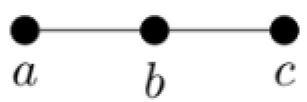
Q1.

Draw a graph with the adjacency matrix

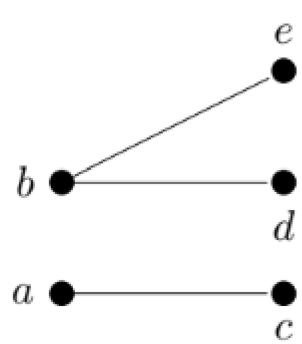
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Ans:



Q2.
(a) Draw a bipartite graph with 5 vertices. (b) Find the adjacency matrix of this graph. (c) Determine whether this graph is a tree. (d) How many simple paths are there in this graph?

Answer:
Possible solution:



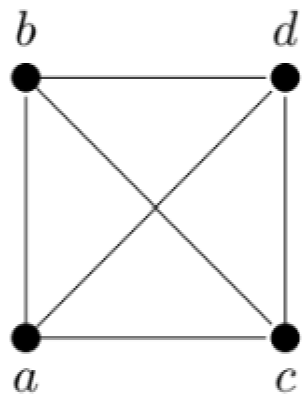
The adjacency matrix is as follows:

	a	b	c	d	e
a	0	0	1	0	0
b	0	0	0	1	1
c	1	0	0	0	0
d	0	1	0	0	0
e	0	1	0	0	0

This graph is not a tree since it is not connected. There are 5 paths consisting of a single vertex, 6 paths consisting of two vertices, and 2 simple paths consisting of 3 vertices, so that the total number of simple paths is 13.

Q3. How many cycles are there in the complete graph on 4 vertices?

Solution. Let the vertices of the graph be a, b, c, d:

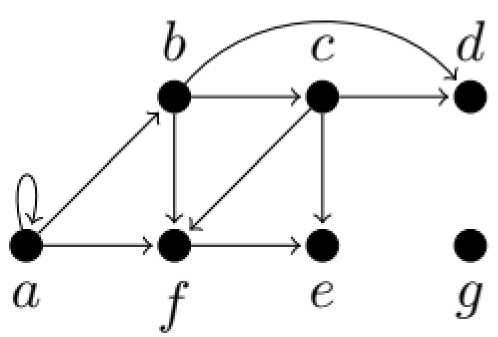


There are 6 cycles that include the vertices a, b, c:

a, b, c, a; a, c, b, a; b, c, a, b; b, a, c, b; c, a, b, c; c, b, a, c.

Similarly, there are 6 cycles including a, b, d, 6 cycles including a, c, d and 6 cycles including b, c, d. So the total number of cycles including three vertices out of four is 24. There are also 24 cycles including all four vertices: one per each permutation of a, b, c, d. So the total number of cycles is 48.

Q4.
(a) Find the in-degree and the out-degree of each vertex in the graph shown in the picture.

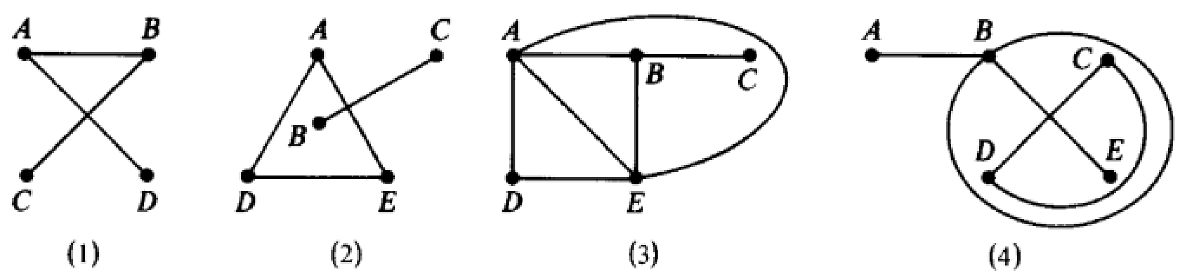


Answer:

Vertex	Indegree	Outdegree
a	1	3
b	1	3
c	1	3
d	2	0
e	2	0
f	3	1
g	0	0

Q5.

Consider the multigraphs in Fig.



(a) Which of them are connected? If a graph is not connected, find its connected components.
(b) Which are cycle-free (without cycles)?
(c) Which are loop-free (without loops)?
(d) Which are (simple) graphs?

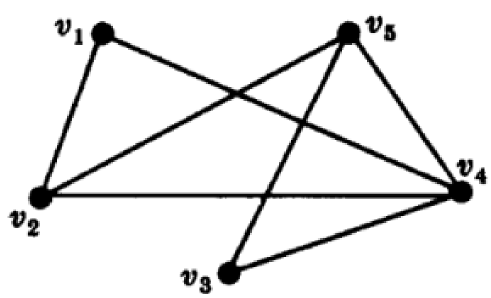
Answer:

(a) Only (1) and (3) are connected, (2) is disconnected; its connected components are {A, D, E} and {B,C}. (4) is disconnected; its connected components are {A, B, E} and {C, D}.
(b) Only (1) and (4) are cycle-free. (2) has the cycle (A, D, E, A), and (3) has the cycle (A, B, E, A).
(c) Only (4) has a loop which is {B,B}.
(d) Only (1) and (2) are graphs. Multigraph (3) has multiple edges {A, E} and {A, E}; and (4) has both multiple edges {C, D} and {C, D} and a loop {B,B}.

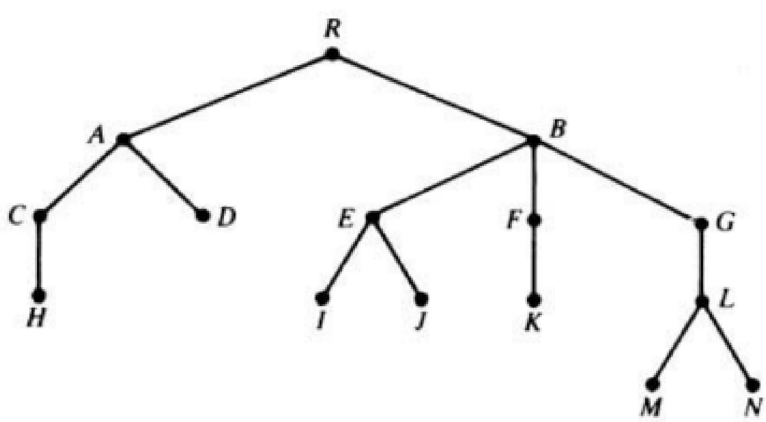
Q6.
Draw the graph G corresponding to each adjacency matrix:

$$(a)\ A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix};$$

Answer:
(a) Since A is a 5-square matrix, G has five vertices, say, v1, v2 ...,v5. Draw an edge from vi to vj when aij = 1.



Q7.
Let T be the rooted tree in Fig.
(a) Identify the path α from the root R to each of the following vertices, and find the level number n of the vertex: (i) H; (ii) F; (iii) M.
(b) Find the siblings of E.
(c) Find the leaves of T .



Answer:
(a) List the vertices while proceeding from R down the tree to the vertex. The number of vertices, other than R, is the
level number:
(i) $\alpha = (R, A, C, H)$, $n = 3$; (ii) $\alpha = (R, B, F)$, $n = 2$; (iii) $\alpha = (R, B, G, L, M)$, $n = 4$.
(b) The siblings of E are F and G since they all have the same parent B.
(c) The leaves are vertices with no children, that is, H,D, I, J,K,M, N.

Q8.
Suppose Friendly Airways has nine daily flights as follows:

1) Atlanta to Houston
2) Boston to Denver
3) Chicago to Miami
4) Houston to Atlanta
5) Denver to Boston
6) Miami to Boston
7) Boston to Chicago
8) Denver to Reno
9) Reno to Chicago

Describe the data by means of a directed graph G.

Answer: