Basic counting principles

Parmutation and combination!

How many different 8-letter passwords are there

abcd   cdab (sequence ~~বদলে~~ -এর উপর depend করে, এবং password change হয়ে যাবে)

same না।

~~Homo~~ How many possible ways are there to pick 11 soccer players out of a 20 players team?

যদি (Rahim, Karim) অথবা (Karim, Rahim) same থাকে- একটা team এ এখানে sequence matter করে না।

The sum rule:

If a task can be done in n_1 ways and a second task in n_2 ways, and if these two tasks cannot be done at the same time, then there are n_1+n_2 ways to do either task.

example:

Suppose that you are in a restaurant and are going to have either soup or salad but not both. There are two soups and four salads on the menu. How many choices do you have?

By the sum rule, you have $2+4=6$ choices.

Generalized sum rules:

If we have tasks $T_1, T_2, \ldots T_m$ that can be done in $n_1, n_2, \ldots n_m$ ways, but cannot be done both ways, then

product rules:

Suppose that a ~~prode~~ procedure can be broken down into two successive tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are $n_1 n_2$ ways to do the procedure

ex: for (i=0; i<5; i++)
{ for (j=0; j<5; j++)
{ print ("Hello"); }

তাহলে; 5×5 = ... হবে

ex: How many different license plates are there that containing exactly three english letters?

Ans: ab c, b x; q, a, a

So একটার জন্য 26 টা Letter আছে

$\frac{\_}{26}\ \frac{\_}{26}\ \frac{\_}{26}$

$(26)^3$ ⟶ way আছে।

The sum and product rules can be also be phrased in terms of set theory.

Sum rule: Let $A_1, A_2, \ldots, A_m$ be disjoint sets. Then the number of ways to choose any element from one of these sets is

$$|A_1 \cup A_2 \cup \cdots \cup A_m| = |A_1| + |A_2| + \cdots + |A_m|$$

product rule: $A_1, A_2, \ldots, A_3 \leftarrow$ finite set

$$|A_1 \times A_2 \times \cdots \times A_m| = |A_1| \cdot |A_2| \cdot |A_3| \cdots |A_m|$$

Inclusion – Exclusion:

~~Qu~~ How many bit strings of length 8 either start with a 1 or end with 00?

$$\overset{0/1}{2} \times \overset{0/1}{2} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$(0/1) \to 2$

2^8 বেশিবার গুণা হবে

$\boxed{1}0\,0\,0\,0\,0\,1\,0\,\boxed{0\,0}$

10000010 বা 00001000

Task 1: Construct a string of length 8 that starts with a 1.

$$\underline{1}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}$$

Product rules: $1 \times 2^7 = 128$ ways
P.R.1

Task 2: Last two digit 0,0 হলে, তাহলে

$$\underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underline{0}\ \underline{0}$$

P.R.2 → $1 \times 2^6 = 64$ ways

Sum rules: $\cancel{2^7 + 2^6 = 128 + 64}$
$\cancel{= 192}$

এখন task 1 solve করেছি এখন নিচের
এর যে 1 হয়েছে কিন্তু last এর fixed
তাহলে হবে না কারণ এর মধ্যে ও last

অন্যআরেকটা string আছে যা same কোন task 2 (মতো আছে?

দুইটাতেই common কিছু string থেকে থাকবে

যদি sum rules করি তাহলে common গুলো double গুনা হয়ে যাবে, ~~common~~ common গুলো বাদ দিতে হবে,

common হয়ে থাকে যেমন

$$\underline{1}\ \ \underset{2}{\_}\ \ \underset{2}{\_}\ \ \underset{2}{\_}\ \ \underset{2}{\_}\ \ \underset{2}{\_}\ \ \underline{0}\ \ \underline{0}$$

$2^5 = 32$ ways of common string থাকে তাই,

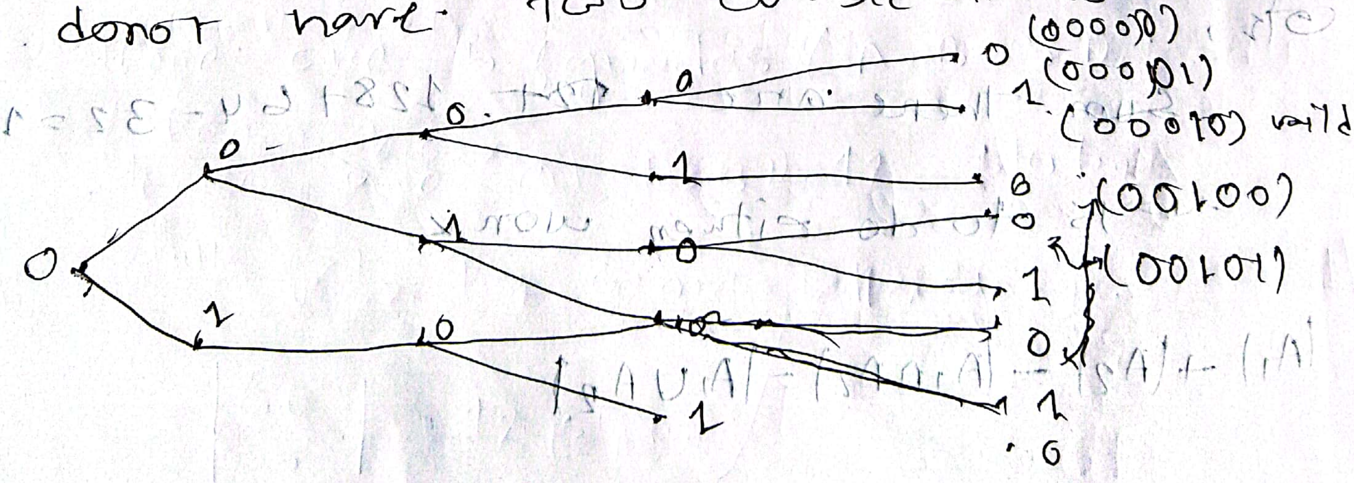
So there are $128 + 64 - 32 = 160$ ways to do either work

$$|A_1| + |A_2| - |A_1 \cap A_2| = |A_1 \cup A_2|$$

Tree diagrams:

A tree diagram is a tool in the fields of general mathematics, probability, and statistics that helps calculate the number of possible outcomes of ~~an~~ an event or problem and to cite those potential outcomes in an organised ways.

How many bit strings of length four donot have two consecutive 1s



The pigeonhole Principle:

If $(K+1)$ or more Objects are placed into K boxes. Then there is at least one box containing two or more of the objects.

ex: If there are 11 players in a soccer team that wins 12-0. There must be ~~be~~ at least one players (in the team) who scored at least twice. [illegible]

ex: If you have ~~a~~ 6 classes from sunday to Thursday. There must be at least one day on which you have at least two classes.

generalized pigeonhole principle: If N Objects are placed into K boxes, then there is at least one box containing at least $\lceil N/K \rceil$ of the Objects.

ceiling

$$\left\lceil \frac{N}{K} \right\rceil = \left\lceil \frac{15}{7} \right\rceil$$

$= 2.14$

$= 3$ হয় at least আমরা

celling function upper limit যায়

floon " lower "

$2.14 \rightarrow 3$ (celling)

$2 \leftarrow$ (floor)

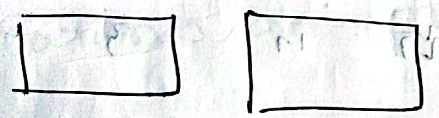
ex) In our 60 student class

there are 5 grades (A, B, C, D, F)

there are at least 12 students will get the same letter grade

$$\left\lceil \frac{60}{5} \right\rceil = 12 \text{ students}$$

ex: Assume you have a drawer containing a random distribution of a dozen brown socks and a dozen black socks. It is dark. so how many socks do you have to pick to be sure that among them there is a matching pair?



b.1 b.2

first এ ২টি black উঠে

2nd → যেহেতু যে black ও হতে পারে brown ও হতে পারে সেহেতু যেটা box এ তুলেছে সেটি brown.

3rd → একটা black ও হতে পারে brown ও হতে পারে, ২টি black বা তাহলে first এ যে black ২টি তোলা আছে মিলে যাবে, আর যদি brown উঠে তাহলে 2nd এ যেগুলো brown

permutations and combinations

~~Q.~~ How many ways are there to pick a set of 3 people from a group of 6?

$$\frac{1}{6} \quad \frac{1}{5} \quad \frac{1}{4}$$

first একজনের জন্য 6টা option ছিল

2nd " " " 5টা "   3

3rd " " " 4টা "   w.d

$6 \times 5 \times 4 = 120$ way ~~to do this~~ ← combination

But এটা correct result না কারণ,

(Rahim, sakib, nati)

(nati, sakib, Rahim)

(sakib, Rahim, nati)

Same rule But এখানে আলাদা হিসাব (কেননা একই লোক ফিরে আসছে)

ওটি common থাকলে বাদ দিতে হবে,

A permutation is an act of arranging objects or number in order. (Order matters হয়।) 1: Example: $^6P_3 \to 120$

Combination's are the ways of selecting objects or numbers from a group of objects or collections; in such a way that the order of the objects does not matter.

set order matter করে না → combination.

Permutation are for lists (order matters)

Combinations are .

Ex: Gmail password এ nabc / abnc > same না

Permutations

The number of r-permutations of a set with n distinct elements is denoted by $P(n, r)$

We can calculated $P(n, r)$ with the product

rule - $P(n, r) = n(n-1)(n-2) \cdots (n-r+1)$

Letter → 26, [illegible] [illegible] তিনটি ~~[illegible]~~

for Pass [illegible]

$n = 26, \; r = 3$

(n choices for the first element, $(n-1)$ for the second one, $(n-2)$ for the ~~thar~~ third one ...)

~~For~~ **Example:**

8 → option/digit/character আছে

3 → টা জায়গায় বসাতে পারবো

ধরি → ~~option~~ (1) (2) 3

option → $\underline{\quad 8 \quad}\ \underline{\quad 7 \quad}\ \underline{\quad 6 \quad}$

$P(8,3) \rightarrow 8.7.6 = 336$

General formula

$$P(n,r) \rightarrow \frac{n!}{(n-r)!}$$

Combinations

General formula

$(r-r)! = 0! = 1$

$1! = 0! = 1$

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{\frac{n!}{(n-r)!}}{\frac{r!}{(r-r)!}} = \frac{n!}{(n-r)!} \times \frac{1}{r!}$$

$$= \frac{n!}{}$$

$$P(n,r) = C(n,r) \times P(r,r)$$

How many ways are there to pick a set of 3 people from a group of 6 (~~disge~~ disregarding the order of picking?)

$$\underset{n,r}{C(6,3)} = \frac{6!}{3! \times (6-3)!} = \frac{\cancel{6} \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$$

$$= 20$$

CamScanner

A soccer club has 8 female and 7 male members. for today's match, the coach wants to have 6 female and 5 male players on the grass. How many possible configurations are there?

~~8~~ female, 7 ~~4~~ male

$$\underbrace{\underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{3}\ \underline{2}}_{\text{female}}\ \underbrace{\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }}_{\text{male}}$$

$\cancel{Cf = \frac{8!}{}}$

$$Cf(8,6) \rightarrow \frac{8!}{6!\,(8-6)!} = 28$$

$$Cm(7,5) \rightarrow \frac{7!}{5! \times (7-5)!} = 21$$

$$Cf(8,6) \cdot Cm(7,5) \rightarrow 28 \cdot 21$$

$$\rightarrow 588$$

Q4: Three people are running for elect. There are 202 people who vote. What is the minimum number of votes needed for someone to win the election?

→ pegeon theorem: $\frac{202}{3} \Rightarrow 67.3$

→ (celling) मान 68

Q5: There are 38 different times periods during which classes at a university can be scheduled. If there are 677 different classes. What is the minimum number of different rooms that will be needed.

$\Rightarrow$ Time slot : 38

different ~~Classe~~ Class $\rightarrow$ 677

$$\rightarrow \frac{677}{38} = 18 \cdot 17 \text{ (ceiling / upper limit)}$$

Q3: I have 7 pairs of socks in ~~my~~ my one each of color of the rainbow. ~~How~~ How many socks do I have to draw out in order to guarantee that I have grabbed at least one pair? What if there are likewise colored pairs of gloves in the and I cannot tell the difference bet-ween gloves and socks and I want a matching set?

Q6. ~~10~~. How many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other?

$\Rightarrow$ Δ | Δ | Δ | Δ | Δ | Δ | Δ | Δ | Δ

এটা permutation এর problem. ~~male~~ wo

~~men~~ ~~P(8,8)~~ ~~P(8,8)~~ ~~P(9,~~ ~~P(9,5)~~

option

~~male P(8,8)~~.

8 7

$P(8,8) \times P(9\times5) \rightarrow 40320 \times 15120 \Rightarrow$

$\frac{8!}{(8-8)!} = 40320$

$P(9,5) \Rightarrow \frac{9!}{(9-5)!} = 15120$

Q7 → How many ways are there to choose a delegation ~~out of~~ 10 males and 10 females if the delegation is made up of 2 males and 3 females.

$C(10,2) \times C(10,3) \rightarrow \frac{10!}{2!(10-2)!} = 45$

$\frac{10!}{3!(10-3)!} = 120$

$C(10,2) \times C(10,3) \rightarrow 5400.$

At a consultant mixer with 42 people shakes everyone else's hand exac once. How many handshakes occur?

$\Rightarrow C(42, 2) \Rightarrow \dfrac{42!}{2!\,(42-2)!}$

Q8. Mark has 5 pants and 7 shirts in his closet. He wants to wear a different pant/shirt combination each day without buying new clothes for as long as he can. How many works can be do this for?

$\Rightarrow$ product rules

~~If $f(x,y) = \frac{xy}{x^2+y^2}$~~

Q4. Twenty Students enter a contest at school. The contest offers a first second and a third prize. How many different combination of 1st 2nd and 3rd place winners can be.

$\underset{1st}{\underline{B}}$ $\underset{2nd}{\underline{C}}$ $\underset{3rd}{\underline{F}}$ $\underset{1st}{\underline{E}}$ $\underset{2nd}{\underline{C}}$ $\underset{3rd}{\underline{B}}$

$\Rightarrow$ permutation; $20 \times 19 \times 18$

$\Rightarrow$

$$^{20}P_3 = \frac{20!}{(20-3)!} \to 6840$$

C.W

permutation and combination

(1). How many ways are there to choose a delegation out of 10 males and 10 females if the delegation is made up of 2 males and 3 females.

$^{10}C_2 = 45$ + $^{10}C_3 = 120$

$= 165$

Q. At a consultant mixer with 42 people. everyone shakes everyone else's hand exactly once. How many handshakes occur?

⇒ combination.

* A committe of 5 people is to be chosen from 6 men and 4 women. In how many can this be done.

(1) if there must be 3 men and 3 women on the committee.

$\Rightarrow$ $6C3 * 4C2 = 20 * 6 = 120$

3 women, 2 men, $\binom{4}{3} \times \binom{6}{2} = 4 * 15 = 60$

4 women, 1 man: $\binom{4}{4} * \binom{6}{1} = 1 * 6$

5 women, 0 men: $\binom{4}{5} * \binom{6}{0}$ ← impossible

Pascal triangle

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 2 & 1 & & \\
 & & 1 & & 3 & & 3 & 1 & \\
 & 1 & & 4 & & 6 & & 4 & 1 \\
\end{array}
$$

- - - - -