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Abstract

The objective of this project is to apply topics learned in Operation research II in everyday scenarios and to solve them using computer programs such as CPLEX and MATLAB. Our group decided to have our problem based on a University. Three problems were being analyzed using various approaches of Operation Research. Problem 1 is an integer programming problem that deals with ATS university planning on renting out the best room(s) at a law firm to promote their new upcoming law school. In order to solve this problem, CPLEX is used. After analyzing the model with the old and new constraints it can be determined that in order for ATS University to make maximum profit, the rooms with lower capacity should be chosen. This was determined by simulating our model on CPLEX and taking a look at the optimal solution. Problem 2 is a dynamic programming problem related to resource allocation. ATS university is trying to determine what the best times are to replace their printers to yield the greatest profit. Using MATLAB, dynamic programming was used to determine the best solution over 12 years given that each printer can be kept for a maximum of 8 years. Finally, Problem 3 was a Markov chain created to analyze statistics on engineering students quitting or passing their mathematical courses. Again, MATLAB was used to solve it. For each problem, alternative solutions were created and analyzed to pick the best possible solutions. Overall, this project helped us further understand and expand our knowledge on these topics while also applying them to real-world problems.

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Introduction

Problem definition 1: Integer Programming

ATS University would like to welcome its new law students to campus by hosting a fundraiser. Due to the size of the fundraiser, the university decides to host the event at a law firm. The law firm has many different rooms to rent out to ATS University. The prices vary for each room due to the room capacity. The University is expecting 2000 guests to attend the event and would like to know which options will allow them to meet all necessary requirements while also minimizing the overall cost. ATS university is using all proceedings from the fundraiser towards providing the new law school students with adequate resources during their academic years.

Problem definition 2: Dynamic Programming

ATS University is looking at buying new printers because the current printers are becoming outdated. Before purchasing new printers, the university would like to conduct market research to ensure that the new printers will last a long time while also being cost-effective. For this problem, dynamic programming is used to determine if after each year they should sell or keep the printer. The goal is to optimize the profit by determining the frequency at which they should be changing the printers. This resource-allocation problem determines the best time for equipment replacement. There is a fixed amount of resources that were considered when determining operating costs, and revenue to solve the problem.

Problem definition 3: Stochastic Process

Universities constantly do surveys and analyze data in order to improve classes and have a higher success rate. In this case, ATS university wants to analyze the success rate in engineering math courses and make sure they are not too demanding compared to other universities. Data were collected regarding the success and failure rates of students for each math course at ATS University. Markov chain is used to analyze this data and provide insight regarding student performance. This study shows the general census of students' success/failure in the math courses of engineering as they progress through university over time.

Problem 1

Problem Definition

The law firm chosen has 12 rooms available for ATS university to use to host their fundraiser. The prices vary for each room due to the room capacity. The University is expecting 2000 guests to attend the event and would like to know which options will allow them to meet all necessary requirements while also minimizing the overall cost. Each room has limitations regarding the room capacity which are as follows: Room 1 has a capacity of 1000 people, Room 2 has a capacity of 550 people, Room 3 has a capacity of 500 people, Room 4 has a capacity of 700 people, Room 5 has a capacity of 250 people, Room 6 has a capacity of 2000 people, Room 7 has a capacity of 1500, Room 8 has a capacity of 1750, Room 9 has a capacity of 1300, Room 10 has a capacity of 1050 people, Room 11 has a capacity of 600 people and Room 12 has a capacity of 350 people. The law firm was able to provide the University with discounted room rental prices which can be seen in Table 1. ATS University would like to know which options will allow them to meet all necessary requirements while also minimizing the overall cost.

As well, the law firm provided ATS University with room restrictions which are the following:

- If room 8 is selected, room 7 cannot be selected
- If room 1 is selected then room 10 cannot be selected
- If room 5 is selected then room 12 has to be selected
- One of Rooms 1, 6, 7, 8, 9, 10 has to be selected
- A maximum of two rooms can be selected between room 2, 3 and 11

Table 1 shows the various different room rental prices along with the admission price per person. It is important to note that the admission price is based on the Room Capacity and the overall price to rent a room.

Table 1. Admission Cost, Rent Cost, and Room Capacity for Each Room

	Room 1	Room 2	Room 3	Room 4	Room 5	Room 6
Admission Price/person ($)	5	5.5	5	4	3.5	5.75
Cost to rent (fixed) ($)	500	260	250	400	175	900
Room capacity	1000	550	500	700	250	2000

Table 1. Admission Cost, Rent Cost and Room Capacity for Each Room

	Room 7	Room 8	Room 9	Room 10	Room 11	Room 12
Admission Price/person ($)	6	5.75	5.75	5.25	5	4
Cost to rent (fixed) ($)	650	775	575	510	300	200
Room capacity	1500	1750	1300	1050	600	350

Design Process Overview

From the information provided in the question above the decisions variables, objective function and constraints can be written as follows:

Decision Variables:
x_i = Admission Price / Person ($)
y_i = 1 if room i is used, 0 otherwise
i = 1,2,3,4,5,6,7,8,9,10,11,12

If $x_1 > 0$, $y_1 = 1$ otherwise 0

Objective Equation:
Min Z = $-5x_1 - 5.5x_2 - 5x_3 - 4x_4 - 3.5x_5 - 5.75x_6 - 6x_7 - 5.75x_8 - 5.75x_9 - 5.25x_{10} - 5x_{11} - 4x_{12} + 500y_1 + 260y_2 + 250y_3 + 400y_4 + 175y_5 + 900y_6 + 650y_7 + 775y_8 + 575y_9 + 510y_{10} + 300y_{11} + 200y_{12}$

It is important to note that it is a minimization problem and not a maximization problem since our primary objective was to minimize the cost and not maximize the profit, making profit was a bonus. If the code yields a negative value, that means the university made profit rather than spending money. However, if the value is positive, the university spends more money than they make through this fundraiser.

Constraints:
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} = 2000$
$x_1 \leq 1000y_1$ or $x_1 - 1000y_1 \leq 0$
$x_2 \leq 550\, y_2$ or $x_2 - 550y_2 \leq 0$

$x_3 \leq 500\ y_3$ or $x_3 - 500 y_3 \leq 0$
$x_4 \leq 700\ y_4$ or $x_4 - 700 y_4 \leq 0$
$x_5 \leq 250\ y_5$ or $x_5 - 250 y_5 \leq 0$
$x_6 \leq 2000\ y_6$ or $x_6 - 2000 y_6 \leq 0$
$x_7 \leq 1500\ y_7$ or $x_7 - 1500 y_7 \leq 0$
$x_8 \leq 1750 y_8$ or $x_8 - 1750 y_8 \leq 0$
$x_9 \leq 1300 y_9$ or $x_9 - 1300 y_9 \leq 0$
$x_{10} \leq 1050\ y_{10}$ or $x_{10} - 1050 y_{10} \leq 0$
$x_{11} \leq 600\ y_{11}$ or $x_{11} - 600 y_{11} \leq 0$
$x_{12} \leq 350\ y_{12}$ or $x_{12} - 350 y_{12} \leq 0$
$x_i \geq 0\ y_i = 0 - 1$

If room 8 is selected, room 7 cannot be selected
$y_8 + y_7 \leq 1$
If room 1 is selected then room 10 cannot be selected
$y_1 + y_{10} \leq 1$
If room 5 is selected then room 12 has to be selected
$y_5 - y_{12} \leq 0$
One of Rooms 1, 6, 7, 8, 9, 10 has to be selected
$y_1 + y_6 + y_7 + y_8 + y_9 + y_{10} \geq 1$
A maximum of two rooms can be selected between room 2, 3 and 11
$y_2 + y_3 + y_{11} \leq 2$

```
// solution (optimal) with objective -10840
// Quality Incumbent solution:
// MILP objective                                -1.0840000000e+04
// MILP solution norm |x| (Total, Max)            2.00200e+03  1.50000e+03
// MILP solution error (Ax=b) (Total, Max)        0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)          0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)            0.00000e+00  0.00000e+00
//

x1 = 0;
x2 = 500;
x3 = 0;
x4 = 0;
x5 = 0;
x6 = 0;
x7 = 1500;
x8 = 0;
x9 = 0;
x10 = 0;
x11 = 0;
x12 = 0;
y1 = 0;
y2 = 1;
y3 = 0;
y4 = 0;
y5 = 0;
y6 = 0;
y7 = 1;
y8 = 0;
y9 = 0;
y10 = 0;
y11 = 0;
y12 = 0;
```

Figure 1: CPLEX Solution for Integer Problem

From the Cplex code we got $y_2 = 1$, $y_7 = 1$ which indicates that both rooms 2 and 7 were the two rooms selected for university ATS to host their event. As well $x_2 = 500 \; and \; x_7 = 1500$, was part of the output of the code which shows that room 2 will host 500 guests while room 7 will host 1500 guests. After analyzing our CPLEX output it was determined that picking rooms 2 and 7 would minimize the cost to rent the rooms while also maximizing admission sales. Since the value is negative, ATS University would make $10840 profit after paying the room rentals for ($260 (Room 2) + $650 (Room 7) = $910) rooms 2 and 7. Thus, ATS University concludes that the best possibility to host the event will be by renting two different rooms.

Alternative Solution

Alternatively, ATS University has been informed that it will cost $3000 extra to set up Audio/Visual equipment for larger rooms. The university wishes to see if it is cheaper to not select rooms greater than 1000. Due to this ATS University decided to compare the price of the rooms if they did not utilize rooms 1,6,7,8,9,10.

The law firm provided ATS University with room restrictions which are the following but due to the new limitations, ATS University can only adhere to the following:

- If Room 5 is selected then Room 12 has to be selected.
  - $y_5 - y_{12} \leq 0$

```
// solution (optimal) with objective -8215
// Quality Incumbent solution:
// MILP objective                                -8.2150000000e+03
// MILP solution norm |x| (Total, Max)            2.00400e+03  6.00000e+02
// MILP solution error (Ax=b) (Total, Max)        0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)          0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)            0.00000e+00  0.00000e+00
//

x2 = 550;
x3 = 500;
x4 = 600;
x5 = 0;
x11 = 0;
x12 = 350;
y2 = 1;
y3 = 1;
y4 = 1;
y5 = 0;
y11 = 0;
y12 = 1;
```

Figure 2. Alternative CPLEX Model Output

From CPLEX (refer to appendix for full code) can be analyzed that Room 2 is selected, allocating 550 people, Room 3 for 500 people, Room 4 for 600 people, and Room 12 for 350 people. Since the value is negative, the profit of this alternative solution is $8125.

Evaluation of Solutions and Selection of the Best Solution/Design

Taking a look at the alternative solution with the newly added constraints it can be noted the profit of this alternative solution is $8,125. The original solution yielded a profit of $10,840 before considering the extra Audio/Visual equipment costs. When considered, the profit is $10840 - $3000 which is $7840. Thus the alternative would be the best option as it makes the most profit.

Planning of the Implementation of the Solution

When analyzing the output of our model the generated solution allows us to determine which rooms would be the best to rent out while following the provided constraints. The solution is feasible and provides a realistic approach to the problem. The optimal solution with the given parameters and constraints allowed for ATS University to make $10,840 in profit by hosting the fundraiser. It was decided that any rooms with a room capacity of 1000 or more would require extra costs, requiring an alternative solution. This limited the rooms that can be chosen as ATS university would like to maximize a profit at the end of the event. By evaluating the alternative solution it can be noted that the profit made by the alternative solution is more than the original solution. This means that if ATS University decided to go with the rooms that have less than 1000 capacity they would yield maximum profit from the fundraiser.

This model may not be the most practical solution as our model only takes into account the room rental prices and various room capacities. When planning such a grand event it would be feasible to take into consideration other factors such as setup costs, staffing, food expenses, etc. In order to accurately implement this model, all the extra costs should be considered to ensure that the event hits its goal of maximizing the profit.

Problem 2

Problem Definition

ATS University determined they require 50 printers accessible to 60000 students. The university got quotes from various universities leading them to go with “Xerox C405” at $800 per printer. This deal requires the university to be exclusive to this printer for 12 years. This printer will stay in running condition until the end of year 8 before needing to be replaced. Salvage value of the printer is 42.5%, 32.5%, 25%, 22.5%, 10%, 7.5%, 5.5%, 3.5% and 1.5%, of the original cost from the end of year 0 to year 8 respectively. Students are charged $0.05 to print pages in black and white, and $0.25 for coloured pages. Table 2.1 shows the average type of pages printed for the next 8 years. Operating costs considered for the printers is the cost for black ink at $59.39, the cost for coloured ink at $229.49, the cost for a pack of 3000 paper at $86, and the warranty offered until the end of year 6 at $75. Each ink cartridge prints 6500 pages on average before running out of ink. Due to the uncertainty of the economy the estimated inflation rates for paper, black ink, and coloured ink are 15%, 18%, and 21% respectively. Determine which years the university should sell and repurchase the printers to make the most revenue.

Table 2.1: Revenue for Each Year

	0 year	1 year	2 years	3 years	4 years
Average black and white printed	7	7	6.9	6.85	6.8
Average Coloured Pages Printed	2.5	2.5	2.45	2.4	2.35

Table 2.1: Revenue for Each Year

	5 year	6 year	7 years	8 years
Average black and white printed	6.8	6.79	6.7	6.6
Average Coloured Pages Printed	2.3	2.3	2.3	2.3

Design Process Overview

Given the data provided, the total revenue for each year was determined as shown in Table 2.2, followed by a Table 2.3 for inflation cost of paper, black ink, and coloured ink per year for 8 years. Based on the inflation costs we determined the operating cost to construct a code for this resource-allocation dynamic programming problem.

Table 2.2 shows the revenue made each year by multiplying the cost to print each paper by the average quantity printed by the students each year. A sample calculation for year 0 can be found below:

Revenue for year 0:

[(0.05 x 7) + (0.25 x 2.5)] x 60000 = $58,500

Table 2.2 Revenue for Each Year

	0 year	1 year	2 years	3 years	4 years
Average black and white printed	7	7	6.9	6.85	6.8
Average Coloured Pages Printed	2.5	2.5	2.45	2.4	2.35
Total cost for 60,000 individuals	$58,500	$58,500	$57,450	$56,550	$55,650

Table 2.2 Revenue for Each Year

	5 year	6 year	7 years	8 years
Average black and white printed	6.8	6.79	6.7	6.6
Average Coloured Pages Printed	2.3	2.3	2.3	2.3
Total cost for 60,000 individuals	$54,900	$54,870	$54,600	$54,300

Table 2.3 represents the operating cost for the printer with the last row representing the overall operating cost. The first thing to determine the operating cost is calculating the quantity of pages printed. To do this, simply refer to Table 2.2. Add the average quantity of black and white pages printed and coloured pages printed and multiply by 60,000 representing the amount of students at ATS university. A sample calculation for year 0 is shown below.

Total amount of pages printed: (7+2.5) x 60,000 = 570,000

After determining the amount of pages printed, the cost for pages, black ink, and coloured ink can be determined. A package of 3000 papers can be bought at $86 with an inflation rate of 15% each year. By dividing total pages printed with 3000 pages you can determine the amount of packages required. Multiply that by the price of pages to determine the cost of paper per year. Ink cartridges can print on average 6500 pages before running out. Price for black ink cartridge is $59.39 with an inflation rate of 18% and price for coloured ink cartridge is 229.49 with an inflation rate of 21%. The inflated prices for each respective year can be found in Table 2.4. Determine how many of each type of page is printed by multiplying the average quantity of pages printed by 60,000 students. Divide the total number of pages printed by 6,500 to determine how many times the cartridges have to be replaced. Multiply the quantity needed by the cost of the respective ink cartridge to determine the overall costs. Sample calculations of year 0 can be shown below.

Cost for pages: (570,000/3000) x 86 = $16,340
Cost of black ink: (7 x 60,000)/6500 x 59.39 = $3837.51
Cost of colour: (2.5 x 60,000)/6500 x 229.49 = $5,295.92

The overall operating cost is the sum of cost for pages, black ink, colour ink and warranty. However, warranty is only considered for the first 6 years as that is how long it is offered.

Table 2.3 Operating cost for each year

	0 year	1 year	2 years	3 years	4 years
Total amount of pages printed	570,000	492,000	402,000	300,000	300,000
Cost for pages ($)	16340	18791	21268.44	24197.12	27525.86
Cost for black ink ($)	3837.51	4528.267	5267.011	6170.04	7227.51
Cost of colour ink ($)	5295.92	6408.06	7598.68	9006.77	10671.14
Warranty Cost ($)	75	75	75	75	75
Total ($)	25,548.44	29,802.33	34,209.14	39,448.93	45,499.51

Table 2.3 Operating cost for each year

	5 year	6 year	7 years	8 years
Total amount of pages printed	288,000	282,000	282,000	270,000
Cost for pages	31481.76	36164.24246	41177.10775	46827.52198

($)				
Cost for black ink ($)	8528.45	10048.77	11700.38	13600.38
Cost of colour ink ($)	12637.35	15291.20	18502.35	22387.85
Warranty Cost ($)	75	75	0	0
Total ($)	52,722.57	61,579.22	71,379.85	82,815.76

Table 2.4 Inflation Cost

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
Pages ($) 15%	86	98.9	113.73	130.79	150.41	172.98	198.92	228.76	263.07
Black ($) 18%	59.39	70.08	82.69	97.58	115.14	135.87	160.33	189.18	223.23
Color ($) 21%	229.49	277.68	335.99	406.55	491.93	595.24	720.24	871.49	1054.50

Table 2.5 Revenue, Operating Cost and Salvage Value for Each Year

	Age of Printer at the Beginning of Each Year				
	0 year	1 year	2 years	3 years	4 years
Revenue ($)	$58,500	$58,500	$57,450	$56,550	$55,650
Operating Costs ($)	25,548.44	29,802.33	34,209.14	39,448.93	45,499.51
Salvage Value at End of Year ($)	42.5%	32.5%	25%	22.5%	10%

Table 2.5 Revenue, Operating Cost and Salvage Value for Each Year

	Age of Printer at the Beginning of Each Year			
	5 year	6 year	7 years	8 years
Revenue ($)	$54,900	$54,870	$54,600	$54,300
Operating Costs ($)	52,722.57	61,579.22	71,379.85	82,815.76
Salvage Value at End of Year ($)	7.5%	5.5%	3.5%	1.5%

After inputting our data into MATLAB we are able to determine when we should replace printers (refer to the appendix for full code) .

Table 2.6 MATLAB Output for Keeping Printers

k11_1 = 8.9952e+04	k10_1 = 1.9903e+04 k10_2 = 3.4649e+04	k9_1 = 4.4601e+04 k9_2 = 4.4601e+04 k9_3 = 5.4890e+04	k8_1 = 6.4842e+04 k8_2 = 6.9298e+04 k8_3 = 6.4842e+04 k8_4 = 7.0991e+04	k7_1 = 8.0943e+04 k7_2 = 8.9539e+04 k7_3 = 8.9539e+04 k7_4 = 8.0943e+04 k7_5 = 7.6142e+04	k6_1 = 9.9491e+04 k6_2 = 1.0564e+05 k6_3 = 1.0978e+05 k6_4 = 1.0564e+05 k6_5 = 8.6093e+04 k6_6 = 7.7319e+04

Table 2.6 MATLAB Output for Keeping Printers

k5_1= 1.1973e+05 k5_2= 1.2419e+05	k4_1 = 1.3583e+05 k4_2 = 1.4443e+05 k4_3 = 1.4443e+05	k3_1 = 1.5438e+05 k3_2 = 1.6053e+05 k3_3 = 1.6467e+05	k2_1 = 1.7462e+05 k2_2 = 1.7908e+05 k2_3 = 1.8077e+05	k1_1 = 1.9072e+05 k1_2 = 1.9932e+05 k1_3 = 1.9932e+05	k0_1 = 2.0927e+05 k0_2 = 2.1542e+05 k0_3 = 2.1956e+05

k5_3 = 1.2588e+05 k5_4 = 1.2588e+05 k5_5 = 1.1079e+05 k5_6 = 8.7271e+04 k5_7 = 6.9810e+04	k4_4 = 1.4198e+05 k4_5 = 1.3103e+05 k4_6 = 1.1197e+05 k4_7 = 7.9761e+04 k4_8 = 5.2230e+04	k3_4 = 1.6053e+05 k3_5 = 1.4713e+05 k3_6 = 1.3221e+05 k3_7 = 1.0446e+05 k3_8 = 6.2182e+04	k2_4 = 1.8077e+05 k2_5 = 165681 k2_6 = 1.4831e+05 k2_7 = 1.2470e+05 k2_8 = 8.6879e+04	k1_4 = 1.9687e+05 k1_5 = 1.8592e+05 k1_6 = 1.6686e+05 k1_7 = 1.4080e+05 k1_8 = 1.0712e+05	k0_4 = 2.1542e+05 k0_5 = 2.0202e+05 k0_6 = 1.8710e+05 k0_7 = 1.5935e+05 k0_8 = 1.2322e+05

In Ki_j, the i represents the year being considered in the question and j represents how long the printers should be kept.

Based on the results received after running MATLAB, it is evident that the best course of action for ATS University is keep the printers for 3 years after buying at year 0, sell, and repurchase again at year 3 for the next 3 years. They should buy printers at years 3, 6, and 9 , so keep them for 3 years before getting replaced. The profit yielded by following this suggestion can be seen below:
$219560+ $164670+ $109780+ $548900=$1,042,910

By doing this, ATS university will profit $1,042,910 over the next 12 years.

Alternative Solution

Alternatively, instead of having the option to keep or sell the printer after each year, ATS University has to keep it for a minimum of 3 years before determining if they keep it or sell it. This will change the course of the problem since the equation will change.

Table 2.7 MATLAB Output for Keeping Printers

k84 = 7.0991e+04	k74 = 7.0991e+04 k75 = 7.6142e+04	k64 = 7.0991e+04 k65 = 7.6142e+04 k66 = 7.7319e+04	k54 = 7.0991e+04 k55 = 7.6142e+04 k56 = 7.7319e+04 k57 = 6.9810e+04

Table 2.7 MATLAB Output for Keeping Printers

k44 = 1.4198e+05 k45 = 7.6142e+04 k46 = 7.7319e+04 k47 = 6.9810e+04 k48 = 5.2230e+04	k34 = 1.4713e+05 k35 = 1.4713e+05 k36 = 7.7319e+04 k37 = 6.9810e+04 k38 = 5.2230e+04	k24 = 1.4831e+05 k25 = 1.5228e+05 k26 = 1.4831e+05 k27 = 6.9810e+04 k28 = 5.2230e+04	k14 = 1.4831e+05 k15 = 1.5346e+05 k16 = 1.5346e+05 k17 = 1.4080e+05 k18 = 5.2230e+04	k04 = 2.1297e+05 k05 = 1.5346e+05 k06 = 1.5464e+05 k07 = 1.4595e+05 k08 = 1.2322e+05

In Ki_j, the i represents the year being considered in the question and j represents how long the printers should be kept.

The MATLAB results above determine the best course of action given the constraint is to replace their current printers every 4 years starting from year 0. The profit yielded by following this suggestion can be seen below:
$212970 +$141980 +$70991 = $425,921

By doing this, ATS university will profit $425,921 over the next 12 years.

Evaluation of Solutions and Selection of the Best Solution/Design

Looking and comparing the original solution to the alternative, we can conclude that the original statement would be the best option. For the original solution, it was found that ATS University has to sell and replace their printers every 3 years, which will result in a profit of $1,042,910 after 12 years. However, for the alternative solution, which is to sell and replace the printers every four years, they will only be making a profit of $425,921. We can see that this alternative lowers their profit by more than half. Thus, ATS university should stick to their original plan and replace their printers every 3 years.

Planning of the Implementation of the Solution

This problem is based on current predictions based on the current performance of the economy. However, the market is highly unreliable and reasonable to change at any time. Thus, the inflation rate might not be as high as predicted in this problem which will change the solution and decision that ATS university will make.

Problem 3

Problem Definition

ATS university is analyzing their failure/success rate in their engineering mathematical courses to analyze where students are struggling or succeeding the most. There are 8 courses being analyzed: MTH 100, MTH 150, MTH 200, MTH 250, MTH 300, MTH 350, MTH 400, and MTH 500. MTH 100 is a prerequisite for MTH 150, MTH 150 is a prerequisite for MTH 200, MTH 250 is a prerequisite for MTH 300, and so forth. Thus, in order for engineering students to receive their diplomas, it is mandatory for them to pass all eight math courses. After going over last year's statistics this is what ATS university analyzed:

- If a student is taking MTH 100, there is a 65% chance that the student will move forward to take MTH 150 and there is a 15 % chance that they will repeat the course.
- 35 % of students taking MTH 150 will repeat the course and 50% will move on to MTH 200
- 20 % of students taking MTH 200 will repeat the course and 70% will move forward to take MTH 250
- 35 % of students taking MTH 250 will repeat the course and 60% will move forward to take MTH 300
- 10 % of students taking MTH 300 will repeat the course and 85% will move forward to take MTH 350
- 15 % of students taking MTH 350 will repeat the course and 80% will move forward to take MTH 400
- 5 % of students taking MTH 400 will repeat the course and 90% will move forward to take MTH 500
- 95% of the students pass all required math courses, while 5% will repeat MTH 500

ATS university wants to analyze 5 main points which can be found below:

- What is the probability that an MTH 100 student passes all the courses?
- What is the probability that a student quits halfway through completing their mathematical courses?
- If a student gets credits from their previous university for MTH 100 and MTH 150, how many semesters can they expect to finish remaining their math courses?
- How long would it take for a student to finish their required math courses?
- ATS university wants to determine if students are more successful at the beginning or near the end. Thus, when students are less prone to quitting.

These questions are important in order for ATS university to determine if these courses are too challenging or easy for all engineering students.

Design Process Overview

From the statistics found from ATS University the following matrix can be formed:

	MTH 100	MTH 150	MTH 200	MTH 250	MTH 300	MTH 350	MTH 400	MTH 500	Quit	Pass
MTH 100	0.15	0.65	0	0	0	0	0	0	0.20	0
MTH 150	0	0.35	0.50	0	0	0	0	0	0.15	0
MTH 200	0	0	0.20	0.70	0	0	0	0	0.10	0
MTH 250	0	0	0	0.35	0.60	0	0	0	0.05	0
MTH 300	0	0	0	0	0.10	0.85	0	0	0.05	0
MTH 350	0	0	0	0	0	0.15	0.80	0	0.05	0
MTH 400	0	0	0	0	0	0	0.05	0.90	0.05	0
MTH 500	0	0	0	0	0	0	0	0.05	0	0.95
Quit	0	0	0	0	0	0	0	0	1	0
Pass	0	0	0	0	0	0	0	0	0	1

*pass = the student has passed all required math courses

*quit = dropping out of the program or taking an extended leave

Taking a look at our matrix we are able to separate the matrix into various different matrices such as the Q and R matrices. Which can be seen below:

$$Q = \begin{bmatrix} 0.15 & 0.65 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.35 & 0.50 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.20 & 0.70 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.35 & 0.60 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.10 & 0.85 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & 0.80 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0.90 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 \end{bmatrix} \quad R = \begin{array}{c} \begin{array}{cc} \text{Quit} & \text{Pass} \end{array} \\ \begin{bmatrix} 0.20 & 0 \\ 0.15 & 0 \\ 0.10 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0 & 0.95 \end{bmatrix} \end{array}$$

From the matrices above ATS's questions may be answered, refer to the appendix for the codes.

a) What is the probability that an MTH 100 student passes all courses?

```
a) Probability that MTH 100 student will pass all of their courses?

pass =

   40.0095


        The probability that a MTH 100 student will pass all courses is 4.000953e+01 percent
```

Refer to the full MATLAB code in the appendix

Looking at $(I - Q)^{-1}R$ (refer to appendix for matrix) we can observe that 40 % of students will pass all courses. ATS University can analyze that it is below average, concluding the courses are quite challenging. In order to increase the passing rate, a quick solution can be to offer a pre-MTH 100 course for incoming students to help them succeed and have a higher success rate.

b) What is the probability that a student quits halfway through completing their mathematical courses?

Since MTH 250 and MTH 300 create the half point, the average of both percentages is taken in order to determine when a student quits halfway through their studies. Looking at $(I - Q)^{-1}R$ (refer to the appendix for matrix)

```
b) Probability that a student will quit halfway through?

Quit_300 =

    0.2227


Quit_250 =

    0.1579

        The probability that a student will quit halfway through the program is 1.902834e+01 percent
```

Refer to the full MATLAB code in the appendix

Thus, 19.05% of students quit halfway through their studies.

c) If a student gets credits from their previous university for MTH 100 and MTH 150, how many semesters can they expect to finish remaining their math courses?

```
c) Starting at MTH 200, how many semesters until they finish remaining math courses?

        If a student starts at MTH 200 then they will finish all remaining math courses in 4.391026e+00 semesters.
```

Refer to the full MATLAB code in the appendix

Due to getting credits for both MTH 100 and MTH 150, the student will have to start at MTH 200. Since the question is asking for an expected number of periods, we would have to look at $(I - Q)^{-1}$(refer to the appendix for matrix). This means that on average a student will finish all math courses in 4.391 semesters.

d) How long would it take for a student to finish their required math courses?

```
d) How long it will take for students to complete their required math courses:

        It would take 5.801601e+00 semesters to finish their required math courses.
>>
```

Refer to the full MATLAB code in the appendix

In order for a student to finish all of their required math courses , they should take on average 5.801 semesters to finish all their math courses at ATS University.

e) ATS university wants to determine if students are more successful at the beginning or near the end. So, when are the students less prone to quitting?

```
e) The university wants to determine if students are more successful in the beginning of their students or near the end.

        The passing rate for the first half of the program is: 5.951967e+01 percent
        The passing rate for the second half of the program is: 9.202786e+01 percent
>>
```

Refer to the full MATLAB code in the appendix

We can analyze that there is a higher success rate for the second half of the program which is expected since students are less prone to want to quit after already having completed most of the courses.

Alternative solution

After analyzing that only 40 % of the students end up passing all mathematical courses, ATS University wants to compare their old curriculum (refer to the appendix) to their current program. They have been following their current program for the past 10 years and want to determine if they should go back to their old curriculum or keep going with their current classes. Previously in the old curriculum, they had two differences. First, one was that MTH 150 was split into two courses to make it easier for the students and the second change is that they had MTH 400 and MTH 500 as one course which made it a bit more demanding near the end of the program. In order to determine if they should keep their current program or go back to their old curriculum, they want to compare the success rates since their goal is to help students succeed while still maintaining a challenging program that will meet the engineering department requirements.

	MTH 100	MTH 150A	MTH 150B	MTH 200	MTH 250	MTH 300	MTH 350	MTH 400/ 500	Quit	Pass
MTH 100	0.15	0.67	0	0	0	0	0	0	0.18	0
MTH 150A	0	0.25	0.60	0	0	0	0	0	0.15	0
MTH 150B	0	0	0.15	0.80	0	0	0	0	0.05	0
MTH 200	0	0	0	0.25	0.70	0	0	0	0.05	0
MTH 250	0	0	0	0	0.10	0.85	0	0	0.05	0
MTH 300	0	0	0	0	0	0.15	0.80	0	0.05	0
MTH 350	0	0	0	0	0	0	0.05	0.90	0.05	0
MTH 400/500	0	0	0	0	0	0	0	0.15	0	0.85
Quit	0	0	0	0	0	0	0	0	1	0
Pass	0	0	0	0	0	0	0	0	0	1

*pass = the student has passed all required math courses
*quit = dropping out of the program or taking an extended leave

$$Q = \begin{bmatrix} 0.15 & 0.67 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.60 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.15 & 0.80 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0.70 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.10 & 0.85 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.15 & 0.80 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0.90 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.15 \end{bmatrix} \quad R = \begin{array}{cc} \text{Quit} & \text{Pass} \\ \end{array} \begin{bmatrix} 0.18 & 0 \\ 0.15 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0.05 & 0 \\ 0 & 0.85 \end{bmatrix}$$

a) **What is the probability that an MTH 100 student passes all the courses ?**

```
What is the Probability that a MTH 100 student passes all their courses?

ans =

    0.4665
```

Figure 3. Probability of MTH 100 student passing all courses
Refer to the full MATLAB code in the appendix

46.6 % of MTH 100 will pass all courses which are higher than before which was 40%. This indicated that more students will have a successful

b) **What is the probability that a student quits halfway through?**

Now, MTH 200 and MTH 250 will create the half point, the average of both percentages is taken in order to determine when a student quits halfway through their studies. Looking at the $(I - Q)^{-1}R$ matrix (refer to Appendix) We can observe that 21.4% of students quit MTH 250 and 15.8% quit MTH 300.

```
What is the probability that a student quits halfway through?

Quit_200 =

    0.2140

Quit_250 =

    0.1579

ans =

    0.1860
```

Figure 4. Probability a student quits halfway through
Refer to the full MATLAB code in the appendix

Thus, 18.6 % of students quit halfway through which is lower than the previous 19.05% of students quitting.

c) How long would it take for a student to finish their required math courses?

```
How long would it take for a student to finish their required math courses?

ans =

    6.0587
```

Figure 5. Student finishing their required math courses
Refer to the full MATLAB code in the appendix

Students take on average 6.057 semesters to complete all their math courses starting from MTH 100. This program will then take on average longer to complete than the previous one which can be a disadvantage for some of the students.

d) The university wants to determine if students are more successful at the beginning of their students or near the end.

```
The university wants to determine if students are more successful in the beginning of their students or near the end.

The passing rate for the first half of the program is: 6.459870e-01
The passing rate for the second half of the program is: 9.202786e-01
>>
```

Figure 6. Determining when the students are successful
Refer to the full MATLAB code in the appendix

We can analyze that there is still a higher success rate for the second half of the program which is expected since students are less prone to want to quit after already having completed most of the courses.

Evaluation of solutions and selection of the best solution/design

After analyzing both curriculums ATS university should determine which curriculum to go with. If their goal is to have a higher success rate then they should select their old curriculum since 46.6% of students pass all courses which are higher than their current program with a 40% success rate. As well, their old program has a lower percentage of students quitting halfway through with an 18.6 % chance of quitting instead of their current programs' 19.05% chance. Thus, if the university's main goal is to have a higher success rate, MTH 150 should be split into 2 courses and combine MTH 400 and MTH 500 into one course. However, the disadvantage of their old curriculum is that on average it takes longer for students to finish all their math courses. This is important to take into account because an engineering program has a limit of 8 years to be completed. So students have to make sure they complete all required courses on time or else they will be required to withdraw from the program. Therefore, whether they should stick to their current program or go back to their old one depends entirely on ATS's main goal as both have advantages and disadvantages.

Planning of the Implementation of the Solution

Both the original and alternative models provide insight into how the math courses are run at ATS University. Taking a look at the data and learning to interpret and implement it can help the University gauge where the students are having difficulties and cater to a solution. When analyzing the data, certain factors that can change the probability were not taken into consideration. These factors include the professors teaching the course, the TA's grading the tests, and the students' performance. These human factors vary from individual to individual, meaning the data is bound to change every year. Overall this can be a practical model when determining the structure of courses and ensuring that all students are learning the necessary materials in order to succeed in subsequent courses.

References

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Appendix

Problem 1: Integer programing CPLEX

```
dvar int+ x1;
dvar int+ x2;
dvar int+ x3;
dvar int+ x4;
dvar int+ x5;
dvar int+ x6;
dvar int+ x7;
dvar int+ x8;
dvar int+ x9;
dvar int+ x10;
dvar int+ x11;
dvar int+ x12;
dvar int+ y1;
dvar int+ y2;
dvar int+ y3;
dvar int+ y4;
dvar int+ y5;
dvar int+ y6;
dvar int+ y7;
dvar int+ y8;
dvar int+ y9;
dvar int+ y10;
dvar int+ y11;
dvar int+ y12;

minimize -5*x1 - 5.5*x2 - 5*x3 - 4*x4 - 3.5*x5 - 5.75*x6 - 6*x7 - 5.75*x8 -
5.75*x9 - 5.25*x10 - 5*x11 - 4*x12+ 500*y1 + 260*y2 + 250*y3 + 400*y4 + 175*y5
+ 900*y6 + 650*y7 + 775*y8 +575*y9 + 510*y10 + 300*y11 + 200*y12;
 subject to
 {
      Constraint1: x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 +x11 +
x12== 2000;

      Constraint2: y8 + y7 <= 1;
      Constraint3: y1 + y10 <= 1;
      Constraint4: y5 - y12 <= 0;
    Constraint5: y1 + y6 + y7 + y8 + y9 + y10 >= 1;
    Constraint6: y2 +y3 +y11 <= 2;

      Constraint7: x1 <= 1000*y1;
      Constraint8: x2 <= 550*y2;
      Constraint9: x3 <= 500*y3;
      Constraint10: x4 <= 700*y4;
      Constraint11: x5 <= 250*y5;
      Constraint12: x6 <= 2000*y6;
```

```
      Constraint13: x7 <= 1500*y7;
      Constraint14: x8 <= 1750*y8;
      Constraint15: x9 <= 1300*y9;
      Constraint16: x10 <= 1050*y10;
      Constraint17: x11 <= 600*y11;
      Constraint18: x12 <= 350*y12;

      Constraint19: y1 <= 1;
      Constraint20: y2 <= 1;
      Constraint21: y3 <= 1;
      Constraint22: y4 <= 1;
      Constraint23: y5 <= 1;
      Constraint24: y6 <= 1;
      Constraint25: y7 <= 1;
      Constraint26: y8 <= 1;
      Constraint27: y9 <= 1;
      Constraint28: y10 <= 1;
      Constraint29: y11 <= 1;
      Constraint30: y12 <= 1;
}
```

Alternative solution CPLEX code for Integer Programming:

```
dvar int+ x2;
dvar int+ x3;
dvar int+ x4;
dvar int+ x5;
dvar int+ x11;
dvar int+ x12;
dvar int+ y2;
dvar int+ y3;
dvar int+ y4;
dvar int+ y5;
dvar int+ y11;
dvar int+ y12;

minimize - 5.5*x2 - 5*x3 - 4*x4 - 3.5*x5 - - 5*x11 - 4*x12 + 260*y2 + 250*y3 +
400*y4 + 175*y5 + 300*y11 + 200*y12;
 subject to
 {

      Constraint1: x2 + x3 + x4 + x5 + x11 + x12 == 2000;

      Constraint2: x2 <= 550*y2;
      Constraint3: x3 <= 500*y3;
      Constraint4: x4 <= 700*y4;
      Constraint5: x5 <= 250*y5;
      Constraint6: x11 <= 600*y11;
      Constraint7: x12 <= 350*y12;
```

```
        Constraint8: y2  <= 1;
        Constraint9: y3 <= 1;
        Constraint10: y4 <= 1;
        Constraint11: y5 <= 1;
        Constraint12: y11 <= 1;
        Constraint13: y12 <= 1;

        Constraint14: y5 - y12 <= 0;
}
```

Problem 2: Dynamic Programming

MATLAB Code

```
table=[58500,58500,57450,56550,55650,54900,54870,54600,54300;
25548.43,29802.33,34209.14,39448.93,45499.51,52722.57,61579.22,71379.85,82815.76;
17000,13000,10000,9000,4000,3000,2200,1400,600];
disp(table);
% what year should you repurchase a printer with the lowest cost over a 12
% year period
% keeping it X amount of years
x=1;
keep1 = -40000 + table(1,1) - table(2,1) + table(3,1);
keep2 = keep1 - table(3,x) +table(3,x+1) - table(2,x+1) + table(1,x+1);
keep3 = keep2 - table(3,x+1) +table(3,x+2) - table(2,x+2) + table(1,x+2);
keep4 = keep3 - table(3,x+2) +table(3,x+3) - table(2,x+3) + table(1,x+3);
keep5 = keep4 - table(3,x+3) +table(3,x+4) - table(2,x+4) + table(1,x+4);
keep6 = keep5 - table(3,x+4) +table(3,x+5) - table(2,x+5) + table(1,x+5);
keep7 = keep6 - table(3,x+5) +table(3,x+6) - table(2,x+6) + table(1,x+6);
keep8 = keep7 - table(3,x+6) +table(3,x+7) - table(2,x+7) + table(1,x+7);
keep9 = keep8 - table(3,x+7) +table(3,x+8) - table(2,x+8) + table(1,x+8);
 c = 40000;
r=2;
counter=0;
x=1;
array = [];
i=0;
while counter==0
   p = table(1,1) - table(2,1) + table(3,1) - c;
   fprintf('Keep 1: %d \n',p);
    while r <= 9
       p = p - table(3,x) +table(3,x+1) - table(2,x+1) + table(1,x+1);
       fprintf('Keep %d: %d \n', r,p);
       r=r+1;
    end
    counter=counter+1;
end
%M = max(table,[],2)
g12 = 0
g11 = keep1 + g12
g10 = max(keep1 +g11, keep2 +g12)
p = max(keep1 +g10, keep2 +g11);
g9 = max(p,keep3 +g12)
o = max(keep1 +g9, keep2 +g10);
u = max(keep3 +g11, keep4 +g12);
g8 = max(o,u)
y = max(keep1 +g8, keep2 +g9);
t = max(keep3 +g10, keep4 +g11);
r = max(y,t);
g7 = max(r, keep5 +g12)
```

```
e = max(keep1 +g7, keep2 +g8);
w = max(keep3 +g9, keep4 +g10);
a = max(keep5 +g11, keep6 +g12);
s = max(e,w);
g6 = max(s,a)
z = max(keep1 +g6, keep2 +g7);
n = max(keep3 +g8, keep4 +g9);
v = max(keep5 +g10, keep6 +g11);
b = max(z,n);
m = max(b,v);
g5 = max(m, keep7+g12)
ki = max(keep1 +g5, keep2 +g6);
k = max(keep3 +g7, keep4 +g8);
h = max(keep5 +g9, keep6 +g10);
f = max(keep7 +g11, keep8 +g12);
q = max(ki,k);
j = max(h,f);
g4 = max(q,j)
ti = max(keep1 +g4, keep2 +g5);
hi = max(keep3 +g6, keep4 +g7);
li = max(keep5 +g8, keep6 +g9);
fi = max(keep7 +g10, keep8+ g11);
com = max(ti, hi);
kom = max(li,fi);
g3 = max(com, kom)
aa = max(keep1 +g3, keep2 +g4);
bb = max(keep3 +g5, keep4 +g6);
cc = max(keep5 +g7, keep6 +g8);
dd = max(keep7 +g9, keep8 +g10);
ab = max(aa, bb);
cd = max(cc,dd);
g2 = max(ab, cd)
ee = max(keep1 +g2, keep2 +g3);
ff = max(keep3 +g4, keep4 +g5);
gg = max(keep5 +g6, keep6 +g7);
hh = max(keep7 +g8, keep8 +g9);
ef = max(ee, ff);
gh = max(gg, hh);
g1 = max(ef, gh)
ii = max(keep1 +g1, keep2 +g2);
jj = max(keep3 +g3, keep4 +g4);
ll = max(keep5 +g5, keep6 +g6);
kk = max(keep7 +g7, keep8 +g8);
ij = max(ii,jj);
lk = max(ll,kk);
g0 = max(ij,lk)
k11_1 = keep1 +g12
k10_1 = keep1 +g11
k10_2 = keep2 +g12
k9_1 = keep1 +g10
k9_2 = keep2 +g11
k9_3 = keep3 +g12
k8_1 = keep1 +g9
k8_2 = keep2 +g10
k8_3 = keep3 +g11
k8_4 = keep4 +g12
k7_1= keep1 + g8
k7_2= keep2 + g9
k7_3= keep3 + g10
k7_4= keep4 + g11
k7_5= keep5 + g12
k6_1= keep1 + g7
k6_2= keep2 + g8
k6_3= keep3 + g9
k6_4= keep4 + g10
k6_5= keep5 + g11
```

```
k6_6= keep6 + g12
k5_1= keep1 + g6
k5_2= keep2 + g7
k5_3= keep3 + g8
k5_4= keep4 + g9
k5_5= keep5 + g10
k5_6= keep6 + g11
k5_7= keep7 + g12
k4_1= keep1 + g5
k4_2= keep2 + g6
k4_3= keep3 + g7
k4_4= keep4 + g8
k4_5= keep5 + g9
k4_6= keep6 + g10
k4_7= keep7 + g11
k4_8= keep8 + g12
k3_1= keep1 + g4
k3_2= keep2 + g5
k3_3= keep3 + g6
k3_4= keep4 + g7
k3_5= keep5 + g8
k3_6= keep6 + g9
k3_7= keep7 + g10
k3_8= keep8 + g11
k2_1= keep1 + g3
k2_2= keep2 + g4
k2_3= keep3 + g5
k2_4= keep4 + g6
k2_5= keep5 + g7
k2_6= keep6 + g8
k2_7= keep7 + g9
k2_8= keep8 + g10
k1_1= keep1 + g2
k1_2= keep2 + g3
k1_3= keep3 + g4
k1_4= keep4 + g5
k1_5= keep5 + g6
k1_6= keep6 + g7
k1_7= keep7 + g8
k1_8= keep8 + g9
k0_1= keep1 + g1
k0_2= keep2 + g2
k0_3= keep3 + g3
k0_4= keep4 + g4
k0_5= keep5 + g5
k0_6= keep6 + g6
k0_7= keep7 + g7
k0_8= keep8 + g8
```

Dynamic Programming Alternative Solution:

MATLAB Code

```
table=[58500,58500,57450,56550,55650,54900,54870,54600,54300;
25548.43,29802.33,34209.14,39448.93,45499.51,52722.57,61579.22,71379.85,82815.76;
17000,13000,10000,9000,4000,3000,2200,1400,600];
disp(table);
% what year should you repurchase a printer with the lowest cost over a 12
% year period
% keeping it X amount of years
x=1;
keep4 = keep3 - table(3,x+2) +table(3,x+3) - table(2,x+3) + table(1,x+3);
keep5 = keep4 - table(3,x+3) +table(3,x+4) - table(2,x+4) + table(1,x+4);
```

```
keep6 = keep5 - table(3,x+4) +table(3,x+5) - table(2,x+5) + table(1,x+5);
keep7 = keep6 - table(3,x+5) +table(3,x+6) - table(2,x+6) + table(1,x+6);
keep8 = keep7 - table(3,x+6) +table(3,x+7) - table(2,x+7) + table(1,x+7);
keep9 = keep8 - table(3,x+7) +table(3,x+8) - table(2,x+8) + table(1,x+8);
 c = 40000;
i=4;
counter=0;
x=1;
while counter==0
   p = table(1,1) - table(2,1) + table(3,1) - c;
   fprintf('Keep 1: %d \n',p);
    while i <= 9
       p = p - table(3,x) +table(3,x+1) - table(2,x+1) + table(1,x+1);
       fprintf('Keep %d: %d \n', i,p);
       i=i+1;
    end
    counter=counter+1;
end
%M = max(table,[],2)
g12 = 0
g11 = 0
g10 = 0
g9 = 0
g8 = keep4 +g12
t = max(keep3 +g10, keep4 +g11);
g7 = max(keep5 +g12,t)
a = max(keep5 +g11, keep6 +g12);
g6 = max(a,keep4 +g10)
v = max(keep5 +g10, keep6 +g11);
b = max(keep7+g12,keep4 +g9);
g5 = max(b, v)
h = max(keep5 +g9, keep6 +g10);
f = max(keep7 +g11, keep8 +g12);
j = max(h,f);
g4 = max(keep4 +g8,j)
hi = keep4 +g7;
li = max(keep5 +g8, keep6 +g9);
fi = max(keep7 +g10, keep8+ g11);
kom = max(li,fi);
g3 = max(kom, hi)
bb = keep4 +g6;
cc = max(keep5 +g7, keep6 +g8);
dd = max(keep7 +g9, keep8 +g10);
cd = max(cc,dd);
g2 = max(bb, cd)
ff =  keep4 +g5;
gg = max(keep5 +g6, keep6 +g7);
hh = max(keep7 +g8, keep8 +g9);
gh = max(gg, hh);
g1 = max(ff, gh)
jj = keep4 +g4;
ll = max(keep5 +g5, keep6 +g6);
kk = max(keep7 +g7, keep8 +g8);
lk = max(ll,kk);
g0 = max(jj,lk)
k84 = keep4 +g12
k74= keep4 + g11
k75= keep5 + g12
k64= keep4 + g10
k65= keep5 + g11
k66= keep6 + g12
k54= keep4 + g9
k55= keep5 + g10
k56= keep6 + g11
k57= keep7 + g12
k44= keep4 + g8
```

```
k45= keep5 + g9
k46= keep6 + g10
k47= keep7 + g11
k48= keep8 + g12
k34= keep4 + g7
k35= keep5 + g8
k36= keep6 + g9
k37= keep7 + g10
k38= keep8 + g11
k24= keep4 + g6
k25= keep5 + g7
k26= keep6 + g8
k27= keep7 + g9
k28= keep8 + g10
k14= keep4 + g5
k15= keep5 + g6
k16= keep6 + g7
k17= keep7 + g8
k18= keep8 + g9
k04= keep4 + g4
k05= keep5 + g5
k06= keep6 + g6
k07= keep7 + g7
k08= keep8 + g8
```

Problem 3: Markov chain

Matrices

$$(I - Q)^{-1} = \begin{bmatrix} 1.176 & 1.176 & 0.735 & 0.792 & 0.528 & 0.528 & 0.445 & 0.421 \\ 0 & 1.538 & 0.962 & 1.036 & 0.690 & 0.690 & 0.581 & 0.551 \\ 0 & 0 & 1.25 & 1.346 & 0.897 & 0.897 & 0.756 & 0.716 \\ 0 & 0 & 0 & 1.538 & 1.026 & 1.026 & 0.864 & 0.818 \\ 0 & 0 & 0 & 0 & 1.111 & 1.111 & 0.936 & 0.886 \\ 0 & 0 & 0 & 0 & 0 & 1.176 & 0.991 & 0.939 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.053 & 0.997 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.053 \end{bmatrix}$$

$$(I - Q)^{-1}R = \begin{bmatrix} 0.599 & 0.4 \\ 0.477 & 0.523 \\ 0.312 & 0.68 \\ 0.223 & 0.777 \\ 0.158 & 0.842 \\ 0.108 & 0.892 \\ 0.053 & 0.947 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix}$$

MATLAB Code

```
m = [0.15,0.65,0,0,0,0, 0, 0, 0.2, 0; 0, 0.35, 0.5, 0, 0, 0, 0, 0, 0.15, 0; 0, 0, 0.20, 0.70, 0, 0, 0,
0, 0.10, 0; 0, 0, 0, 0.35, 0.60, 0, 0, 0, 0.05, 0; 0, 0, 0, 0, 0.10, 0.85, 0, 0, 0.05, 0; 0, 0, 0, 0,
0, 0.15, 0.80, 0, 0.05, 0; 0, 0, 0, 0, 0, 0, 0.05, 0.90, 0.05, 0; 0, 0, 0, 0, 0, 0, 0, 0.05, 0 0.95; 0,
0, 0, 0, 0, 0, 0, 0, 1, 0; 0, 0, 0, 0, 0, 0, 0, 0, 0, 1];
disp(m);
Q=[0.15,0.65,0,0,0,0,0,0;0,0.35,0.50,0,0,0,0,0;0,0,0.2,0.7,0,0,0,0;0,0,0,0.35,0.60,0,0,0;
0,0,0,0,0.10,0.85,0,0; 0,0,0,0,0,0.15,0.80,0; 0,0,0,0,0,0,0.05,0.9; 0,0,0,0,0,0,0,0.05];
R = [0.2,0; 0.15,0; 0.10,0; 0.05,0; 0.05,0; 0.05,0; 0.05,0; 0,0.95];
I=[1,0,0,0,0,0,0,0;0,1,0,0,0,0,0,0; 0,0,1,0,0,0,0,0; 0,0,0,1,0,0,0,0; 0,0,0,0,1,0,0,0; 0,0,0,0,0,1,0,0;
0,0,0,0,0,0,1,0; 0,0,0,0,0,0,0,1];
A = (inv(I-Q));
B = A*R;
disp('a) Probability that MTH 100 student will pass all of their courses?');
pass = B(1,2)*100
fprintf("\n\tThe probability that a MTH 100 student will pass all courses is %d percent\n\n",pass)
disp('b) Probability that a student will quit halfway through?');
Quit_300 = B(4,1)
Quit_250 = B(5,1)
quit = ((Quit_250+Quit_300)/2)*100;
fprintf("\n\tThe probability that a student will quit halfway through the program is %d
percent\n\n",quit)
disp('c) Starting at MTH 200, how many semesters until they finish remaining math courses?');
% math courses
sum = A(3,3) + A(3,4) + A(3,5) + A(3,6);
fprintf("\n\tIf a student starts at MTH 200 then they will finish all remaining math courses in %d
semesters.\n\n",sum)
disp('d) How long it will take for students to complete their required math courses:')
ans = A(1,1)+A(1,2)+A(1,3)+A(1,4)+A(1,5)+A(1,6)+A(1,7)+A(1,8);
fprintf("\n\tIt would take %d semesters to finish their required math courses.\n",ans)
fprintf("\ne) The university wants to determine if students are more successful in the beginning of
their students or near the end.\n")
first = ((B(1,2)+B(2,2)+B(3,2)+B(4,2))/4)*100;
fprintf("\n\tThe passing rate for the first half of the program is: %d percent",first)
second = ((B(5,2)+B(6,2)+B(7,2)+B(8,2))/4)*100;
fprintf("\n\tThe passing rate for the second half of the program is: %d percent",second)
```

Markov Chain Alternative Solution:

Matrices

$(I - Q)^{-1}$:

$$\begin{bmatrix} 1.176 & 1.051 & 0.742 & 0.791 & 0.615 & 0.615 & 0.518 & 0.549 \\ 0 & 1.333 & 0.941 & 1.004 & 0.781 & 0.781 & 0.658 & 0.696 \\ 0 & 0 & 0.176 & 1.255 & 0.976 & 0.976 & 0.822 & 0.870 \\ 0 & 0 & 0 & 1.333 & 1.037 & 1.037 & 0.873 & 0.925 \\ 0 & 0 & 0 & 0 & 1.111 & 1.111 & 0.936 & 0.991 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1.176 & 0.991 & 1.049 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.053 & 1.115 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.176 \end{bmatrix}$$

$(I - Q)^{-1}R =$

$$\begin{bmatrix} 0.534 & 0.466 \\ 0.408 & 0.592 \\ 0.260 & 0.740 \\ 0.214 & 0.786 \\ 0.158 & 0.842 \\ 0.108 & 0.892 \\ 0.053 & 0.947 \\ 0 & 1 \end{bmatrix}$$

MATLAB Code

```
alt=[0.15,0.67,0,0,0,0,0,0,0.18,0;0,0.25,0.6,0,0,0,0,0,0.15,0;0,0,0.15,0.80,0,0,0,0,0.05,0;
0,0,0,0.25,0.70,0,0,0,0.05,0;0,0,0,0,0.10,0.85,0,0,0.05,0;0,0,0,0,0,0.15,0.80,0,0.05,0;
0,0,0,0,0,0,0.05,0.90,0.05,0; 0,0,0,0,0,0,0,0.15,0,0.85; 0,0,0,0,0,0,0,0,1,0; 0,0,0,0,0,0,0,0,0,1];
disp(alt)
Q=[0.15,0.67,0,0,0,0,0,0;0,0.25,0.6,0,0,0,0,0;0,0,0.15,0.80,0,0,0,0;0,0,0,0.25,0.70,0,0,0;
0,0,0,0,0.10,0.85,0,0; 0,0,0,0,0,0.15,0.80,0; 0,0,0,0,0,0,0.05,0.90; 0,0,0,0,0,0,0,0.15];
I = [1,0,0,0,0,0,0,0; 0,1,0,0,0,0,0,0;0,0,1,0,0,0,0,0;0,0,0,1,0,0,0,0;0,0,0,0,1,0,0,0; 0,0,0,0,0,1,0,0;
0,0,0,0,0,0,1,0; 0,0,0,0,0,0,0,1];
R = [0.18,0; 0.15,0; 0.05,0; 0.05,0; 0.05,0; 0.05,0; 0.05,0; 0,0.85];
A = (inv(I-Q))
C = A*R
fprintf("What is the Probability that a MTH 100 student passes all their courses?\n")
ans = C(1,2)
fprintf("What is the probability that a student quits halfway through?\n")
Quit_200 = C(4,1)
Quit_250 = C(5,1)
ans = ((Quit_250+Quit_200)/2)
fprintf("How long would it take for a student to finish their required math courses?\n")
ans = A(1,1)+A(1,2)+A(1,3)+A(1,4)+A(1,5)+A(1,6)+A(1,7)+A(1,8)
fprintf("The university wants to determine if students are more successful in the beginning of their
students or near the end.\n")
first = ((C(1,2)+C(2,2)+C(3,2)+C(4,2))/4);
fprintf("\nThe passing rate for the first half of the program is: %d %",first)
second = ((C(5,2)+C(6,2)+C(7,2)+C(8,2))/4);
fprintf("\nThe passing rate for the second half of the program is: %d %",second)
```