MATH THEORY

Independent and Dependent Events(PDF:01):

Independent Event: When multiple events occur, if the outcome of one event "DOES NOT" affects the outcome of the other events, they are called independent events.
Say, a die is rolled twice. The outcome of the first roll doesn't affect the second outcome. These two are independent events.
Dependent Event: When two events occur, if the outcome of one event affects the outcome of the other, they are called dependent events.

Compound probability(PDF:02):

Compound probability is when the problem statement asks

for the likelihood of the occurrence of more than one outcome.

P(A or B) = P(A) + P(B) − P(A and B)

Where, A and B are any two events.
P(A or B) is the probability of the occurrence of at least one of the events.
P(A and B) is the probability of the occurrence of both A and B at the same time.

Mutually Exclusive Events:

When two events cannot occur at the same time, they are considered mutually exclusive.For a mutually exclusive event, P(A and B) = 0. P(A or B) = P(A) + P(B)

Conditional probability:

Conditional probability is calculating the probability of an event given that another event has already occurred.
Conditional probability is defined as P (A|B), read as P (A given B) is

P (A|B) = P (A and B) / P (B)

Complement of an event:

A complement of an event A can be stated as that which does NOT contain the occurrence of A.
A complement of an event is denoted as P (A').

P (A’) = 1 – P (A)
Or, it can be stated, P (A) + P (A’) = 1

Baye’s Theorem(PDF:03):

If B1,B2, B3, ⋯ ⋯ ,Bn are mutually exclusive events
with P(Bi) ≠ 0, i = 1,2,3, ⋯ ⋯ , n of a random experiment then for any arbitrary event A of the sample space of the above experiment with P(A) > 0, we have,

$$P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}$$

Say, if $n = 3$,

$$P(B_1/A) = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2) + P(B_3)P(A/B_3)}$$

$$P(B_2/A) = \frac{P(B_2)P(A/B_2)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2) + P(B_3)P(A/B_3)}$$

$$P(B_3/A) = \frac{P(B_3)P(A/B_3)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2) + P(B_3)P(A/B_3)}$$

Probability Distribution: Any statement of a function associating each of a set of mutually exclusive and exhaustive classes or class intervals with its probability is a probability distribution.

Probability Mass Function (p.m.f.) (PDF:04):

If a random variable X has a discrete
distribution, the probability distribution of X is defined as the function f such that for any real number x, f(x) = P(X = x). The function
f(x) must satisfy the following conditions to be a probability mass function.

i. $f(x) \geq 0$
ii. $\sum_x f(x) = 1$
iii. $P(X = x) = f(x)$

Probability Density Function (PDF:05):

In probability function, when
the random variable X is continuous variable, then the corresponding function f(x) is called probability density function. A probability

density function is a non-negative function. The function f(x) must satisfy the following conditions to be a probability density function.

i. $f(x) \geq 0$

ii. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

iii. $P(a < X < b) = \int_a^b f(x)\, dx$

Expected Value or Mathematical Expectation:

If X is a discrete random variable, with a probability function f(x), then the expected value or the mathematical expectation of X is denoted by E(X) and is defined as,

$$E(X) = \sum_x x f(x)$$

If X is a continuous random variable, with a probability density function $f(x)$, then,

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Variance: The variance of a random variable X is defined as

$$V(X) = E(X - E(X))^2 = E(X^2) - \{E(X)\}^2$$

Standard Deviation: $\sigma = \sqrt{E(X - E(X))^2}$

19(a) Description of Binomial Distribution

When an experiment has two possible outcomes, success and failure and the experiment is repeated n times independently and the probability p of success of any given trial remains constant from trial to trial, the experiment is known as binomial experiment. The binomial distribution is defined as

$$b(x; n, p) = \begin{cases} n_{C_x} p^x (1-p)^{n-x}, & x = 0,1,2,3, \dots \dots, n \\ 0, & elsewhere \end{cases}$$

Properties:

i. The total probability is $p + q = 1$; where p is success and q is failure.

ii. The mean of a Binomial Distribution, $\mu = np$

iii. The Standard Deviation of a Binomial Distribution, $\sigma = \sqrt{npq}$

iv. The Variance of a Binomial Distribution, $\sigma^2 = npq$

Poisson Distribution(PDF:06):

Let, µ be the mean or expected number of success in a specified time or space and the random variable X designate the number of success in a given time interval or specified region. Then the Poisson distribution is defined as

$$f(x,\mu) = \frac{e^{-\mu}\mu^{x}}{x!}, x = 0, 1, 2, 3, \dots \dots \dots, \infty$$

Where $e = 2.71828$ is a constant and x is any positive value and

$\mu = np$ where n is the number of occurrence in the distribution and p is the success.

Properties:

i. Poisson Distribution is a probability mass function.

ii. The mean of a Poisson Distribution, $\mu = np$

iii. The Standard Deviation of a Poison Distribution, $\sigma = \sqrt{\mu}$

iv. The Variance of a Poisson Distribution, $\sigma^2 = \mu$

27(a) Definition of Normal and Standard Normal Distribution

A continuous random variable X has a normal distribution with mean μ and variance $\sigma^2(-\infty < \mu < \infty \; and \; \sigma^2 > 0)$. The normal distribution can be defined as

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], -\infty < x < \infty$$

Where $e = 2.71828$ and $\pi = 3.1416$ are two constants.

Properties:

i. Normal distribution is a probability density function.
ii. The mode of the normal distribution is $\frac{\sigma}{\sqrt{2\pi}}$

Standard Normal Distribution

If a random variable X has a normal distribution with mean μ and variance σ^2, then the variable $z = \frac{x-\mu}{\sigma}$ will be called a standard normal variate (or Z score) and its distribution is referred to as the standard normal distribution having the following density function

$$f(x;0,1) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}; -\infty < z < \infty$$

Properties:

i. Standard Normal distribution is a probability density function.
ii. The mean of the standard normal distribution is 0.
iii. The variance of the standard normal distribution is 1.