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MTH

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Ans: 1

ii) $\lim_{(x,y)\to(0,0)} f(x,y) = 0 \quad (x=0)$

$f(x,y) = \frac{xy}{x^2+y^2}$

Sol^n: Given that,

$$\lim_{(x,y)\to(0,0)} f(x,y) = \frac{xy}{x^2+y^2}$$

$$= \frac{0 \times y}{0^2+y^2}$$

$$= \frac{0}{y^2} = 0$$

Again,

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0$$

which is unique and finite.

(i) $f(x,y) = \frac{xy}{x^2+y^2}$, where limit $(x,y)\to(0,0)$ along $y=0$

Sol^n: when, $y=0$

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} = \frac{x \times 0}{x^2+0}$$

$$= \frac{0}{x^2} = \frac{0}{0} = 0$$

which is unique and finite.

(iii) $\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y) = 0$ where, $f(x,y) = \frac{xy}{x^2+y^2}$

$y = x^2$

Soln: when, $y = x^2$

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$= \frac{x \cdot x^2}{x^2+x^4}$$

$$= \frac{x^3}{x^2(1+x^2)}$$

$$= \frac{x}{1+x^2}$$

Now,

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x}{1+x^2}$$

$$= \frac{0}{1+0}$$

$$= \frac{0}{1}$$

$$= 0$$

Since $f(x,y) = 0$ so this is unique and finite. The limit exists.

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(iv) Given that,

$$\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y) = \frac{xy}{x^2+y^2} \qquad [y = x]$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y) = \frac{x \times x}{x^2+x^2}$$

$$= \frac{x^2}{2x^2}$$

$$= 1/2$$

$f(x,y) = 1/2$ which is unique and finite

So the limit exist.

(v) Given that,

$$\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y) = \frac{xy}{x^2+y^2}$$

From (i), (ii), (iii) and (iv) we found that,

$\lim_{(x,y)\to(0,0)} f(x,y) = 0$ along $y = 0$

$\lim_{(x,y)\to(0,0)} f(x,y) = 0$ along $x = 0$

$\lim_{(x,y)\to(0,0)} f(x,y) = 0$ along $y = x^2$

$\lim_{(x,y)\to(0,0)} f(x,y) = 1/2$ along $y = x$

$f(x,y)$ is differenent along $(x,y) \to (0,0)$

$y=0,\ x=0,\ y=x^2$ and $y=x$

So, $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

Ans: 2

(i) $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y) = \dfrac{x^3+y^3}{x^3+y}$; along $y=mx$

$$= \frac{x^3+(mx)^3}{x^3+mx}$$

$$= \frac{x^3(1+m^3)}{x(x^2+m)}$$

$$= \frac{x^2(1+m^3)}{x^2+m}$$

$$\lim_{(x,y)\to(0,0)} = \frac{0(1+m^3)}{0^2+m} = \frac{0}{m} = 0$$

$f(x,y)=0$ which is unique and finite

Hence $f(x,y)$ exists along $y=mx$

(ii) $\lim_{\substack{x\to 0\\ y\to 0}} f(x,y) = \frac{x^3+y^3}{x^3+y}$ along $y=-x^2e^x$

$$= \frac{x^3+(-x^2e^x)^3}{x^3+(-x^2e^x)}$$

$$= \frac{x^3-x^6e^{3x}}{x^3-x^2e^x}$$

$$= \frac{x^3(1-x^3e^{3x})}{x^3(1-e^x/x)}$$

$$= \frac{(1-x^3e^{3x})}{(1-e^x/x)}$$

Now,

$$(x,y)\to(0,0) \quad \frac{1-0^3 e^{3\times 0}}{1-e^0/0}$$

$$= \frac{1}{1}$$

$$= 1$$

$f(x,y) = 1$ is unique and finite. Here

$\lim_{(x,y)\to(0,0)} f(x,y)$ exists along $y=-x^2e^x$

(iii) $\lim\limits_{\substack{x \to 0 \\ y \to 0}} f(x,y)$ from (i) and (ii) we found that $f(x,y) = 0$; $f(x,y) = 1$.

~~Limit~~ $f(x,y)$ is different along $(x,y) \to (0,0)$ $y = mx$ and $y = -x^2 e^x$ passing throug $(0,0)$. So, $f(x,y)$ does not exist.