Seperation of variables Technique :-

A first order differential equation of the form

$$\frac{dy}{dx} = g(x)h(y) \quad \text{——— (1)}$$

is said to be seperable.

Method of solution :-

(1) Make sure at each side we can have only one variables. That is when there is x in L.H.S of a equation, there can't be y.

Then in seperable way, (1) can be written as

$$\frac{dy}{h(y)} = g(x)\,dx \quad \text{——— (2)}$$

(2) Then Integrate both side of equation (2), then we get,

$$\int \frac{dy}{h(y)} = \int g(x)\,dx$$

which will provide you the required result.

Malthusian Model Solution :-

The model is

$$\frac{dP}{dt} = ap$$

$$\Rightarrow \frac{dP}{P} = a\,dt$$

$$\Rightarrow \ln p = at + c$$

$$\Rightarrow P = e^{(at+c)}$$

$$\Rightarrow P(t) = e^{at} \cdot e^{c}$$

$$\Rightarrow P(t) = C_1 e^{at}$$ where $C_1 = e^{c}$ = constan

$\therefore$ The required solution is

$$P(t) = C_1 e^{at} \quad \text{——— (1)}$$

Now if at initial time that is at time $t=0$, the populatio is P_0 i.e. $P(0) = P_0$ ——— (2)

Then (1) can be written as at $t=0$

$$P(0) = C_1 e^{a \times 0}$$

$$\Rightarrow P_0 = C_1 \times 1 \quad [\text{using (2)}]$$

$$\Rightarrow P_0 = C_1 \quad \text{——— (3)}$$

Then (1) becomes,

$$P(t) = P_0 e^{at}.$$

which is the solution of the model as an initial value probler

Problem-01:

The population of a town grows at a rate proportional to the population at time t.

The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years.

Solution:- Let $P(t) :=$ Total population at time t.

Then, the problem follows Malthusian model

$$\frac{dP}{dt} = aP \qquad \text{with } P(0) = 500$$

$$P(10) = 500 + 500 \times 0.15$$
$$= 500 + 75 = 575$$

Now, $$\int \frac{dP}{P} = \int a\,dt$$

$$\Rightarrow \ln P = at + c$$

$$\Rightarrow P(t) = e^{at+c} = e^{at} \cdot e^{c} = c_1 e^{at} \qquad \text{where, } c_1 = e^{c}$$

$$\Rightarrow P(t) = c_1 e^{at} \quad \text{———} \; (1)$$

Since at $t = 0$ $P(t) = 500$

$$\therefore P(0) = c_1 e^{0 \times a}$$

$$\Rightarrow 500 = c_1$$

$\therefore$ (1) becomes. $$P(t) = 500 e^{at}. \quad \text{———} \; (2)$$

~~Also, $P(10) = 500 e$~~

Again, $P(10) = \cancel{e}\, 500\, e^{a \times 10}$

$$\Rightarrow 575 = 500\, e^{a \times 10}$$

$$\Rightarrow e^{10a} = \frac{\cancel{50}\,575}{500}$$

$$\Rightarrow a = \frac{1}{10} \ln\left(\frac{575}{500}\right) = 0.014$$

Then, when, $\cancel{p}$ $t = 30$, then ② implies.

$$P(30) = 500 \times e^{0.014 \times 30} \simeq 760$$