Q1. Determine if each function is one-to-one.
(a) To each person on the earth assign the number which corresponds to his age.
(b) To each country in the world assign the latitude and longitude of its capital.
(c) To each book written by only one author assign the author.
(d) To each country in the world which has a prime minister assign its prime minister.

Ans:
(a) No, (b) yes, (c) no, (d) yes.

Q2. Let functions f, g, h from V = {1, 2, 3, 4} into V be defined by: f (n) = 6 − n, g(n) = 3,
h = {(1, 2), (2, 3), (3, 4), (4, 1)}. Decide which functions are:
(a) one-to-one; (b) onto; (c) both; (d) neither.

Ans:
(a) f, h; (b) f, h; (c) f, h; (d) g.

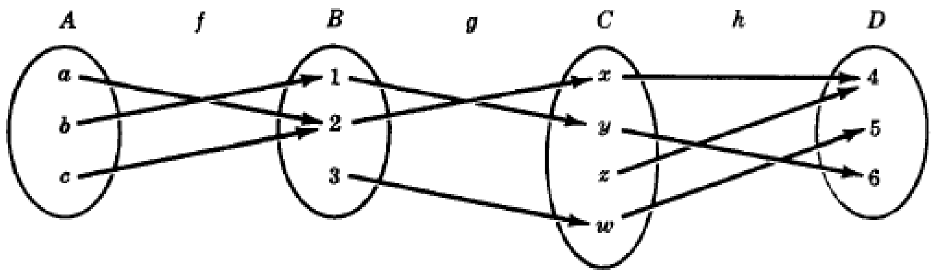
Q3. Let functions f, g, h from **N** into **N** be defined by f (n) = n + 2, (b) g(n) = 2^n;
h(n) = number of positive divisors of n. Decide which functions are:
(a) one-to-one; (b) onto; (c) both; (d) neither;

Ans:
(a) f, g; (b) h; (c) none; (d) none;

integer *n*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
f(n)	3	4	5	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	17	18
g(n)	2	4	8	16												
h(*n*)	1	2	2	3	2	4	2	4	3	4	2	6	2	4	4	5

Q4.

Let the functions $f: A \to B$, $g: B \to C$, $h: C \to D$ be defined by Fig. 3-9. Determine if each function is: (*a*) onto, (*b*) one-to-one, (*c*) invertible.



Ans:

(*a*) The function $f: A \to B$ is not onto since $3 \in B$ is not the image of any element in A.

The function $g: B \to C$ is not onto since $z \in C$ is not the image of any element in B.

The function $h: C \to D$ is onto since each element in D is the image of some element of C.

(*b*) The function $f: A \to B$ is not one-to-one since a and c have the same image 2.

The function $g: B \to C$ is one-to-one since 1, 2 and 3 have distinct images.

The function $h: C \to D$ is not one-to-one since x and z have the same image 4.

(*c*) No function is one-to-one and onto; hence no function is invertible.

Q5. Consider the following recurrence relation and the initial condition:

Base case: $a_0 = 0$

Recursive case: $a_n = a_{n-1} + 2$ for n >= 1

List the 5 elements $a_1 \ldots\ldots a_5$ of the sequence defined by this recurrence relation.

Answer:

$$a_1 = 2;\ a_2 = 4;\ a_3 = 6;\ a_4 = 8;\ a_5 = 10.$$

Q6. Let a and b be positive integers, and suppose Q is defined recursively as follows:

Q(a, b) = 0 if a<b

Q(a − b, b) + 1 if b ≤ a

(a) Find: (i) Q(2, 5); (ii) Q(12, 5).

(b) What does this function Q do? Find Q(5861, 7).

Ans:

(a) (i) Q(2, 5) = 0 since 2 < 5.

(ii) $Q(12, 5) = Q(7, 5) + 1$

$= [Q(2, 5) + 1] + 1 = Q(2, 5) + 2$
$= 0 + 2 = 2$

(b) Each time b is subtracted from a, the value of Q is increased by 1. Hence Q(a, b) finds the quotient when a is divided by b. Thus Q(5861, 7) = 837.

Q7. Find the domain and range of the following function f. It assigns to each string of bits (that is, of 0s and 1s) the number equal to twice of the number of zeros in that string. For example f(011001010101010) = 16.

Solution
By definition f assigns a value to “each string of bits” thus the domain is the set of all bit strings. It’s easy to see that the range is the set of all positive even numbers. An odd number can not be a value of f since the value is twice the number of zeros. Any even number 2k has a preimage: a string of k zeros.

Q8. Let D be the set of days in the week. Let $f: D \to Z^+$ be the function that assigns to each day the number of letters in its English name. Does f define a one-to-one function from Dto Z+?

Ans: No, f is not one-to-one because f(Monday) = 6 = f(Sunday), but Monday = Sunday.