(203)

Dine

Mathematical Induction

Rony

(1) Use mathematical induction to prove $2^n < n!$ for every (+)ve integer n with $n \geq 4$

Ans let, $p(n)$ be a proposition that $2^n < n!$

Basis step: let $p(4)$ will be true for this statement i.e,

$L.H.S = 2^n$ $\qquad$ $R.H.S = 4!$ $\qquad$ Kutusu

$= 2^4$ $\qquad$ $= 24$

$= 16$

L.H.S < R.H.S, so $P(4)$ is true.

Inductive step: Assume that $p(n)$ is true that is, assume that $2^n < n!$ under this assumption it must be shown that $p(n+1)$ is true namely $2^{n+1} < (n+1)! \cdots (1)$ multiplying both sides of the inequality $2^n < n!$ by 2 it follows that, $2 \cdot 2^n < 2n!$

$$\Rightarrow 2^{n+1} < (n+1) n!$$

This shows that $p(n+1)$ is true when $p(n)$ is true. Hence it follows that $2^n < n!$ is true for all integers n with $n \geq 4$.

→11

Use the mathmatical induction to prove the inequa- $n<2^n$ for all positive integer.॥

Let $p(n)$ be the proposition "$n<2^n$"

Basis step: $P(1)$ is true, since $1<2^1=2$

Induction step; Assume that $p(k)$ in true for the positive integer k. That is assume that $k<2^k$. we have to show that $p(k+1)$ is true i.e we have to show that $k+1<2^{k+1}$, adding 1 to both sides of $k<2^k$ and then noting that $1\leq 2^k$ gives,

$$k+1<2^k+1\leq 2^k+2^k=2^{k+1}$$

we have shown that $p(k+1)$ is true, namely the $k+1<2^{k+1}$ based on the assumption that $p(k)$ is true. There $n<2^n$ is true for all positive integer n.

→16

Use mathematical induction to prove that n^3-n is divisible by 3 whenever n is a positive integer.

Let $p(n)$ be the proposition n^3-n is divisible by 3.

Basis step: $P(1)$ is true, since $1^3-1=0$ is divisible by 3.

Inductive step: Assume that $P(n)$ is true that is assume n^3-n is divisibe by 3. Under this assumption it must be shown that $P(n+1)$ is true namely that, $(n+1)^3-(n+1)$ is divisible by 3

$$\text{Now, } (n+1)^3-(n+1)=(n^3+3n^2+3n+1)-(n+1)$$
$$=(n^3-n)+3(n^2+n) \quad \text{--}(1)$$

since both terms of (1) are divisible by 3 it follows that $(n+1)^3-(n+1)$ is also divisible by 3. This complete the induction step. Thus by the principle of mathematical induction, n^3-n is divisible by 3 when n is a positive integer.

→ 16+17

Q. prove that if n is a positive integer then n has a prime divisor less than or equal to $\sqrt{n}$.

A. Let a and b are two positive integer. If n is composible it has a factor with $1<a<n$. since a/n then we can write $n=ab$.

we see that $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

otherwise $a.b > \sqrt{n}.\sqrt{n}$

$ab > n$

Hence n has a prime divisor not exceding $\sqrt{n}$. This divisor is either prime or by the fundamental of orithmatic has a prime divisor. In either case n has a prime divisor less than or equal $\sqrt{n}$.

2 ✓

★ composite number: A positive integer that is greater than 1 and is not prime is called composite number.

11+16

what do you mean by vacuous and trivial proof math

most

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Let S be a universal set, $p(x)$ and $Q(x)$ be two open sentences. For $x \in S$, it can be shown that $Q(x)$ is true regardless of the true value of $P(x)$. Then $P(x) \to Q(x)$ is true of all $a \in S$. In this case $P \to Q$ is true trivially. such a proof called trivial proof.

Result: Let $x \in R$. If $x>0$ then $x^2+5>0$

proof: since $x^2 \geq 0$, for all $x \in R$, it follows that

$$x^2+5 > x^2 \geq 0$$

Hence $x^2+5 \geq 0$

Note that we did not use the fact that $x>0$. since it holds for all $a \in R$.

Now, if it can be shown that $P(x)$ is false all $x \in S$, Then $P(x) \to Q(x)$ is true for all values of $x \in S$.

That is $P \to Q$ is true vacuously such a proof is called a vacuous proof.

Result : let $x \in \mathbb{R}$. If $x^2+1<0$ then $x^5 \geq 4$

proof : observe that $x^2+1 > x^2 \geq 0$.

Thus $x^2+1<0$ is false for all $x \in S$ and so the implication is true.

soln : $a \equiv b \pmod m$ if and only if m divides $a-b$ see that $a-a=0$ is divisible by m

because $0 = 0.m$

Hence $a \equiv a \pmod m$ so congruence modulo m is reflexive.

suppose that $a \equiv b \pmod m$

$\Rightarrow a-b = km$ (where k is an integer)

$\Rightarrow b-a = (-k)m$

so $b \equiv a \pmod m$. Hence congruence modulo m is symmetric

Next $a \equiv b \pmod{m}$

and $b \equiv c \pmod{m}$ [m divides both a-b and b-c]

$a-b = km$

and $b-c = lm$ [where k and l integer]

Adding these two equation

$a-b+b-c = km+lm$

$\Rightarrow a-c = km+lm$

$\Rightarrow a-c = m(k+l)$

$\Rightarrow a \equiv c \pmod{m}$

so congruence modulo m is transitive.