

$$\min z = 2x_1 + 3x_2 + 4x_3$$

s.t.

$$x_1 - x_2 + x_3 \geq 10$$

$$x_1 - 2x_2 + 3x_3 \geq 6$$

$$3x_1 - 4x_2 + 5x_3 \geq 15$$

$$x_1, x_2, x_3 \geq 0$$

convert to max

→ problem

convert to
slack form

$$\max z = -2x_1 - 3x_2 - 4x_3$$

$$-x_1 + x_2 - x_3 \leq -10$$

$$-x_1 + 2x_2 - 3x_3 \leq -6$$

$$-3x_1 + 4x_2 - 5x_3 \leq -15$$

$$x_1, x_2, x_3 \geq 0$$

convert to dual
problem

$$\max z = -2x_1 - 3x_2 - 4x_3$$

$$s.t. \quad S_1 = -10 + x_1 - x_2 + x_3$$

$$S_2 = -6 + x_1 - 2x_2 + 3x_3$$

$$S_3 = -15 + 3x_1 - 4x_2 + 5x_3$$

* Starting point is infeasible,
primal simplex method cannot solve
this problem, 2-phase method need
to be used, but I will use duality
since I want to.

$$\min z_d = -10y_1 - 6y_2 - 15y_3$$

$$s.t. \quad -y_1 - y_2 - 3y_3 \geq -2$$

$$y_1 + 2y_2 + 4y_3 \geq -3$$

$$-y_1 - 3y_2 - 5y_3 \geq -4$$

$$y_1, y_2, y_3 \geq 0$$

convert to max problem

pivot y_3

$$\max z_d = 10y_1 + 6y_2 + 15y_3$$

s.t.

$$y_1 + y_2 + 3y_3 \leq 2$$

$$-y_1 + 2y_2 - 4y_3 \leq 3$$

$$y_1 + 3y_2 + 5y_3 \leq 4$$

$$\max z_d = \frac{2}{3}y_1 + \frac{1}{3}y_2 - \frac{5}{3}S_1 + 10$$

$$s.t. \quad y_3 = \frac{2}{3} - \frac{1}{3}y_1 - \frac{1}{3}y_2 - \frac{1}{3}S_1$$

$$S_2 = \frac{17}{3} - \frac{1}{3}y_1 + \frac{2}{3}y_2 - \frac{4}{3}S_1$$

$$S_3 = \frac{2}{3} + \frac{2}{3}y_1 - \frac{4}{3}y_2 + \frac{5}{3}S_1$$

$$y_1, y_2, y_3, S_1, S_2, S_3 \geq 0$$

convert
to slack form

$$\max z_d = 10y_1 + 6y_2 + 15y_3$$

$$s.t. \quad S_1 = 2 - y_1 - y_2 - 3y_3 \quad (1, 3)$$

$$S_2 = 3 + y_1 + 2y_2 + 4y_3$$

$$S_3 = 4 - y_1 - 3y_2 - 5y_3$$

$$y_1, y_2, y_3, S_1, S_2, S_3 \geq 0$$

$$\max z_d = 20 - 4y_2 - 15y_3 - 10S_1$$

$$s.t. \quad y_1 = 2 - y_2 - 3y_3 - S_1 \quad \text{done}$$

$$S_2 = 5 + y_2 + y_3 - S_1$$

$$S_3 = 2 - 2y_3 - 2y_2 + S_1$$

$$\max z = 20$$

$$y_1 = 2, y_2 = 0, y_3 = 0$$