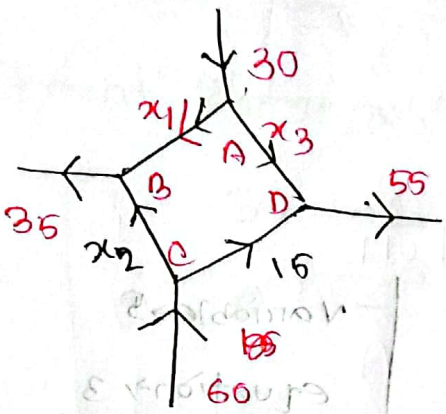
* Network analysis:



NODE	flow in	flow out
A	$\cancel{x_1+x_3}$ 30	$= x_1+x_3$
B	x_1+x_2	$= 35$
C	$\cancel{x_2+15}$ 60	$= x_2+15$
D	x_3+15	$= 55$

$x_1+x_3 = 30 \Rightarrow$

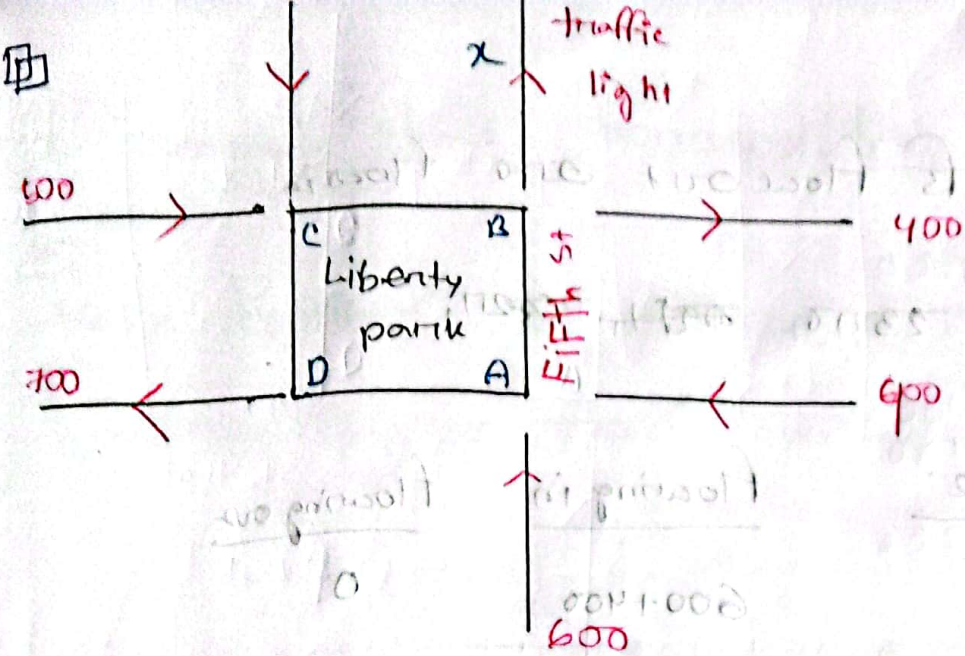
$x_1+x_2 = 35 \Rightarrow$

$x_2+15 = 60 \Rightarrow x_2 = 60-15 = 45$

$x_3+15 = 55 \Rightarrow x_3 = 55-15 = 40$

$\therefore x_1+45 = 35$

$x_1 = -10;$



average number of vehicle per hour that are expected to flow in and out of the streets that border the complex. All street are one way.

(a) How many vehicles per hour should the traffic light let through to ensure that the average number of vehicles per hour flowing into the complex is the same as the average number of vehicles flowing out?

Sol: এখানে রাস্তা গুলো one way মানে যে রাস্তা দিয়ে car একবার ঢুকে গেলে আর সেই রাস্তা দিয়ে back করে যাবে না। অর্থাৎ যে রাস্তা দিয়ে car আসবে সেখান দিয়ে গাড়ি গুলো শুধু আসবে। কোনো গাড়ি back করবে না। আর যে গাড়ি গুলো flow out হচ্ছে সেগুলো ওই রাস্তা দিয়ে flow-in করবে না,

তাই এখানে সবটি flow out এবং flow in

এদের এটা same হওয়া ~~কথা~~ কথা,

Sol:

Node	flowing in	flowing out
A	600+400	0
B	0	400+x
C	500+200	0
D	0	700
	1700	1100+x

$1100 + x = 1700$

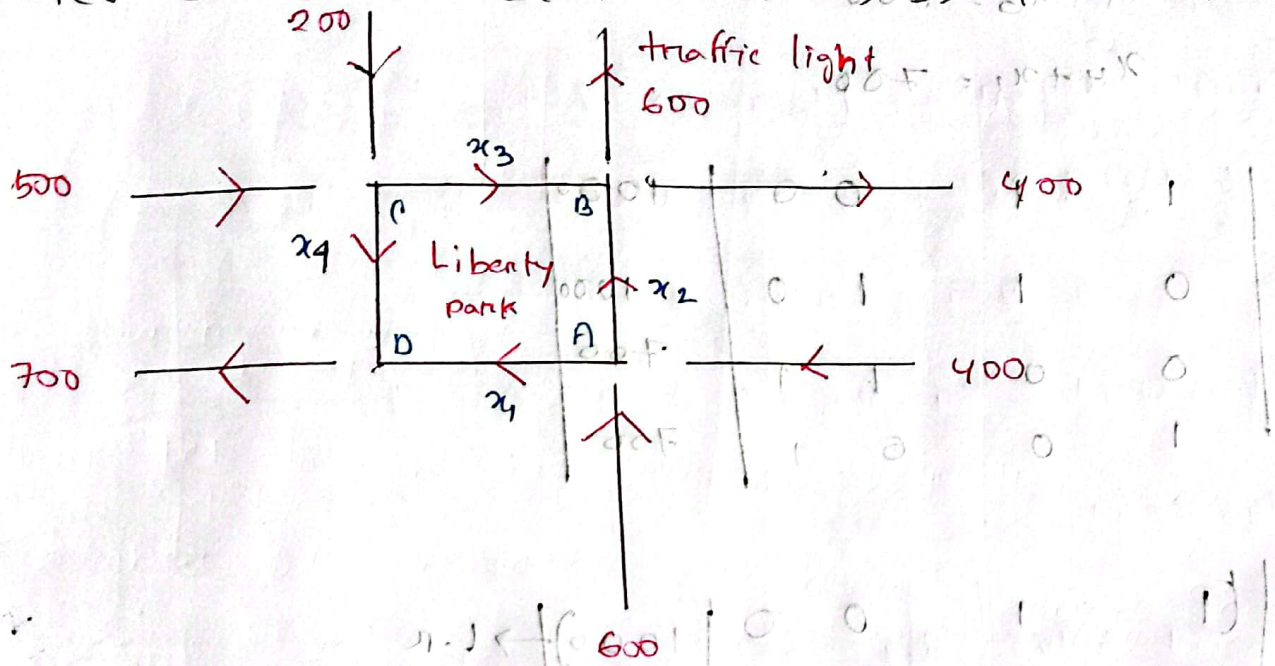
$x = 1700 - 1100 = 600$

(b) Assuming that the traffic light has been set to balance the total flow in and out of the complex. What can you say about the average number of vehicles per hour that will flow along

the streets that border the complex?

Sol:

এখানে border line গুলোতে কি পরিমাণ গাড়ি যাবে তা দেওয়া হবে, x_1, x_2, x_3, x_4 এর value



Node	flowing in	flowing out
A	$600+400$	x_1+x_2
B	x_3+x_2	$600+400$
C	$500+200$	x_4+x_3
D	x_4+x_1	700

$x_1+x_2 = 1000$ $\quad$ $x_4+x_3 = 700$

$x_3+x_2 = 1000$ $\quad$ $x_4+x_1 = 700$

System of linear equations

$$x_1 + x_2 = 1000$$
$$x_3 + x_2 = 1000$$
$$x_4 + x_3 = 700$$
$$x_4 + x_1 = 700$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 1 & 0 & 0 & 1 & 700 \end{array}\right|$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 1-1 & 0-1 & 0 & 1 & 700-1000 \end{array}\right| \begin{array}{l} \to L.R \\ \\ \\ R_4' = R_4 - R_1 \end{array}$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 0 & -1 & 0 & 1 & -300 \end{array}\right| \begin{array}{l} \\ \to L.R \\ \\ \\ \end{array}$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 0 & -1+1 & 0+1 & 1 & -300+1000 \end{array}\right| \begin{array}{l} \\ \rightarrow L.n \\ \\ \\ \end{array}$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 0 & 0 & 1 & 1 & 700 \end{array}\right| \begin{array}{l} \\ \\ \rightarrow L.n. \\ \\ \end{array}$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 0 & 0 & 1-1 & 1-1 & 700-700 \end{array}\right| \begin{array}{l} \\ \\ \rightarrow L.n \\ \\ \end{array}$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1000 \\ 0 & 1 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 1 & 700 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right|$$

equation : 3
variable : 4

$x_4 = t$

$x_3 = 700 - x_4$

$x_3 = 700 - t$

$x_1 + x_2 = 1000$

$x_2 + x_3 = 1000$

$x_3 + x_4 = 700$

$x_2 = 1000 - x_3$

$x_2 = 1000 - 700 + t$

$= 300 + t$

$x_1 = 1000 - x_2$

$x_1 = 1000 - 300 - t = 700 -$

where $0 \le t \le 700$.

এই matn এ x_1, x_2, x_3, x_4 এ

দিকে বের করার জন্য আমরা আগে ~~NODE~~

যে NODE গুলোতে flow in হচ্ছে সেই

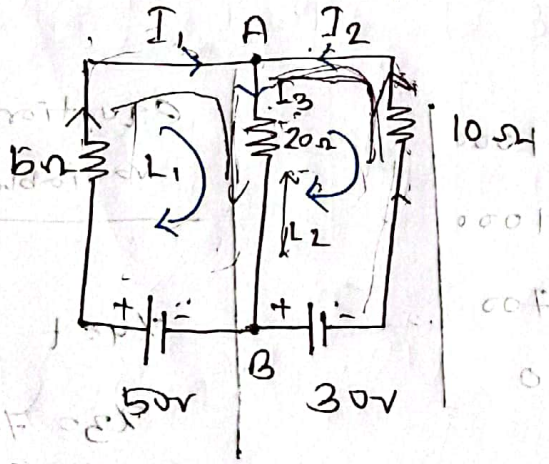
গুলো যেহেতুকরে ~~0~~ সেই NODE গুলো flow out

mark করতে হবে। যেমন A, $\longleftarrow$

এর flow out x_4, x_3 এবং C এর flow out

x_1, x_2।

(3) circuit solve By network analysis:



NODE A তে Kirchhoff's current Law $I_1 + I_2 = I_3$

এখানে Kirchhoff voltage law তে loop

যে দিকে current এর flow এর দিক

দিকে সেই এই current কে (-) হয়

লিখতে হয়,

$$20I_3 - 50 + 5I_1 = 0$$

Loop 1: $20I_3 + 5I_2 = 50$

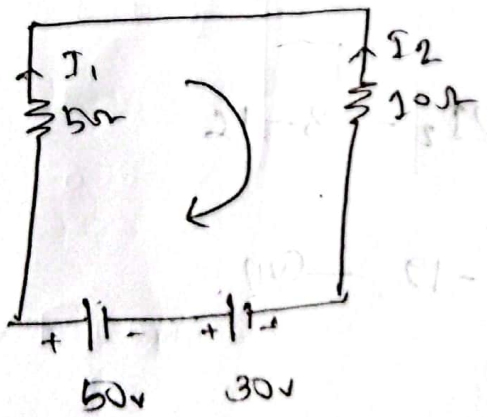
Loop 2: $-10I_2 - 20I_3 - 30 = 0$

$-10I_2 - 20I_3 = 30$

$10I_2 + 20I_3 = -30$

এখানে Loop 2 এর Loop এর দিক, ~~[illegible]~~ ~~এখানে~~ 10Ω এবং 20Ω দিয়ে যে current flow করে ~~যে দিকে~~ তার দিকের এর উল্টো। তাই $-10I_2$, এবং $-20I_3$ হয়েছে। তার voltage কে '+' ই ধরা হয়েছে।

এবং খেয়াল রাখতে হবে কোন এই যে 20V এর এর তার ধরানোর পর যে close circuit হবে, তার একটা Loop এর equation কে করবো।



$-80 + 5I_1 - 10I_2 = 0$

বা

$I_1 + I_2 - I_3 = 0$ —(i)

$5I_1 + 20I_3 = 50$

$I_1 + 4I_3 = 10$ —(ii)

$10I_2 + 20I_3 = -30$

$I_2 + 2I_3 = -3$ —(iii)

$5I_1 - 10I_2 = 80$

$I_1 - 2I_2 = 16$ —(iv)

একটা variable এর equation থেকে equation +/- করে এটা নিয়ে আসতে হবে।

(ii) + (i),

$I_1 + 4I_3 + I_2 + 2I_3 = 10 - 3$

$I_1 + I_2 + 6I_3 = 7$ —(v)

(iii) + (iv),

$I_2 + 2I_3 + I_1 - 2I_2 = -3 + 16$

$I_1 + 2I_3 - I_2 = 13$ —(vi)

application of system of equation.

$I_1 + I_2 - I_3 = 0$ —(I)

$I_1 + I_2 + 6I_3 = 7$ —(V)

$I_1 - I_2 + 2I_3 = 13$ —(VI)

<u>System of Linear equations</u>

$$\left|\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 1 & 6 & 7 \\ 1 & -1 & 2 & 13 \end{array}\right|$$

$$= \left|\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1-1 & 1-1 & 6+1 & 7 \\ 1-1 & -1-1 & 2+1 & 13 \end{array}\right|$$

$R_2' = R_2 - R_1$

$R_3' = R_3 - R_1$

$$= \left|\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 7 & 7 \\ 0 & -2 & 3 & 13 \end{array}\right|$$

$$= \left|\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 3 & 13 \\ 0 & 0 & 7 & 7 \end{array}\right|$$

$R_3 \to R_2$

$\cancel{x_1 + x_2 - x_3}$

$I_1 + I_2 - I_3 = 0$

$-2I_2 + 3I_3 = 13$

$7I_3 = 7$

$I_3 = 1$

$-2I_2 + 3 \times 1 = 13$

$-2I_2 = 13 - 3$

$I_2 = \frac{10}{-2}$

$= -5$

$I_1 + I_2 - I_3 = 0$

$\cancel{I_1 + 11 - 1 = 0}$

$\cancel{I_1 = -10}$

$I_1 - 5 - 1 = 0 \Rightarrow I_1 = 6$

$\cancel{(Ans: I_1, I_2, I_3 = -10, +1, 1)}$

(Ans: 6, −5, 1)

Math → Chemg

$CH_4 + O_2 \rightarrow CO_2 + H_2O$

এখানে ২টা মূলের মোল বিক্রিয়া চলার যে পরিমাণ দেওয়া হয়েছে সে পরিমাণের output হিসাবে হচ্ছে না। তাই বিক্রিয়কে O এর পরিমাণ 2

But output এ উৎপাদে O এর পরিমাণ 3.

math-chem 2

~~$x_1(CH_4) + O_2 \rightarrow CO_2 + H$~~

$$x_1(CH_4) + x_2(O_2) \rightarrow x_3(CO_2) + x_4(H_2O)$$

বিক্রিয়ক – উৎপাদ

⇒ C (কার্বন) এর ক্ষেত্রে $x_1 = x_3 \rightarrow x_1 - x_3 = 0$

⇒ H (হাইড্রোজেন) " " $4x_1 = 2x_4 \rightarrow 4x_1 - 2x_4 = 0$

$2x_1 - x_4 = 0$

⇒ O (অক্সিজেন) " " $2x_2 = 2x_3 + x_4 \rightarrow 2x_2 - 2x_3 - x_4 = 0$

এখানে বিক্রিয়ক এবং উৎপাদে যেহেতু কোনো মৌল কি পরিমাণ আছে, সেটা count করবো না এবং x_1, x_2, x_3, x_4 এর সাথে গুণ দিবো।

$x_1 - x_3 = 0$

~~$2x_4 = 0$~~

$2x_1 - x_4 = 0$

$2x_2 - 2x_3 - x_4 = 0$

$$= \left|\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right|$$

$$= \left|\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right|$$

$$= \left|\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2-2 & 0 & 0+2 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right|$$

$R_2' = R_2 - 2R_1$

$$= \left|\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right|$$

$$= \left|\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{array}\right|$$

$x_1 - x_3 = 0$

$2x_2 - 2x_3 - x_4 = 0$

$2x_3 - x_4 = 0$

equation 3

variable 4

1

~~$x_4 = t$~~

$x_3 = t$

$x_1 = x_3$

$x_1 = t$

$x_4 = 2x_3$

$x_4 = 2t$

$2x_2 = 2x_3 + x_4$

$= 2t + 2t$

$x_2 = \frac{4t}{2} = 2t$

$x_1 = t$

$x_3 = t$

$x_4 = 2x_2 = 2t$

যেহেতু t arbitary, এর একটা fixed value count করতে হবে,

ধরি Let $t = 1$;

$\therefore x_1 = x_3 = 1$, $x_4 = x_2 = 2$,

$x_1(CH_4) + x_2(O_2) \rightarrow x_3(CO_2) + x_4(H_2O)$

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$.

$HCl + Na_3PO_4 \rightarrow H_3PO_4 + NaCl$

$x_1(HCl) + x_2(Na_3PO_4) \rightarrow x_3(H_3PO_4) + x_4(NaCl)$

$\rightarrow H$ $\quad x_1 = 3x_3 \quad \rightarrow 3x_3 - x_1 = 0$

$\rightarrow Cl$ $\quad x_1 = x_4 \quad \rightarrow x_1 - x_4 = 0$

$\rightarrow Na$ $\quad 3x_2 = x_4 \quad \rightarrow 3x_2 - x_4 = 0$

$\rightarrow P$ $\quad x_2 = x_3 \quad \rightarrow x_2 - x_3 = 0$

$\rightarrow O$ $\quad 4x_2 = 4x_3 \quad \rightarrow x_2 - x_3 = 0$.

$$3x_3 - x_1 = 0$$
$$x_1 - x_4 = 0$$
$$3x_2 - x_4 = 0$$
$$x_2 - x_3 = 0$$

$$\left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right| \rightarrow L_n$$

$$= \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ -1-1 & 0 & 0+3 & -1 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right| \quad R_2 \to R_2 + R_1$$

$$= \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right|$$

$$= \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right| \rightarrow L_n \Rightarrow \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 3-3 & -3 & 0+1 & 0 \end{array}\right| \quad R_4 \to 3R_4 - R_2$$

$$= \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & [3 & -1 & 0] \\ 0 & 0 & -3 & 1 & 0 \end{array}\right| \rightarrow L.n$$

$$= \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & -3+3 & 1-1 & 0 \end{array}\right| \quad R_4' = R_4 + R_3$$

$$= \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left|\begin{array}{cccc|c} -1 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right| \end{array} \Rightarrow \begin{array}{l} -x_1 + 3x_3 = 0 \\ 3x_2 - x_4 = 0 \\ 3x_3 - x_4 = 0 \end{array} \quad \left| \begin{array}{l} \text{equation} = 3 \\ \text{variable} = 4 \\ \hline 1 \end{array} \right.$$

$\cancel{x_4 = t}$

$$= \left|\quad\right|$$

$x_4 = t$

$3x_3 - x_4 = 0.$

$\Rightarrow x_3 = \frac{t}{3}$

$\cancel{\Rightarrow x_3 = \frac{t}{3}}$

$x_2 = \frac{x_4}{3}$

$\Rightarrow x_2 = \frac{t}{3}$

$x_1 = x_4 = t.$

$x_2 = x_3 = \frac{t}{3}$

$-x_1 + 3x_3 = 0$

$\Rightarrow -x_1 + 3 \times \frac{t}{3} = 0$

$\Rightarrow x_1 = t$

$t = 3$

$x_1 = x_4 = 3$

$x_2 = x_3 = \frac{t}{3} = \frac{3}{3} = 1$

$$x_1(HCl) + x_2(Na_3PO_4) \rightarrow x_3(H_3PO_4) + x_4(NaCl)$$

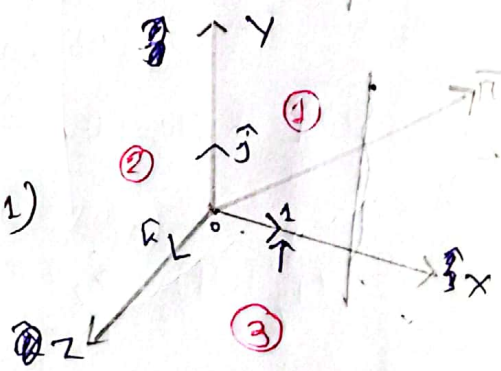
$$3HCl + Na_3PO_4 \rightarrow H_3PO_4 + 3NaCl$$

(Ans)

Vector space:

3D Vector: যে সব variable এর মানের সাথে দিক থাকে

ex: $\vec{r}$



3D: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $(\hat{i} = \hat{j} = \hat{k} = 1)$

$\vec{r} = (x, y, z)$ ← position vector

$= \langle x, y, z \rangle$ এভাবেও লিখা যায়

3D এর, 3D এর space হলো 3.

ND: $\vec{A} = \langle x_1, x_2, x_3, \ldots, x_n \rangle$

এর Dymansion এবং Space n.

Vector space: কোনো set এর Domain বা element হিসেবে vector থাকলে তাকে বা সেই set কে vector space বলে।

linear combination:

If $S = \{\vec{v_1}, \vec{v_2}, \vec{v_3}, \ldots, \vec{v_n}\}$ is a set of vectors in n-space, and $\vec{v_1}, \vec{v_2}, \vec{v_3}, \ldots, \vec{v_n}$ can be expressed by one vector $\vec{v}$ such that

$\vec{v} = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} + \ldots + c_n\vec{v_n}$, where $c_1, c_2, c_3, \ldots, c_n$ are constants then v is called the linear combination of $\vec{v_1}, \vec{v_2}, \vec{v_3}, \ldots, \vec{v_n}$

(*) Consider $\vec{u} = (1,2,-1)$, $\vec{v} = (6,4,2)$ in R^3.

Show that $\vec{w} = (9,2,7)$ is a linear combination of $\vec{u}$ & $\vec{v}$. also show that $w' = (4,-1,8)$ is a linear combination of $\vec{u}$ & $\vec{v}$.

$\Rightarrow$ From the definition of linear combination

$$\vec{w} = c_1\vec{u} + c_2\vec{v}$$

$$(9,2,7) = c_1(1,2,-1) + c_2(6,4,2)$$

$$(9,2,7) = (c_1, 2c_1, -c_1) + (6c_2, 4c_2, 2c_2)$$

$$(9,2,7) = (c_1+6c_2, 2c_1+4c_2, -c_1+2c_2)$$

$c_1 + 6c_2 = 9$ —(1)

$2c_1 + 4c_2 = 2$

$\Rightarrow c_1 + 2c_2 = 1$ —(ii)

$-c_1 + 2c_2 = 7$ —(iii)

~~linear eq~~ system of linear equation:

$$\left|\begin{array}{cc|c} 1 & 6 & 9 \\ 1 & 2 & 1 \\ -1 & 2 & 7 \end{array}\right|$$

$c_1 + 6c_2 = 9$

$-4c_2 = -8$

$c_2 = 2$

$$= \left|\begin{array}{cc|c} 1 & 6 & 9 \\ 1-1 & 2-6 & 1-9 \\ -1+1 & 2+6 & 7+9 \end{array}\right| \quad \begin{array}{l} R_2' = R_2 - R_1 \\ R_3' = R_3 + R_1 \end{array}$$

$c_1 + 12 = 9$

$c_1 = -3$

$\vec{w} = -3\vec{u} + 2\vec{v}$

$$= \left|\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -4 & -8 \\ 0 & 8 & 16 \end{array}\right|$$

$$= \left|\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -4 & -8 \\ 0 & 8-8 & 16-16 \end{array}\right| \quad R_3' = R_3 + 2R_2$$

$$= \left|\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -4 & -8 \\ 0 & 0 & 0 \end{array}\right|$$

(2) $\vec{w}' = (4, -1, 8)$

$\vec{w}' = c_1\vec{u} + c_2\vec{v}$

$(4, -1, 8) = c_1(1, 2, -1) + c_2(6, 4, 2)$

$\Rightarrow (4, -1, 8) = (c_1, 2c_1, -c_1) + (6c_2, 4c_2, 2c_2)$

$\Rightarrow (4, -1, 8) = (c_1 + 6c_2, 2c_1 + 4c_2, -c_1 + 2c_2)$

$c_1 + 6c_2 = 4$

$2c_1 + 4c_2 = -1$

$-c_1 + 2c_2 = 8$

$$\left[\begin{array}{cc|c} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 8 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 2-2 & 4-12 & -1-8 \\ -1+1 & 2+6 & 8+4 \end{array}\right] \quad \begin{array}{l} R_2' = R_2 - 2R_1 \\ R_3' = R_3 + R_1 \end{array}$$

$$= \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 12 \end{array}\right]$$

$$\left|\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 0 & 3 \end{array}\right|$$

$c_1 + 6c_2 = 4$

$-8c_2 = -9$

$0 \neq 3$ (which is not possible)

This system has no solution, so, $\vec{w}$ can not be expressed as a linear combination of $\vec{u}$ & $\vec{v}$.

Dependency and Independency:-

Dependency: এটা আসে linear combination থেকে,

$S = \{\vec{v_1}, \vec{v_2}, \vec{v_3}, \ldots\ldots, \vec{v_n}\}$

linear combination: $\vec{v} = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} + \ldots + c_n\vec{v_n}$

If; $c_1 = c_2 = c_3 = \ldots\ldots = c_n = 0$ হয় then vector are independent.

যদি $(c_1 = c_2 = c_3 = \ldots\ldots = c_n) \neq 0$ হয় তাহলে vector dependend.

(1) Determine wheather $\vec{V_1} = (1, 2, 2, -1)$, $\vec{V_2} = (4, 9, 9, -4)$, $\vec{V_3} = (5, 8, 9, -5)$; are linearly dependent or independent.

Soln

$\vec{V_1} = (1, 2, 2, -1)$

$\vec{V_2} = (4, 9, 9, -4)$

$\vec{V_3} = (5, 8, 9, -5)$

$c_1\vec{V_1} + c_2\vec{V_2} + c_3\vec{V_3} = 0$

$\Rightarrow c_1(1, 2, 2, -1) + c_2(4, 9, 9, -4) + c_3(5, 8, 9, -5) = 0$

$\Rightarrow (c_1, 2c_1, 2c_1, -c_1) + (4c_2, 9c_2, 9c_2, -4c_2) + (5c_3, 8c_3, 9c_3, -5c$

$\Rightarrow (c_1 + 4c_2 + 5c_3,\ 2c_1 + 9c_2 + 8c_3,\ 2c_1 + 9c_2 + 9c_3,\ -c_1 - 4c_2 - 5c_3$

$\Rightarrow$ $c_1 + 4c_2 + 5c_3 = 0$

$2c_1 + 9c_2 + 8c_3 = 0$

$2c_1 + 9c_2 + 9c_3 = 0$

$-c_1 - 4c_2 - 5c_3 = 0$

$$= \left|\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 2 & 9 & 8 & 0 \\ 2 & 9 & 9 & 0 \\ -1 & -4 & -5 & 0 \end{array}\right|$$

$$= \left|\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 2-2 & 9-8 & 8-10 & 0 \\ 2-2 & 9-8 & 9-10 & 0 \\ -1+1 & -4+4 & -5+5 & 0 \end{array}\right|$$

$R_2' = R_2 - 2R_1$

$R_3' = R_3 - 2R_1$

$R_4' = R_4 + R_1$

$$= \left|\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right| \rightarrow 1.n$$

$$= \left|\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 1-1 & -1+2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

$R_3' = R_3 - R_2.$

$$= \left|\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

$c_1 + 4c_2 + 5c_3 = 0$

$c_2 - 2c_3 = 0$

$c_3 = 0$

$c_2 - 0 = 0$

$c_1 = c_2 = 0$

$c_1 = c_2 = c_3 = 0$

$c_1 = c_2 = c_3 = 0$, so the vectors are Independent.

(2) Determine wheather $\vec{v_1} = (0, 3, 1, -1)$, $\vec{v_2} = (6, 0, 5, 1)$, $\vec{v_3} = (4, -7, 1, 3)$ are linearly dependent or Independent.

Sol:- $c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = 0$

$\cancel{c_1(0, 3, 1, -1) + c_2(4, -7, 1, 3) + c_3}$

$c_1(0, 3, 1, -1) + c_2(6, 0, 5, 1) + c_3(4, -7, 1, 3) = 0$

or, $(0, 3c_1, c_1, -c_1) + (6c_2, 0, 5c_2, c_2) + (4c_3, -7c_3, c_3, 3c_3) = 0$

or, $(6c_2 + 4c_3, 3c_1 - 7c_3, c_1 + 5c_2 + c_3, -c_1 + c_2 + 3c_3) = 0$

$6c_2 + 4c_3 = 0$

$3c_1 - 7c_3 = 0$

$c_1 + 5c_2 + c_3 = 0$

$-c_1 + c_2 + 3c_3 = 0$

$$= \left|\begin{array}{ccc|c} 0 & 6 & 4 & 0 \\ 3 & 0 & -7 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right|$$

$$= \left|\begin{array}{ccc|c} 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right|$$

$$= \left|\begin{array}{ccc|c} 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ 3-3 & 15 & 3+7 & 0 \\ -3+3 & 3 & 9-7 & 0 \end{array}\right|$$

$R_3' = 3R_3 - R_1$

$R_4' = 3R_4 + R_1$

$$= \left|\begin{array}{ccc|c} 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 15 & 10 & 0 \\ 0 & 3 & 2 & 0 \end{array}\right| \Rightarrow \left|\begin{array}{ccc|c} 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 90-90 & 60-60 & 0 \\ 0 & 6-6 & 4-4 & 0 \end{array}\right|.$$

$R_3' = 6R_3 - 15$

$R_4' = 2R_4 +$

$$= \left|\begin{array}{ccc|c} 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

equation = 2

variable = 3

$c_3 = t$

$3c_1 - 7c_3 = 0$

$6c_2 - 4c_3 = 0$

$6c_2 = 4c_3$

$c_2 = \frac{4t}{6}$

$= \frac{2t}{3}$

$c_1 = \frac{7c_3}{3}$

$= \frac{7t}{3}$

$c_1 \neq c_2 \neq c_3 \neq 0$

So, vectors are ~~not dependent~~ dependent.

Eigen value and Eigen vectors

A = Square matrix ex: $\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_2 \end{bmatrix}$

I = Same size এর Identity matrix থাকবে

ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

এই I matrix কে একটা constant λ দিয়ে গুণ দিলে যে একটা matrix λI পাবো তা থেকে A matrix কে বাদ দেওয়ার পর যে matrix পাবো তার Determination বের করলে যে value পাবো তা যদি zero হয় তাহলে λ হলো Eigen value.

$|\lambda I - A| = 0$ Then λ is eigen value.

প্রত্যেকটা Eigen value এর একটা Eigen vector পাবো। যদি same value দুইবার / তিনবার পাই ~~same~~ তাহলে same value এর কয়টা ~~দুইটা vector~~ আলাদা আলাদা vector পাবো।

$$\begin{bmatrix} 0 & 0 & \lambda \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \lambda \\ 0 & \lambda & 0 \\ \lambda & 0 & 0 \end{bmatrix}$$

find the eigen value and eigen vector of the matrix

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

<u>Sol:-</u> for eigen value, λ

Characterestics equation

$$= |\lambda I - A|$$

$$= \left| \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix} \right|$$

$$= \left| \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} \right| = 0$$

$$= \begin{vmatrix} \lambda & 0 & +2 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda-3 \end{vmatrix}$$

$\cancel{= \lambda(\lambda-2)(\lambda-3) - 0 + (2 \times -1 \times (\lambda-2))}$

$= \lambda\{(\lambda-2)(\lambda-3) - 0\} - 0 + 2\{0 - (-1 \times (\lambda-2))\}$

$= \lambda(\lambda-2)(\lambda-3) + 2(\lambda-2)$

$= (\lambda-2)(\lambda^2 - 3\lambda + 2)$

$\lambda = 2,$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda-2) - 1(\lambda-2) = 0$

$(\lambda-2)(\lambda-1) = 0$

$\lambda = 2, \lambda = 1$

$\lambda = 1, 2, 2$

So, eigen values are 1 and 2,

let the eigen vector is $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$[\lambda I - A] \cdot X = 0$

X একটা matrix এ column এবং row গুলো element এর এই গুলো row থাকবে,

$[\lambda I - A] \cdot X = 0$

$$\Rightarrow \begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda-3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

putting $\lambda = 2$ in (1)

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1-2 & -1 \\ -1 & 0 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\left.\begin{aligned} x + 2z &= 0 \\ -x - y - z &= 0 \\ -x + 2z &= 0 \end{aligned}\right\} \quad \begin{aligned} x + 2z &= 0 \\ x + y + z &= 0 \end{aligned}$$

equation - 2

Variable 3

$z = t$

$x = -2t$

$y - 2t + t = 0$

$y = t$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t \\ t \\ t \end{bmatrix} = t \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ is an eigen vector for $\lambda = 1$

Putting $\lambda = 2$ in (1)

$$\begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\left.\begin{aligned} 2x + 2z &= 0 \\ -x - z &= 0 \\ -x - z &= 0 \end{aligned}\right\} \quad x + z = 0$$

variable $\to 3$

equation $\to 1$

2

$x = t$

$z = -t$

$y = s$

$$X = \begin{bmatrix} t \\ s \\ -t \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ -t \end{bmatrix} + \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix}$$

$$= t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(2). find eigen value and eigen vectors

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

<u>Sol:</u>- for eigen value, λ

Characterestics equation : $|\lambda I - A| = 0$

$$\Rightarrow \left| \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{vmatrix} \right| = 0$$

$$\Rightarrow \left| \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{vmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{vmatrix} = 0$$

$$\Rightarrow \{\lambda-4\}\{(\lambda-1)(\lambda-1)\} - 0 + (-1 \times \{0 - 2 \times (\lambda-1)\} = 0$$

$$\Rightarrow (\lambda-4)\cdot(\lambda-1)^2 + 2(\lambda-1) = 0$$

$$\Rightarrow (\lambda-1)\{(\lambda-4)\cdot(\lambda-1)+2\} = 0$$

$$\Rightarrow (\lambda-1) = 0 \quad \{\lambda^2 - 4\lambda - \lambda + 4 + 2\} = 0$$

$\lambda = 1$

$$\Rightarrow \lambda^2 - 5\lambda + 6$$

$$\cancel{\Rightarrow \lambda^2 - 6\lambda + \lambda + 6}$$

$$\Rightarrow \lambda^2 - 3\lambda - 2\lambda + 6 \Rightarrow \lambda(\lambda-3) - 2(\lambda-3)$$

$$\Rightarrow (\lambda-3)(\lambda-2)$$

$\lambda = 1, \lambda = 2, \lambda = 3$

$\lambda = 1, 2, 3,$

So, eigen values are 1, 2, 3

let the eigen vector is $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$[\lambda I - A] . X = 0$

$$\begin{bmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \text{—(1)}$$

putting $\lambda = 1$, in (1)

$$\begin{bmatrix} 1-4 & 0 & -1 \\ 2 & 1-1 & 0 \\ 2 & 0 & 1-1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-3x - z = 0$

$2x = 0$

$2x = 0$

$-3x - z = 0$

equation 2

variable 3

1

$y = t$

$x = z = 0$

$y = t$

$-3x - z = 0$

$2x = 0$

$x = 0$

$z = 0$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

~~(3) Find eigen value and eigen vectors~~

putting $\lambda = 2$, in (1)

$$\begin{bmatrix} 2-4 & 0 & -1 \\ 2 & 2-1 & 0 \\ 2 & 0 & 2-1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -2 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x - z = 0$

~~$-2x + z = 0$~~

$\Rightarrow 2x + z = 0$

$2x + y = 0$

$2x + z = 0$

$2x + y = 0$

$2x + z = 0$

$-2x + z = 0$

$\Rightarrow 2x - z = 0$

$2x + y = 0$

$2x + z = 0$

$2x - z = 0$

$2x - y = 0$

equation 2

variable = 3

1

$x = t$

$z = -2t$

$-2t = y$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ -2t \\ -2t \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$$

~~Put~~ Putting $\lambda = 3$, in (1)

$$\begin{vmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{vmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{vmatrix} 3-4 & 0 & -1 \\ 2 & 3-1 & 0 \\ 2 & 0 & 3-1 \end{vmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$= \begin{vmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-x - z = 0 \Rightarrow x + z = 0$

$= \quad 2x + 2y = 0$

$2x + 2z = 0$

$x + z = 0$

$x + z = 0$

$x + y = 0$

$x = t$

$z = y = -t$

equation 2

variable 3

2

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ -t \\ -t \end{bmatrix} = t \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

) Find eigen value and eigen vector

$$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Sol: for eigen value, λ

characteristics equation $|\lambda I - B|$

$$\Rightarrow \left| \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} \right| = 0$$

$$\Rightarrow \left| \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} \right| = 0$$

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$$\Rightarrow \begin{vmatrix} \lambda-0 & 0-1 & 0-1 \\ 0-1 & \lambda-0 & 0-1 \\ 0-1 & 0-1 & \lambda-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \lambda & -1 & -1 \\ -1 & \lambda & -1 \\ -1 & -1 & \lambda \end{vmatrix} = 0 \quad \text{—(1)}$$

$$\Rightarrow \lambda\{(\lambda^2-1) - (-1\times\{-\lambda-1\}) + (-1\times\{1+\lambda\}] = 0$$

$$\Rightarrow \lambda^3 - \lambda - \lambda - 1 - 1 - \lambda = 0$$

$$\Rightarrow \lambda^3 - 3\lambda - 2 = 0$$

$$\cancel{= \lambda^2(\lambda-3)}.$$

$$\cancel{\Rightarrow \lambda^3 - 3\lambda^2 + 3\lambda^2 - 9\lambda + 6\lambda}$$

$$\cancel{\Rightarrow \lambda^2(\lambda-3) + 3\lambda(\lambda-3) + 6(\lambda-3)}$$

$$\lambda^3 - 3\lambda - 2 = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 + 2\lambda^2 - 4\lambda + \lambda - 2$$

$$\Rightarrow \lambda^2(\lambda-2) + 2\lambda(\lambda-2) + 1(\lambda-2) = 0$$

$$\Rightarrow (\lambda-2)(\lambda^2+2\lambda+1) = 0$$

$$(\lambda-2)(\lambda+1)^2 \qquad \therefore \lambda = -1, -1, 2$$

so, eigen values are -1 and 2

let the eigen vector is $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$[\lambda I - B] \cdot X = 0$$

$$\begin{bmatrix} \lambda & -1 & -1 \\ -1 & \lambda & -1 \\ -1 & -1 & \lambda \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \text{——— (11)}$$

putting $\lambda = -1$, in (11);

$$B = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left.\begin{aligned} -x - y - z &= 0 \\ -x - y - z &= 0 \\ -x - y - z &= 0 \end{aligned}\right\} \quad x + y + z = 0$$

variable 3, equation 1

$x = t$

$z = s$

$y = -t - s$

$$\cancel{X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ -t - s \\ s \end{bmatrix}} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ -t - s \\ s \end{bmatrix}$$

চরণ

X কে আলাদা matrix করলে,

$$X = \begin{bmatrix} t \\ -t-s \\ s \end{bmatrix} = \begin{bmatrix} t \\ -t \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -s \\ s \end{bmatrix}$$

$$= t\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

So, $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ are eigen vectors.

putting $\lambda = 2$, in (II)

$$\begin{vmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$$

$\cancel{2x - z - y}$ $\quad 2x - y - z = 0$ —(I)

$-x + 2y - z = 0$ —(II)

$-x - y + 2z = 0$ —(III)

$\cancel{y = -x + 2z}$

$\cancel{2x - y - z = 0}$

$\cancel{2x -}$

$$\begin{cases} 2x - y - z = 0 & (I) \\ -2x + 4y - 2z = 0 & (II) \\ 3y - 3z = 0 \end{cases}$$

$(I)\times 2+$

$$y = z$$

$$x = y = z = 0$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ הוא הפתרון הטריוויאלי.

$(I) + 2\times(II)$

$$\begin{cases} -x + 2y - z = 0 \\ (+)\ -2x - 2y + 4z = 0 \\ -3x + 3z = 0 \end{cases}$$

$$x = z$$

$$\begin{cases} y = x + 2z \\ 2x - y - z = 0 \\ 2x - x + 2z - z = 0 \end{cases}$$

$x + z = 0$. $(II)\times 2+(I)$

$$\begin{cases} 2x - y - z = 0 & (I) \\ -2x + 4y - 2z = 0 & (II) \end{cases}$$

$(II)\times 2+$

$$\begin{cases} 3y - 3z = 0 \\ y = z \end{cases}$$

$$x = y = z = 0$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ הוא הפתרון הטריוויאלי.