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}}$$

$$= \frac{26.6 - 13.8}{\sqrt{\frac{(0.1)^2}{25} + \frac{(0.5)^2}{25}}}$$

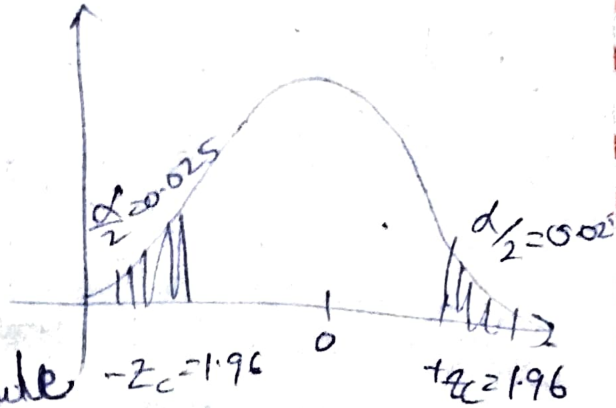
$$= \frac{12.8}{0.102}$$

$$\boxed{z\text{-score} = 125.51}$$

Now using z-table

$\boxed{Z_{critical} = 1.96}$

$\boxed{P\ value = 0}$



On basis of decision rule

for critical value

if Z-critical < Z-score

1.96 < 125.51

Null hypothesis gets rejected

for critical value

P-value < significance value.

0.0000 < 0.05

$\boxed{\therefore \text{Null hypothesis rejected}}$

The distance from lens to the object and distance from the lens to the real image are not same.

$\boxed{\mu_A \neq \mu_B}$