



**Department of Mathematics and Natural Sciences**  
**MAT215 : Complex Variables and Laplace Transformations**  
**Assignment 2**  
**Date: February 18, 2024**

Deadline : February 27, 2023

Spring 2024

Total Marks: 60

**Name:**

**Section:**

**ID:**

**Use this page as the cover page of your assignment. No late submission will be graded.**

1. Evaluate  $\int_{(0,1)}^{(2,5)} (3x+y)dx + (2y-x)dy$  along (a) the curve  $y = x^2 + 1$ , (b) the straight line joining (0, 1) and (2, 5), (c) the straight lines from (0, 1) to (0, 5) and then from (0, 5) to (2, 5), (d) the straight lines from (0, 1) to (2, 1) and then from (2, 1) to (2, 5). [1.25 x 4 = 5]
2. (a) Evaluate  $\oint_C (x+2y)dx + (y-2x)dy$  around the ellipse  $C$  defined by  $x = 4\cos\theta, y = 3\sin\theta, 0 \leq \theta < 2\pi$  if  $C$  is described in a counterclockwise direction.  
(b) What is the answer to (a) if  $C$  is described in a clockwise direction? [2 x 2.5 = 5]
3. Evaluate  $\oint_C |z|^2 dz$  around the square with vertices at (0, 0), (1, 0), (1, 1), (0, 1). [5]
4. Evaluate  $\int_i^{2-i} (3xy + iy^2) dz$  (a) along the straight line joining  $z = i$  and  $z = 2 - i$ , (b) along the curve  $x = 2t - 2, y = 1 + t - t^2$ . [5]
5. Evaluate  $\lim_{z \rightarrow 0} (\cos z)^{1/z^2}$ . [5]
6. Determine whether  $|z|^2$  has a derivative anywhere. [5]
7. Determine whether the function  $u = e^{-2xy} \sin(x^2 - y^2)$  is harmonic. If  $u$  is a harmonic function, find the conjugate harmonic function  $v$  and express  $u + iv$  as an analytic function of  $z$ . [5]
8. (a) Prove that the function  $u = 2x(1 - y)$  is harmonic. (b) Find a function  $v$  such that  $f(z) = u + iv$  is analytic [i.e., find the conjugate function of  $u$ ]. (c) Express  $f(z)$  in terms of  $z$ . [2.5 x 2 = 5]
9. Evaluate (a)  $\lim_{z \rightarrow 0} \frac{z - \sin z}{z^3}$ , (b)  $\lim_{z \rightarrow m\pi i} (z - m\pi i) \left( \frac{e^z}{\sin z} \right)$ . [2.5 x 2 = 5]
10. Evaluate  $\lim_{z \rightarrow 0} \left( \frac{\sin z}{z} \right)^{1/z^2}$ . [5]
11. Given  $C$  is the circle  $|z| = 1$ . Find the value of (a)  $\oint_C \frac{\sin^6 z}{z - \pi/6} dz$  (b)  $\oint_C \frac{\sin^6 z}{(z - \pi/6)^3} dz$  [5]
12. Evaluate  $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{(z^2 + 1)^2} dz$  when  $t > 0$  and  $C$  is the circle  $|z| = 3$ . [5]