1. Determine whether the given formula is true or false. Justify your answers.
(a) $\forall m \exists n (2 \mid m + n)$. [2 divides m+n]

Solution: True. Choose $n = -m$. Then $m + n = 0$; 0 is even.

(b) $\exists m \forall n (m - 5 \mid n)$. [m-5 divides n]

Solution: True. Choose $m = 6$. Then $m - 5 = 1$; every integer
is a multiple of 1.

2. Simplify the given formula. Justify your answers.
(a) $n > 4 \land n^2 < 30$.

Answer: $n = 5$.

(b) $n > 4 \lor (n^2)! = 0$.

Answer: $n > 4$.

(c) $x > 3 \lor x < 3$.

Answer: $x \neq 3$.

(d) $\neg(x > 10)$.

Answer: $x \leq 10$.

(e) $n > 4 \land n < 6$.

Answer: $n = 5$.

3.

Determine whether the given formula is true. If it is, prove it. If not, find a counterexample.

(a) $\forall n (1 \leq n \leq 4 \rightarrow 4 \cdot 2^{2-n} > 1)$.

Answer: false; counterexample: $n = 4$. Indeed, $4 \cdot 2^{2-4} = 1$.

(b) $\forall n (1 \leq n \leq 4 \rightarrow n! \leq 2^{n+1})$.

Answer: true. Proof by exhaustion:

n	$n!$	2^{n+1}	$n! \leq 2^{n+1}$
1	1	4	T
2	2	8	T
3	6	16	T
4	24	32	T

4. Translate into logical notation:
(a) There exists a positive integer that is less than 5.

Answer: $\exists n(0 < n < 5)$.

(b) The square of every negative real number is positive.

Answer: $\forall x(x < 0 \rightarrow x^2 > 0)$.

5. Determine whether the given formula is true or false. Justify your answers.

(a) $\forall n \exists x(n < x < n + 1)$.

Answer: true; take $x = n + ½$

(b) $\forall n \exists x(n < x^2 < n + 1)$.

Answer: false; $n = -1$ is a counterexample. Indeed, there is no real number x such that $-1 < x^2 < 0$, because x^2 is nonnegative.

(c) $\forall x \exists y(y^3 + 1 = x)$.

Answer: true; take $y = (x - 1)^{1/3}$.

(d) $\exists x \forall y(x + 4 < y^4)$.

Answer: true; take $x = -5$. Indeed, for all y, $-1 < y^4$, because y^4 is nonnnegative.

(e) $\exists xy \forall z(xz = y)$.

Answer: true; take $x = y = 0$. It is clear that for all z, $0 \cdot z = 0$.

6. Determine whether the given formula is true or false. If it is false then find a counterexample:
(a) $\forall n(2n > 1 \lor n < 0)$.

Answer: false; counterexample: $n = 0$.

(b) $\forall n\,(n^2 > 2^{-1/2})$

Answer: false; counterexample: $n = 0$.

(c) $\forall xy(x^2 + y^2 = x^3 + y^3)$.

Answer: false; counterexample: $x = y = 2$.

(d) (b) $\forall n(n = 10^2 + 11^2 + 12^2 \leftrightarrow n = 13^2 + 14^2)$.

Answer: true, because $10^2 + 11^2 + 12^2$ and $13^2 + 14^2$ both equal 365.

7. Translate into logical notation:

(a) There exists a pair of negative integers such that their product is 6.
Find an example showing that this assertion is true.

Answer: $\exists mn(m < 0 \land n < 0 \land mn = 6)$; witness: $m = -2$, $n = -3$.

b) The sum of any two positive integers is greater than 1.

Answer: $\forall mn(m > 0 \land n > 0 \rightarrow m + n > 1)$.

8. Determine if the following argument is valid and explain why.
Every CS major takes CMSC203. Joel is taking CMSC203, therefore he is a CS major.

Answer: Invalid: $p \rightarrow q$ and q do not support the conclusion of p

9. Use a truth table to decide whether this argument is valid.
(a)
$p \lor q$ —-----------premise 1
$q \lor r$ —----------- premise 2
$\therefore\ p \lor r$ conclusion

SOLUTION: NOT valid.

(b) $p \lor \neg q$ ——————-premise 1
$p \land \neg r$ ——————-premise 2
$\therefore\ r \rightarrow q$ conclusion

SOLUTION: Valid.

10. Let p be “It is cold” and let q be “It is raining”. Give a simple verbal sentence which describes each of the following statements: (a) $\neg p$; (b) $p \land q$; (c) $p \lor q$; (d) $q \lor \neg p$.

Answers:
(a) It is not cold. (c) It is cold or it is raining.
(b) It is cold and raining. (d) It is raining or it is not cold.

11) Use the laws to show that $\neg(p \lor q) \lor (\neg p \land q) \equiv \neg p$.

12) Let A = {1, 2, 3, 4, 5}. Determine the truth value of each of the following statements:
(a) $(\exists x \in A)(x + 3 = 10)$
(b) $(\forall x \in A)(x + 3 < 10)$
(c) $(\exists x \in A)(x + 3 < 5)$
(d) $(\forall x \in A)(x + 3 \leq 7)$

Answer:
(a) False. For no number in A is a solution to $x + 3 = 10$.
(b) True. For every number in A satisfies $x + 3 < 10$.
(c) True. For if $x0 = 1$, then $x0 + 3 < 5$, i.e., 1 is a solution.
(d) False. For if $x0 = 5$, then $x0 + 3$ is not less than or equal 7. In other words, 5 is not a solution to the given condition.

13. Simplify the expression $(x + y)(x + z)$ using the laws of boolean algebra.
Solution:

Let $Q = (x + y)(x + z)$

Using the distributive law, we can write;

$Q = x.x + x.z + y.x + y.z$

By applying the idempotent law $A.A = A$.

$Q = x + x.z + y.x + y.z$

$Q = x(1 + z) + y.x + y.z$ [Using distributive law]

Applying identity OR law (1 +A = 1), we can write

Q = x. 1 + y.x + y.z

Q = x + y.x + y.z

Again using the distributive law, we get

Q = x. (1 + y) + y.z

Q = x. 1 + y.z (By applying identity OR Law)

Q = x + y.z.

Therefore, the simplification of the expression (x + y)(x + z) is x + y.z.

14. Write the logic expression for the following statement:

a) **"*You can purchase this book if you have $20 or $10 and a discount coupon.*"**

Let *a*, *b*, *c,* and *d* represent the sentences "*You can purchase this book.*", "*You have $20.*", "*You have $10.*", and "*You have a discount coupon.*" respectively. Then the given sentence can be translated to

(b ∨ (c ∧ d) -> a,

which simply means that "if you either have $20 or $10, along with a discount coupon, then you can purchase the book."

b) "If the computer is within the local network or it is not within the local network but has a valid login id or it is under the use of administrator, then the Internet is accessible to the computer."

Let a, b, c, and d represent the sentences "The computer is within the local network.", "The computer has a valid login id.", "The computer is under the use of administrator.", and "Internet is accessible to the computer."

(a ∨ (¬a ∧ b) ∨ c) -> d