

30 Sept 23

MTH

H.M. Tahir Shah  
22201243

ii)  $\lim_{(n,y) \rightarrow (0,0)} f(n,y) = 0$  (Given) (v)  
 $f(n,y) = \frac{ny}{n^2+y^2}$   
 $\therefore \lim_{(n,y) \rightarrow (0,0)} f(n,y) = \lim_{(n,y) \rightarrow (0,0)} \frac{ny}{n^2+y^2} = 0$

Soln: Given that,

$$\begin{aligned} \lim_{n \rightarrow 0} f(n,y) &= \lim_{(n,y) \rightarrow (0,0)} \frac{ny}{n^2+y^2} \\ &= \lim_{(n,y) \rightarrow (0,0)} \frac{0 \cdot y}{0^2+y^2} \\ &= \lim_{(n,y) \rightarrow (0,0)} 0 = 0 \end{aligned}$$

Again,

$$\lim_{(n,y) \rightarrow (0,0)} f(n,y) = 0$$

which is unique and finite.

(i)  $f(n,y) = \frac{xy}{n^2+y^2}$ , where  $\lim_{(n,y) \rightarrow (0,0)}$   
along  $y=0$

Soln:

$$\begin{aligned} \lim_{(n,y) \rightarrow (0,0)} f(n,y) &= \lim_{(n,y) \rightarrow (0,0)} \frac{xy}{n^2+y^2} \\ &= \lim_{(n,y) \rightarrow (0,0)} \frac{n \cdot 0}{n^2+0} = 0 \end{aligned}$$

which is unique and finite.

$$(iii) \lim_{\substack{n \rightarrow 0 \\ y \rightarrow 0}} f(x,y) = 0 \quad \text{where, } f(x,y) = \frac{xy}{x^2+y^2}$$

$y = x^2$  off

Soln: when  $y = x^2$   $\Rightarrow$  (both)

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$\therefore L.H.S = \frac{nx^2}{x^2+x^4} = \frac{x^3}{x^2+x^4} = \frac{x^3}{x^2(1+x^2)} = \frac{x}{1+x^2}$$

if we divide by  $x^3$  then (L.H.S)  $\Rightarrow$

$$= \frac{x}{x^2(1+x^2)} = \frac{1}{x^2+1}$$

if we divide by  $x^2$  then (R.H.S)  $\Rightarrow$

$$= \frac{x}{1+x^2}$$

Hence  $L.H.S = R.H.S$

Now,

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) \leftarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x}{1+x^2}$$

0 - B mode.

$$0 = \left(\frac{0}{1}\right) \frac{0}{1} = 0$$

Since  $f(x,y) = 0$  so this is unique and finite. The limit exists.

0 - B mode,  $\therefore \lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$

(iv) Given that

$$\lim_{\substack{n \rightarrow 0 \\ y \rightarrow 0}} f(n,y) = \frac{ny}{n^2+y^2}$$

$$[y=n]$$

$$\lim_{\substack{n \rightarrow 0 \\ y \rightarrow 0}} f(n,y) = \frac{nxn}{n^2+n^2}$$

$$f_y = n \cdot \frac{2n^2}{n^2+n^2}$$

$f(n,y) = \frac{ny}{n^2+y^2} = \frac{ny}{n^2+n^2}$  which is unique and finite  
So the limit exist.

(v) Given that

$$\lim_{\substack{n \rightarrow 0 \\ y \rightarrow 0}} f(n,y) = \frac{xy}{n^2+y^2}$$

From (i), (ii), (iii) and (v) we found that,

$$\lim_{(ny) \rightarrow (0,0)} f(n,y) = 0 \quad \text{along } y=0$$

$$\lim_{(n,y) \rightarrow (0,0)} f(n,y) = 0 \quad \text{along } n=0$$

$$\lim_{(n,y) \rightarrow (0,0)} f(n,y) = \infty \quad \text{along } y=n$$

$$\lim_{(ny) \rightarrow (0,0)} f(n,y) = \frac{1}{2} \quad \text{along } y=n$$

$f(n,y)$  is different along  $(n,y) \rightarrow (0,0)$

$y=0, n=0 \Rightarrow y=n^2$  and  $y=0$

So,  $\lim_{(n,y) \rightarrow (0,0)} f(n,y)$  does not exist.

### Ques 12

(i)  $\lim_{\substack{n \rightarrow 0 \\ y \rightarrow 0}} f(n,y) = \frac{n^3 + y^3}{n^3 + y}$ ; along  $y=mn$

$$\begin{aligned} &= \frac{n^3 + (mn)^3}{n^3 + mn} \\ &\stackrel{\text{Q.E.D.}}{=} \frac{n^3(1+m^3)}{n(n^2+m)} \leftarrow (\text{R.H.S.}) \\ &= \frac{n^2(1+m^3)}{n^2+m} \end{aligned}$$

$$\lim_{(n,y) \rightarrow (0,0)} = \frac{0(1+m^3)}{0+m} = \frac{0}{m} = 0$$

$f(n,y) = 0$  which is unique and finite

Hence  $f(n,y)$  exists along  $y=mn$

$\therefore \lim_{(n,y) \rightarrow (0,0)} f(n,y) = 0$

$$(ii) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} f(x,y) = \frac{x^3 + y^3}{x^3 + y} \text{ along } y = -x^2 e^x$$

$$\begin{aligned} & \text{Now take } y = -x^2 e^x \quad \text{and} \\ & = \frac{x^3 + (-x^2 e^x)^3}{x^3 + (-x^2 e^x)} \\ & = \frac{x^3 - x^6 e^{3x}}{x^3 - x^2 e^x} \end{aligned}$$

$$\begin{aligned} & \text{Now } \lim_{x \rightarrow 0} \frac{e^x + x^3(1 - x^3 e^{3x})}{e^x x^3 (1 - e^{3x}/x)} \quad (i) \\ & = \frac{e^x + (1 - x^3 e^{3x})}{e^x (1 - e^{3x}/x)} \\ & = \frac{e^x + (1 - e^{3x}/x)}{e^x (1 - e^{3x}/x)} \end{aligned}$$

$$\begin{aligned} & \text{Now, } \\ & (x,y) \rightarrow (0,0) \quad \frac{1 - 0^3 \cdot e^{3 \times 0}}{e^0 / 0} \\ & = \frac{1}{0} = \infty \end{aligned}$$

$f(x,y)$  is unique and finite. Here

$\lim_{(x,y) \rightarrow (0,0)} f(x,y)$  exists. Hence  $f(x,y)$  exists along  $y = -x^2 e^x$ ,

(iii)  $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} f(x,y)$  from (i) and (ii) we found that  $f(x,y) = 0$ ;  $f(x,y) = 1$ .  $\lim f(x,y)$  is different along  $(x,y) \rightarrow (0,0)$ ,  
 $y = mx$  and  $y = -x^2 e^x$  passing through  $(0,0)$ . So,  $f(x,y)$  does not exist.