Curves Defined by parametric equations.

Parametric equation :-

If x and y co-ordinates of a point of any curve be expressed by third variable, then the third variable is called parameter.

The equation with parameter is called parametric equation.

For example.

If x and y are both changes with respect to time t, then. we can say that

$$\left.\begin{array}{ll} x = f(t) & ;\ \text{here, } t \text{ is independent} \\ \text{and}\quad y = g(t) & ;\ \text{here, } t \text{ is independent} \end{array}\right\} \text{——— (I)}$$

Here, the equation (I) is called parametric equation.

Curves Defined by parametric Equations:-

When the path of a particle moving in the plane is not the graph of a function, we can not describe it using a formula that express y directly in terms of x or x directly in terms of y. Instead, we need to use a third variable t, called a parameter and write

$$x = f(t)$$

and $$y = g(t)$$

(*) The set of points $(x, y) = (f(t), g(t))$ described by these equations when t varies in an interval I, form a curve, called a parametric curve, and $x = f(t)$, $y = g(t)$ are called the parametric equations of the curve.

Often, t respresent time and therefore, we

* can think of

$(x, y) = (f(t), g(t))$ as the position of a particle at time t.

(*) If I is a closed interval, $a \le t \le b$, the point $(f(a), g(a))$ is the initial point and the point $(f(b), g(b))$ is the terminal point.

Problem-01

Draw and identify the parametric curve given by the parametric equations

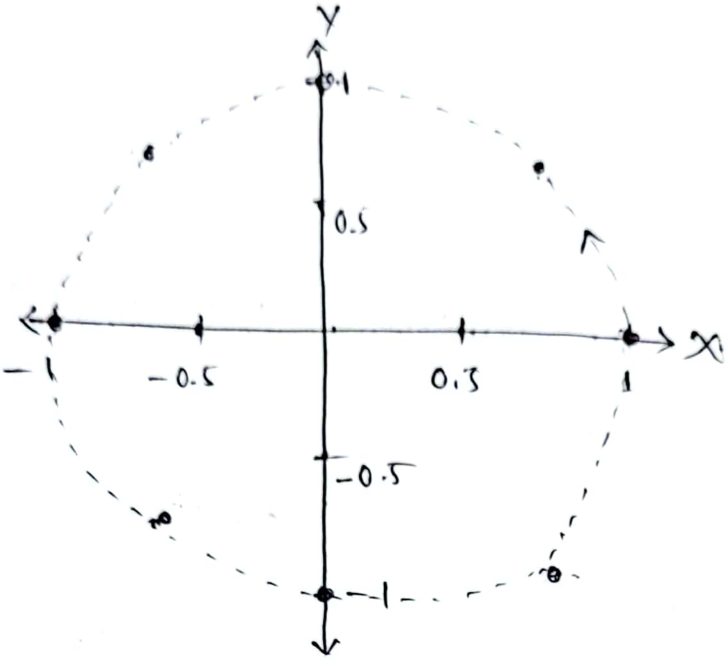
$$x = \cos t \quad ; \quad y = \sin t \quad : \quad 0 \le t \le 2\pi .$$

Solution:-

1. First look at points on the curve for particular values of t. (in table)

t	$x = \cos t$	$y = \sin t$
0	1	0
$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\pi/2$	0	1
$3\pi/4$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0
$3\pi/2$	0	-1
2π	1	0

2. Plotting these points, we get points on a circle as shown below.



3. Filling in the details, we see that as t increases from 0 to 2π, the point traces out a circle of radius 1 in a anti-clockwise direction with Initial point $(1,0)$ and terminal point $(1,0)$.

<u>H.W.</u> Describe the parametric curve represented by the parametric equations

$$x = \sin 2t \quad ; \quad y = \cos 2t \quad ; \quad 0 \le t \le 2\pi$$

H.W: Sketch the graph of the curve describe by the following set of parametric equations

$$x = t^3 - t \quad ; \quad y = t^2 \quad ; \quad 0 \le t < \infty$$

Converting Parametric to cartesian :-

In this case, we try to remove t and combine both equations. Then we obtain an equation involving x and y only. not any t.

Ex. Convert the following parametric equation to an equation relating x and y.

$$x = 2t + 1 \quad ; \quad y = t - 2 \quad ; -\infty < t < \infty$$

Solution :- Here, $y = t - 2$

$\therefore t = y + 2$ ——— (1)

Then, $x = 2t + 1$ becomes

$\Rightarrow x = 2 \times (y+2) + 1$ [using (1)]

$\Rightarrow x = 2y + 4 + 1$

$\therefore x = 2y + 5$

which is the required cartesian equation.

H.W.

① convert the following parametric equation to an equation relating x and y

$x = 2\cos t$

$y = 3\sin t \quad ; \quad 0 \le t \le 4\pi$

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