Q1:
Use a truth table to decide whether this argument is valid.

$p \lor \neg q$ premise 1
$p \land \neg r$ premise 2
$\therefore$ $r \to q$ conclusion

SOLUTION: Valid.

				prem 1		*prem 2*	*conclu*
p	q	r	$\neg q$	$p \lor \neg q$	$\neg r$	$p \land \neg r$	$r \to q$
T	T	T	F	T	F	F	
T	T	F	F	T	T	T	$T*$
T	F	T	T	T	F	F	
T	F	F	T	T	T	T	$T*$
F	T	T	F	F	F	F	
F	T	F	F	F	T	F	
F	F	T	T	T	F	F	
F	F	F	T	T	T	F	

Q2:
Use a truth table to decide whether this argument is valid.

$p \lor q$ premise 1
$q \lor r$ premise 2
$\therefore$ $p \lor r$ conclusion

SOLUTION: NOT valid.

p	q	r	$\overbrace{p \vee q}^{prem\ 1}$	$\overbrace{q \vee r}^{prem\ 2}$	$*$	$\overbrace{p \vee r}^{conclu}$
T	T	T	T	T	$*$	T
T	T	F	T	T	$*$	T
T	F	T	T	T	$*$	T
T	F	F	T	F		
F	T	T	T	T	$*$	T
F	T	F	T	T	$*$	$\boxed{F}$
F	F	T	F	T		
F	F	F	F	F		

Q3: Test the validity of the following argument:

If two sides of a triangle are equal, then the opposite angles are equal.
Two sides of a triangle are not equal.
The opposite angles are not equal.

Answer: this argument is a fallacy.

Q4: Test the validity of the following argument:
If I study, then I will not fail mathematics.
If I do not play basketball, then I will study.
But I failed mathematics.
Therefore I must have played basketball.

Answer:
Let p be “I study,”
q be “I failed mathematics,” and
r be “I play basketball.”
The argument has the form:
$p \rightarrow \neg q$,
$\neg r \rightarrow p$,
q
—------
r
Valid

p	q	r	$\neg q$	$p \rightarrow \neg q$	$\neg r$	$\neg r \rightarrow p$
T	T	T	F	F	F	T
T	T	F	F	F	T	T
T	F	T	T	T	F	T
T	F	F	T	T	T	T
F	T	T	F	T	F	T
F	T	F	F	T	T	F
F	F	T	T	T	F	T
F	F	F	T	T	T	F

1. q
2. p -> ¬q
3. ¬p [modus tollens]
4. ¬r -> p
5. ¬p -> r [¬r -> p ≡ ¬p -> r]

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r [modus ponens on step 3 and 5]

Q: Construct an argument using rules of inference to show that the hypotheses

- All humans are mortal,
- All philosophers are humans, and
- Socrates is a Greek philosopher

imply the conclusion that

- Socrates is mortal.

Solution:

Predicates: H(x), M(x), P(x), and G(x), for x is a human, x is mortal, x is a philosopher, and x is Greek, respectively.

Step 1: "x(H(x) → M(x))	hypothesis
Step 2: "x(P(x) → H(x))	hypothesis
Step 3: G(Socrates) ∧ P(Socrates)	hypothesis
Step 4: P(Socrates)	simplification on Step 3
Step 5: P(Socrates) →H(Socrates)	universal instantiation on Step 2
Step 6: H(Socrates)	modes ponens on Steps 4 & 5
Step 7: H(Socrates) → M(Socrates)	universal instantiation on Step 1
Step 8: M(Socrates)	Modes ponens on Steps 6 & 7