(6.

*find the eqn. of tangent at (1, 2) for the function $y = x^2 + 3$.

$m = \frac{dy}{dx} = 2x$

$m = 2 \times 1 = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 1)$

* Parametric ? x ও y t এর function হয়,

① Find the eqn. of tangent at $t = 2$, ~~for Paramet~~
For Parametric eqn.: $x = t$, $y = t^2 + 3$

$\therefore \frac{dx}{dt} = 1 \quad \ldots ①$

$\therefore \frac{dy}{dt} = 2t \quad \ldots ②$

② ÷ ①,

$\cancel{\frac{dy}{dx} = \frac{2t}{1}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{1}$

$\Rightarrow \frac{dy}{\cancel{dt}} \times \frac{\cancel{dt}}{dx} = 2t$

$\therefore \frac{dy}{dx} = 2t$

$t = 2$, $x = 2$, $y = 7$ $\quad (x_1, y_1) = (2, 7)$

at $t = 2$,

$m = \frac{dy}{dx} = 4$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 7 = 4(x - 2)$

$\Rightarrow y = 4x - 8 + 7$

$\therefore y = 4x - 1$

↳ cartesian eqn.

Ex-02: Find an eqn. of tangent to the curve

$x = t^2 - 2t$; $y = t^3 - 3t$ when $t = 2, -2$.

$\Rightarrow \frac{dy}{dx} = \frac{dx}{dt} = 2t - 2 \quad \cdots (1)$

$\frac{dy}{dt} = 3t^2 - 3 \quad \cdots (2)$

$\cancel{(1) \div} (2) \div (1)$

$$\frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}} = \frac{3t^2 - 3}{2t - 2}$$

$$\therefore \frac{dy}{dx} = \frac{3t^2 - 3}{2t - 2}$$

$t = 2$,

$m = \frac{3t^2 - 3}{2t - 2} = \frac{3.2^2 - 3}{2.2 - 2} = \frac{12 - 3}{4 - 3} = \frac{9}{1} = 9$

$x = 4 - 2.2 = 0$

$y = 8 - 6 = 2$.

$m = \frac{dy}{dx}$

$\cancel{y - y_1 = m(x-x_1)}$

$\Rightarrow$ $t = -2$, $\frac{dy}{dx} = \frac{3t^2-3}{2t-2} = \frac{12-3}{-4-2} = \frac{9}{-6} = \frac{-3}{2}$

$(x_1, y_1) = \cancel{4-(-2-2)} = (8, -2)$

$\cancel{= -8-3.(-2) = -8+6}$

$y - y_1 = m(x - x_1)$

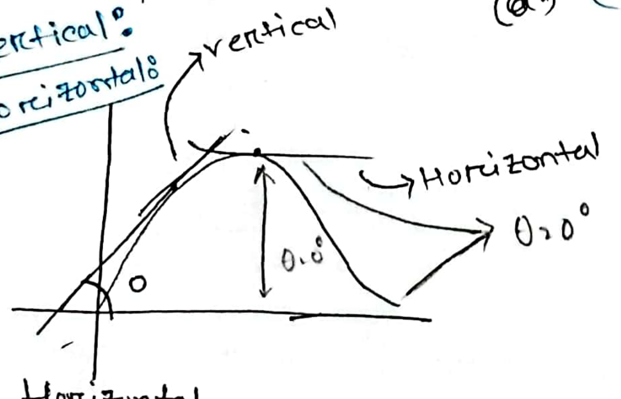
$y + 2 = \frac{-3}{2}(x-8)$

$\Rightarrow 2y + 4 = -3x + 24$

$\Rightarrow 3x + 2y - 20 = 0$ (a) (ans.)

Vertical:

Horizontal:



Horizontal,

$\frac{dy}{dx} = \tan\theta = \tan 0 = 0$

~~In which point~~

(3) Find the value of t when tangent of $x = t^2 - 2t$; $y = t^3\ 3t$ is horizonatal.

Since the tangent is horizontal, note-1

$\Rightarrow$ ~~t=0 হবে~~ $\frac{dy}{dx} = 0$ হবে।

$\Rightarrow \dfrac{3t^2-3}{2t-2} = 0$

$\Rightarrow 3t^2 - 3 = 0$

$\Rightarrow t^2 = 1$

$\therefore t = \pm 1$

A if tangent vertical,

$\dfrac{dy}{dx} = \tan 90^\circ$

$= \infty$ ← note-2

$\dfrac{dy}{dx} = \dfrac{1}{0}$

$\Rightarrow \dfrac{3t^3-3}{2t-2} = \dfrac{1}{0}$

$\therefore t = 1$

H.W. -

21.8.23

Ex
(5) Find the point on the parametric curve where

$x = t^2 - 2t$

$y = t^3 - 3t$

the tangent is horizontal.

$\Rightarrow$

$$\frac{dx}{dt} = 2t - 2 \quad \cdots\cdots (1)$$

$$\frac{dy}{dt} = 3t^2 - 3 \quad \cdots\cdots (2)$$

$(2) \div (1)$ implies that,

$$\frac{dy}{dt} \times \frac{dt}{dx} = \frac{3t^2-3}{2t-2} \quad ; \quad 2t-2 \neq 0$$

$$\frac{dy}{dx} = \frac{3t^2-3}{2t-2}$$

Since the tangent is horizontal,

$$\frac{dy}{dx} = 0$$

$$\frac{3t^2-3}{2t-2} = 0$$

$\Rightarrow 3t^2 = 3$

$\Rightarrow t^2 = 1$

$\therefore t = +1, -1$

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, 0^{∞}

for, $t = 1$, $\frac{dy}{dx} = \frac{0}{0}$ (indeterminant form)

$t = -1$, $\frac{dy}{dx} = \frac{0}{-4} = 0$

$\frac{dy}{dx} = \frac{0}{-4} = 0$

$x_1 = 2 \times (-1) - 2 = -4$

$y_1 = (-1)^2 - 3 = 0$

$\frac{3t^2 - 3}{2t - 2} = \frac{1}{0}$

$(x_1, y_1) = (0, 0)$

Hence, the horizontal tangent exists when $t = -1$ and the point is

$x_1 = (-1)^2 - 2(-1) = 1 + 2 = 3$

$y_1 = (-1)^3 - 3(-1) = -1 + 3 = 2$

$(x_1, y_1) \equiv (3, 2) \Rightarrow$ tangent horizontal

(c) 2. Find the point on the parametric curve where,

$x = t^2 - 2t$

$y = t^3 - 3t$

the tangent is vertical.

$\Rightarrow$ $\frac{dx}{dt} = 2t - 2 \dots (1)$

$\frac{dy}{dt} = 3t^2 - 3 \dots (2)$

② ÷ ① implies that,

$$\frac{dy}{dt} \times \frac{dt}{dx} = \frac{3t^2-3}{2t-2} \quad ; \; 2t-2 \neq 0$$

$$\frac{dy}{dx} = \frac{3t^2-3}{2t-2}$$

Since the tangent is vertical

$$\frac{dy}{dx} = \frac{1}{0}$$

$$\Rightarrow \frac{3t^2-3}{2t-2} = \frac{1}{0}$$

$$\Rightarrow 2t-2 = 0$$

$$\Rightarrow t = 1$$

for, $t = 1$, $\frac{dy}{dx} = \frac{0}{0}$

Thus we cannot take the value $t = 1$.

Hence, the vertical tangent of the parametric curve does not exist.

$(x_1, y_1) =$

* vertical এর ক্ষেত্রে t indeterminant form ও zero হতে পারবে না। note-3