GPS-Based Tracking Control for a Car-Like Wheeled Mobile Robot With Skidding and Slipping

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***Abstract*—Many wheeled mobile robot (WMR) controllers are developed based on nonskidding and nonslipping assumptions; however, these assumptions are usually violated due to wheel tire deformation. As a result, the performance of these controllers is not guaranteed. This paper presents a GPS-based tracking controller for a car-like WMR in the presence of wheel skidding and slipping. The controller exploits Real-Time Kinematic (RTK)-GPS and other aiding sensors to measure the WMR's posture, velocities, and perturbations due to wheel skidding and slipping for control compensation. The reported experimental results validate the control scheme.**

***Index Terms*—Inertial sensors, mobile robot, outdoor vehicles, Real-Time Kinematic (RTK)-GPS, slipping, tracking control, wheel skidding.**

I. Introduction

Many solutions have been proposed for wheeled mobile robots (WMRs) control problems; however, these solutions assume that WMRs satisfy nonslipping and nonskidding conditions [1]–[5]. In reality, these assumptions cannot be met due to tire deformation and other reasons; hence, stability and control performance of these existing controllers are not guaranteed in real running.

Several controllers have been proposed for type (2,0) WMR based on a kinematic model constructed in [6] to address the skidding effect represented by unknown bounded perturbation. Under the assumption of the unknown perturbation being state vanishing, an exponentially stable robust stabilizing controller was proposed for WMR [6]. A uniform boundedness solution was proposed in [7] for tracking problem without assuming the perturbation to be constant or state vanishing. The skidding problem is addressed in [8] by designing robust tracking and regulation controllers using kinematic model. Similarly, other robust control methodologies are applied to path-following and trajectory-tracking problems of a farm tractor [9], [10]. Robust control offers uniform boundedness solutions; however, if a high-precision control performance is desirable, these control laws would require fast-switching rate control actions. These methods can be constrained by implementation and mechanical issues.

Another framework was proposed to study the WMR tracking control problem in the presence of skidding and slipping effects [11], [12]. This framework relies on an accurate measurement of a parameter ϵ that is difficult to obtain in practice.

In [14] and [15], researchers deploy Real-Time Kinematic (RTK)-GPS to address the path-following problem of a farm tractor with wheel skidding effects. Unfortunately, the perturbation estimation approach utilized in the control compensation suffers from several deficiencies. First, the algorithm utilizes lateral deviation that requires accurate instantaneous curvilinear information to compute. A mild error in the positioning measurement could possibly lead to erroneous and noisy estimates. The perturbation estimator should be developed based on vehicle's kinematic description rather than restrictive path following models since the skidding perturbations are properties of the WMR and should be problem independent. These limitations can deteriorate the performance of the control laws.

Recently, WMR kinematic models with wheel skidding and slipping have been developed from control perspectives [16]. The models describe skidding and slipping perturbations using explicit descriptions, and this enables better control designs. In this paper, a GPS-based tracking control scheme is developed based on the kinematic models to achieve exponential convergence of tracking errors with a practical perturbation measurement system.

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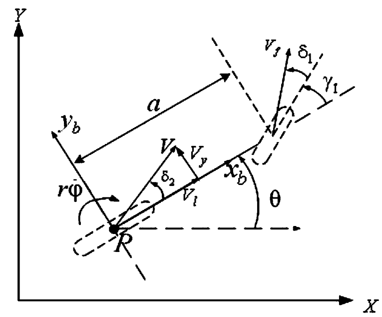

Fig. 1. Type (1,1) car-like WMR in the presence of skidding effects.

II. WMR Kinematic Model with Skidding and Slipping

The WMR considered in this paper is a type (1,1) car-like WMR where its bicycle model is shown in Fig. 1. The WMR has a body frame $\{X_b, Y_b\}$ attached to the reference point P with coordinates $\xi = (x, y)^T$ in the global coordinate frame $\{X, Y\}$. γ_1 denotes the WMR's front steering angle and a denotes the WMR's wheelbase. θ denotes the WMR's orientation. The posture of the WMR is defined as $q = [x \;\; y \;\; \theta]^T$. This class of WMR is characterized by a front-centered steerable wheel and fixed parallel wheels on the WMR's rear axis. The kinematic perturbations due to wheel skidding are characterized by slipping angles $\{\delta_1, \delta_2\}$. V denotes the velocity of the WMR. The velocity is resolved into *lateral velocity* V_y and *longitudinal linear velocity* V_l. The linear velocity can be modeled as $V_l = r\dot{\varphi} - d$, where d denotes the longitudinal slippage and $r\dot{\varphi}$ represents the WMR's controllable linear velocity. The control input of the WMR is $\mathbf{U} = [r\dot{\varphi} \;\; \gamma_1]^T$. It has been shown in our recent work [16] that the kinematic model of this WMR is as follows:

$$\dot{x} = V\cos(\theta + \delta_2) \tag{1}$$

$$\dot{y} = V\sin(\theta + \delta_2) \tag{2}$$

$$\dot{\theta} = \frac{V\tan(\gamma_1 + \delta_1)\cos(\delta_2)}{a} - \frac{V\sin(\delta_2)}{a}. \tag{3}$$

The lateral velocity V_y is related to the rear slip angle δ_2 via geometric relations

$$V_y = V\sin\delta_2 \qquad V_y = V_l\tan\delta_2. \tag{4}$$

The perturbations are classified as: 1) δ_1 and d are input additive and 2) V_y is unmatched.

Assumption 1: The perturbations $\{\delta_1, \delta_2, d\}$ satisfy $|\delta_1| < \rho_1 < \pi/2$, $|\delta_2| < \rho_2 < \pi/2$, and $|d| < \rho_3$, where ρ_i's, $i = 1, 2, 3$, are positive constants.

Remark 1: The boundedness of the wheels' slip angles, longitudinal slippage, and linear velocity holds in most applications.

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III. GPS-Based Tracking Control Design

The undertaken tracking problem is to find control laws $\mathbf{U}$ such that the WMR tracks a *reference trajectory* $q_r = (x_r, y_r, \theta_r)^T$ that is described by

$$\begin{aligned}\dot{x}_r &= v_r \cos\theta_r\\ \dot{y}_r &= v_r \sin\theta_r\\ \dot{\theta}_r &= \omega_r.\end{aligned} \tag{5}$$

We also assume that $v_r(t), \omega_r(t)$, and $V_y(t)$ are *smooth and bounded* and that the reference velocity v_r is positive and $\inf_{\forall t>t_0} v_r(t) > 0$. In this tracking problem, we choose the *posture tracking error* as (see [1])

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r - x\\ y_r - y\\ \theta_r - \theta\end{bmatrix}. \tag{6}$$

Differentiating the posture error (6) leads to the following error dynamics:

$$\dot{\tilde{x}} = v_r \cos\tilde{\theta} - V_l + \omega\tilde{y} \tag{7}$$

$$\dot{\tilde{y}} = v_r \sin\tilde{\theta} - V_y - \tilde{x}\omega \tag{8}$$

$$\dot{\tilde{\theta}} = \omega_r - \frac{V_l}{a}\tan(\gamma_1 + \delta_1) + \frac{V_y}{a}. \tag{9}$$

Based on the error dynamics (7)–(9), we can show that the point error convergence $\tilde{\xi} \to 0$ implies a nonzero steady-state orientation error. This characteristic motivates us to formulate the tracking problem with wheel skidding and slipping as follows.

Trajectory tracking problem: Find control laws for $r\dot{\varphi}$ and γ_1 such that for small initial errors $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$, the following are satisfied.

1) The tracking errors $\tilde{\xi}$ and $\tilde{\theta}$ are uniformly bounded.
2) The point tracking error $\tilde{\xi}$ converges to zero as $t \to \infty$.
3) $\tilde{\theta}$ converges to a neighborhood about $\tilde{\theta} = 0$ as $t \to \infty$. Moreover, the steady-state orientation error $\lim_{t\to\infty}\tilde{\theta}(t) = 0$ if $\lim_{t\to\infty}\delta_2(t) \to 0$.

A. GPS-Based Tracking Control Scheme

This paper aims to propose a practical control scheme to solve the tracking control problem without imposing restrictive assumptions. We use an exteroceptive RTK-GPS and other aiding sensors to measure the skidding and slipping perturbations for compensation. The key attribute of these sensors is that the measurements provided by these devices are not affected by wheel skidding and slipping. The relations between the sensors measurements and the perturbations are provided by the kinematic model (1)–(3). Recently, we applied this approach to address the *path following problem* in the presence of wheel skidding and slipping [17], and some promising results have been achieved.

B. WMR Posture, Velocities, and Perturbations Measurements

The perturbation measurements are based on the kinematic model (1)–(3) and a suite of suitably chosen sensors. First, we express the velocity $\dot{\xi}$ of the WMR as $\dot{\xi} = A(\theta)\eta$, where

$$A(\theta) = \begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix} \qquad \eta = \begin{bmatrix}V_l\\ V_y\end{bmatrix}. \tag{10}$$

Suppose that the orientation θ and the velocity $\dot{\xi}$ are measurable, then the perturbation V_y and the WMR's linear velocity V_l can be uniquely determined by $\eta = A^T(\theta)\dot{\xi}$. By differentiating $\dot{\xi} = A(\theta)\eta$ and by some algebraic manipulations, we obtain $\dot{\eta} = u_a + [V_l\ V_y]^T\dot{\theta}$. $\ddot{\xi}$ can be provided by the GPS Doppler velocity measurements. u_a is the acceleration of the reference point expressed in the body coordinates $\{X_b, Y_b\}$ that can be measured by a strap-down accelerometer. The WMR's orientation θ can be monitored by a magnetic compass or a gyrocompass where their measurements are not affected by wheel skidding and slipping, and the robot's yaw rate $\dot{\theta}$ can be measured by a single-axis gyroscope. With $\{V_l, V_y\}$ measurements, the perturbations $\{\delta_1, \delta_2, d\}$ can be computed using

$$\begin{aligned}d &= r\dot{\varphi} - V_l\\ \delta_1 &= \tan^{-1}\left(\frac{a}{V_l}\left\{\dot{\theta} + \frac{V_y}{a}\right\}\right) - \gamma_1\\ \delta_2 &= \tan^{-1}(V_y/V_l).\end{aligned}$$

C. Backstepping Control Design

First, we eliminate the additive perturbations $\{\delta_1, d\}$ by exploiting the measurements. To achieve this, we define the auxiliary inputs

$$V_{ld} = r\dot{\varphi} - d \tag{11}$$

$$\omega = \frac{V_l}{a}\tan(\gamma_1 + \delta_1) - \frac{V_y}{a}. \tag{12}$$

With these input transformations, the error dynamics becomes

$$\dot{\tilde{x}} = v_r \cos\tilde{\theta} - V_{ld} + \omega\tilde{y} \tag{13}$$

$$\dot{\tilde{y}} = v_r \sin\tilde{\theta} - V_y - \omega\tilde{x} \tag{14}$$

$$\dot{\tilde{\theta}} = \omega_r - \omega. \tag{15}$$

These input transformations are invertible, i.e., we can convert $\{V_{ld}, \omega\}$ to original control inputs $\{r\dot{\varphi}, \gamma_1\}$.

Let $V_1 = (1/2)\tilde{x}^2 + (1/2)\tilde{y}^2$. Taking $\alpha = \sin\tilde{\theta}$ as the virtual control, the derivative of V_1 becomes $\dot{V}_1 = \tilde{x}\{v_r\cos\tilde{\theta} - V_l + \omega\tilde{y}\} + \tilde{y}\{v_r\alpha - V_y - \tilde{x}\omega\}$. By choosing

$$V_{ld} = v_r\cos\tilde{\theta} + k_1\tilde{x} \tag{16}$$

$$\alpha = \frac{V_y - k_2\tilde{y}}{v_r} \tag{17}$$

$\dot{V}_1 = -k_1\tilde{x}^2 - k_2\tilde{y}^2$. Let $\chi = [\tilde{x}\ \tilde{y}\ z]^T$ with $z = \sin\tilde{\theta} - \alpha$. The error dynamics (13)–(15) can be expressed as

$$\dot{\tilde{x}} = v_r\cos\tilde{\theta} - V_{ld} + \omega\tilde{y} \tag{18}$$

$$\dot{\tilde{y}} = v_r(z + \alpha) - V_y - \tilde{x}\omega \tag{19}$$

$$\dot{z} = \beta_2\omega + \beta_1 \tag{20}$$

where

$$\beta_1 = -\frac{\partial\alpha_1}{\partial\tilde{y}}\{v_r\sin\tilde{\theta} - V_y - \tilde{x}\omega_r\} - \frac{\partial\alpha_1}{\partial V_y}\dot{V}_y - \frac{\partial\alpha_1}{\partial v_r}\dot{v}_r \tag{21}$$

$$\beta_2 = \cos\tilde{\theta} - \frac{\partial\alpha_1}{\partial\tilde{y}}\tilde{x}$$

$$\frac{\partial\alpha_1}{\partial\tilde{y}} = -\frac{k_2}{v_r}, \quad \frac{\partial\alpha_1}{\partial V_y} = \frac{1}{v_r}, \quad \frac{\partial\alpha_1}{\partial v_r} = \frac{k_2\tilde{y} - V_y}{v_r^2}. \tag{22}$$

Consider a Lyapunov function $V_2 = (1/2)\|\chi\|^2$ and its derivative as $\dot{V}_2 = -k_1\tilde{x}^2 + \tilde{y}\{v_r(z+\alpha) - V_y\} + z\{\omega\beta_2 + \beta_1\}$. By choosing ω as

$$\omega = \frac{-\beta_1 - k_3 z - v_r\tilde{y}}{\beta_2} \tag{23}$$

$\dot{V}_2 = -k_1\tilde{x}^2 - k_2\tilde{y}^2 - k_3 z^2$.

The performance of the control system is stated as follows.

Theorem 1: For small initial conditions $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$, there exist positive constants k, k_4, and c_1 such that the tracking errors of the closed-loop system (13)–(16), (23) are governed by the following statements.

1) The tracking errors $\{\tilde{\xi}(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2) The point tracking error $\tilde{\xi}$ exponentially converges to zero and satisfies the inequality

$$\|\tilde{\xi}(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \qquad \forall t \geq t_0 \geq 0. \tag{24}$$

3) The orientation error $\tilde{\theta}$ exponentially converges to a neighborhood containing $\tilde{\theta} = 0$ and satisfies the inequality

$$|\sin\tilde{\theta}(t)| \leq k\|\chi(t_0)\| e^{-k_4(t-t_0)} + c_1 \qquad \forall t \geq t_0 \geq 0. \tag{25}$$

4) The steady-state orientation error satisfies

$$\lim_{t\to\infty} (\tilde{\theta}(t) - \delta_2(t)) = 0. \tag{26}$$

Proof: The quadratic nature of V_2 and its derivative implies the existence of a positive constant $k_4 = \min\{k_1, k_2, k_3\}$ such that $\dot{V}_2 \leq -k_4\|\chi\|^2$. Consequently, we have $\dot{V}_2 \leq -2k_4 V_2$. By comparison lemma [18], we obtain $V_2(t) \leq V_2(t_0)e^{-2k_4(t-t_0)}$, which implies

$$\|\chi(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \qquad \forall t \geq t_0 \geq 0. \tag{27}$$

Inequality (27) implies that the tracking errors $(\tilde{\xi}, \tilde{\theta})$ are uniformly bounded. Since $\tilde{\xi}$ is a subvector of χ, inequality (27) implies

$$\|\xi(t)\| \leq \|\chi(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \qquad \forall t \geq t_0 \geq 0 \tag{28}$$

and this establishes condition (24). Note that the virtual control law α satisfies $|\alpha| \leq |V_y/v_r| + |k_2/v_r||\tilde{y}| \leq c_1 + c_2|\tilde{y}|$ where $\{c_1, c_2\}$ are positive constants such that $|V_y/v_r| < c_1$ and $|k_2/v_r| < c_2$. Furthermore, definition $z = \sin\tilde{\theta} - \alpha$ leads to

$$\begin{aligned} |z| &= |\sin\tilde{\theta} - \alpha| \geq |\sin\tilde{\theta}| - |\alpha| \\ |\sin\tilde{\theta}| &\leq |z| + |\alpha| \leq |z| + c_1 + c_2|\tilde{y}| \\ &\leq k\|\chi(t_0)\| e^{-k_4(t-t_0)} + c_1 \end{aligned}$$

where $k = (1 + c_2)$.

To show (26), we first establish that $V \to v_r$ as $t \to \infty$. Inequality (28) indicates that the point tracking error $\tilde{\xi}$ is uniformly bounded and approaches zero. Since v_r, ω_r, and V_y are smooth by assumption, the convergence of $\tilde{\xi}$ and differentiability of the control input $\omega(t)$ imply the boundedness of $\ddot{\tilde{\xi}}$; hence, $\dot{\tilde{\xi}}$ is uniformly continuous. Since $\tilde{\xi}$ is finite, by Barbalat lemma [18], we conclude that $\dot{\tilde{\xi}} \to 0$ as $t \to \infty$, and this means $V \to v_r$ as $t \to \infty$. The convergence of z implies that $\lim_{t\to\infty}\{\sin\tilde{\theta} - (V_y/V)\} = 0$. Geometric relation (4) leads the equation to $\lim_{t\to\infty}(\tilde{\theta}(t) - \delta_2(t)) = 0$ and this completes the proof. □

Remark 2: Theorem 1 solves the tracking problem based on the perturbation measurements and the backstepping control laws. The results provide estimates for the effect of control gains $\{k_1, k_2, k_3\}$. For instance, the convergence rate and the maximum overshoot of the tracking errors can be estimated using inequalities (24) and (25).

Remark 3: Equation (26) implies $\lim_{t\to\infty}\tilde{\theta}(t) = 0$ if $\lim_{t\to\infty}\delta_2(t) = 0$. Due to the limited maneuverability of this WMR, $\tilde{\theta}$ does not converge to zero if δ_2 is nonzero [16]; nevertheless, in many practical cases, a zero converging point tracking error and a well-behaved orientation error are sufficient.

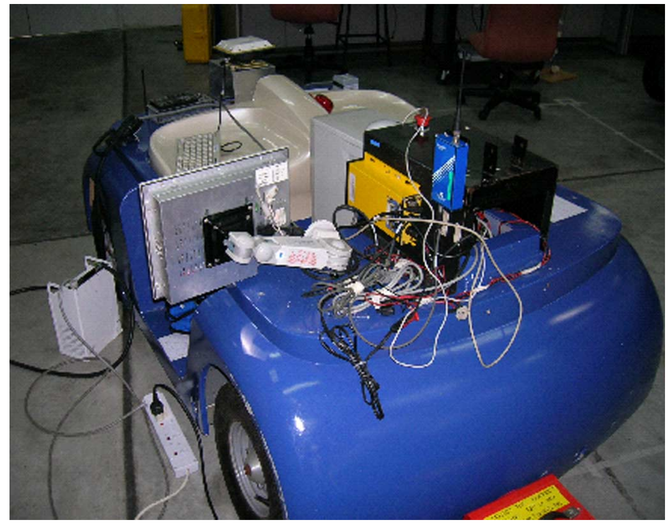

Fig. 2. Cycab.

IV. Experimental Results

A. Experimental Configuration

The developed GPS-based tracking controller was implemented on a type (1,1) car-like WMR, as shown in Fig. 2. The WMR has a wheelbase of $a = 1.2$ m and is equipped with four dc motors and a hydraulic steering system to provide the driving and steering actuations. The computer computes the control algorithms at a frequency of 10 Hz and return the desired values to two well-tuned low-level PID control systems to control the vehicle's steering angle and velocity input $\{\gamma_1, r\dot{\varphi}\}$. The WMR is equipped with a high-grade MS-750 RTK-GPS receiver that provides the position and velocity measurements simultaneously. An absolute encoder is installed on the WMR's steering system to measure the steering angle γ_1. Additionally, a low-noise KVH RA1100 fiber-optic gyroscope and a CXL01LF3 tri-axial accelerometer are attached on the WMR's reference point P to provide the acceleration and turning rate measurements. In this setup, we integrate the measured yaw rate $\dot{\theta}$ using trapezoidal integration to estimate θ. Readers may refer to [17] for a more detailed hardware description.

B. Experimental Results

1) Experiments Without Perturbations Compensation: To validate the GPS-based tracking control scheme, the WMR was first controlled by the proposed backstepping control laws without any compensation action, i.e., $(\delta_1 = 0, V_y = 0, d = 0)$. The control gains chosen in the experiments are $k_1 = 0.1, k_2 = 0.1$, and $k_3 = 0.3$. The WMR begins at initial tracking errors of $(\tilde{\xi}, \tilde{\theta}) = (0, 0)$. The reference trajectory is chosen as $(v_r, \omega_r) = (0.8\ \mathrm{m{\cdot}s^{-1}}, -0.08\ \mathrm{rad{\cdot}s^{-1}})$.

The control scheme was activated to perform the tracking control task. The tracking errors are shown in Fig. 3 (dash-dot lines). We see that the tracking errors $(\tilde{\xi}, \tilde{\theta})$ remain in a neighborhood about the point $(\tilde{\xi}, \tilde{\theta}) = (0, 0)$ with $\tilde{x}$ converging to a steady-state value of 0.05 m, $\tilde{y}$ converging to a nonzero bias of -0.06 m, and the orientation error $\tilde{\theta}$ converging to a low amplitude close to zero.

2) Experiments With Perturbation Compensation: For the given same initial errors and control gains $\{k_1, k_2, k_3\}$, the wheel skidding compensation was activated by including the perturbation terms in the control laws. Fig. 4 depicts the perturbations computed using the GPS measurements. Fig. 5 shows the WMR's linear velocity V_l and the smooth steering input recorded during the experiment. The longitudinal

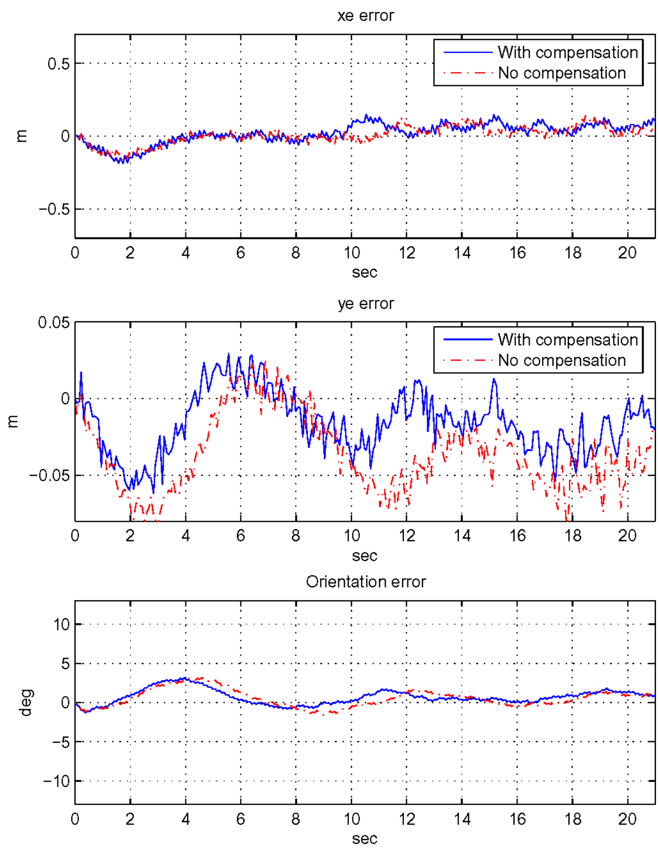

Fig. 3. Tracking performances.

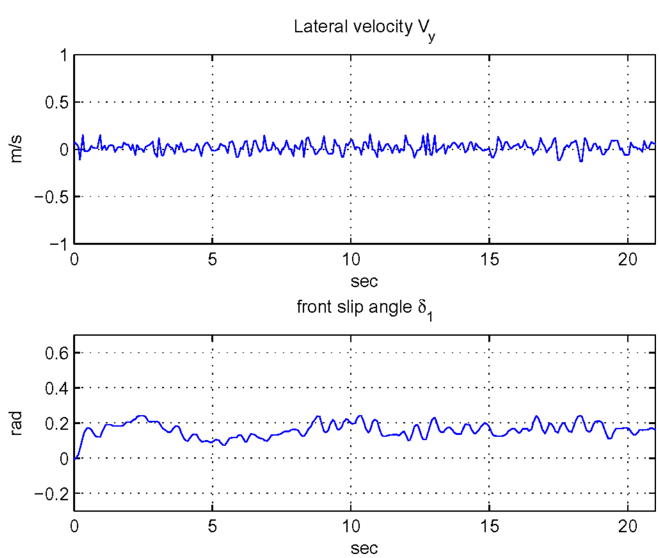

Fig. 4. Measured perturbations.

slipping velocity d is assumed to be insignificant and additional velocity compensation is not required. The estimated V_y shown in Fig. 4 has a zero mean. In contrast, the front slip angle δ_1 is more dominant compared with V_y and d. From Fig. 3, it is clear that introducing perturbation compensation reduces the steady-state bias of $\tilde{y}$. This reduction of the bias is due to the compensation of perturbation δ_1 by the control laws. The mild nonzero steady-state orientation error could be due to the accumulated integration error of the orientation estimate.

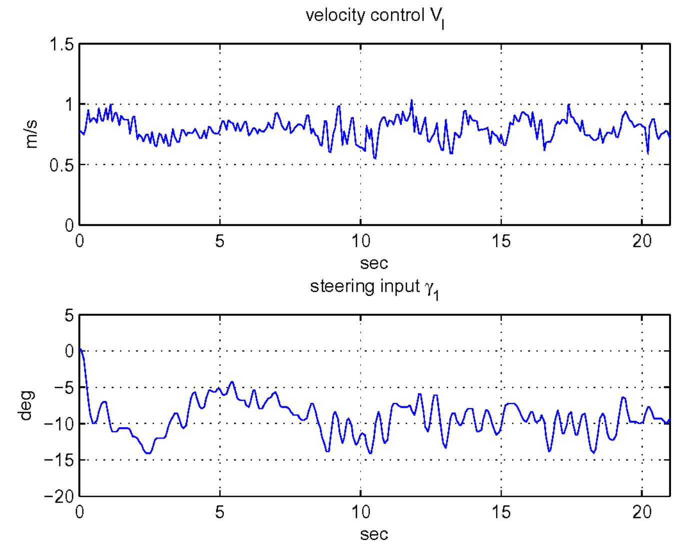

Fig. 5. Control inputs.

V. Conclusion

In this paper, a tracking control problem was formulated and solved using a GPS-based control scheme for a car-like WMR with wheel skidding and slipping. A nonlinear backstepping tracking controller is developed to achieve zero exponentially converging tracking performance, and it has been validated in experiments. It is clear that the potentials of the control system could be limited by the low-update rate and occasional outages of the GPS measurements. Recently, we have looked into the issues, and a method has been proposed to increase the updating rate and reliability of the information required by the control laws [19]. Hence, the GPS-based tracking control system developed in this paper should be able to perform effectively in high-speed maneuvers.

References

[1] Y. Kanayamaa, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for an autonomous mobile robot," in *Proc. 1990 Int. Conf. Robot. Autom.*, 2008, pp. 384–389.
[2] Z. P. Jiang and H. Nijmeijer, "Tracking control of mobile robots: A case study in backstepping," *Automatica*, vol. 33, no. 7, pp. 1393–1399, 1997.
[3] D. Wang and G. Xu, "Full-state tracking and internal dynamics of nonholonomic wheeled mobile robot," *IEEE/ASME Trans. Mechatronics*, vol. 8, no. 2, pp. 203–214, Jun. 2003.
[4] C. Samson, "Path following and time-varying stabilization of wheeled mobile robot," in *Proc. 2nd Int. Conf. Autom., Robot. Comput. Vis.*, Singapore, 1992, pp. RO13.1.1–RO13.1.5.
[5] A. Astolfi, "Discontinuous control of nonholonomic systems," *Syst. Control Lett.*, vol. 27, pp. 37–45, 1996.
[6] C. C. de Wit and H. Khennouf, "Quasi-conjtinuous stabilizing controllers for nonholonomic systems: Design and robustness considerations," in *Proc. 3rd Eur. Control Conf.*, Rome, Italy, Sep. 1995, pp. 2630–2635.
[7] M. L. Corradini, T. Leo, and G. Orlando, "Experimental testing of a discrete-time sliding mode controller for trajectory tracking of a wheeled mobile robot in the presence of skidding effects," *Int. J. Robot. Syst.*, vol. 19, pp. 177–188, 2002.
[8] W. E. Dixon, D. M. Dawson, and E. Zergeroglu, "Robust control of a mobile robot system with kinematic disturbances," in *Proc. 2000 IEEE Int. Conf. Control Appl.*, pp. 437–442.
[9] H. Fang, R. Lenain, B. Thuilot, and P. Martinet, "Robust adaptive control of automatic guidance of farm vehicles in the presence of skidding," in *Proc. IEEE Conf. Robot. Autom.*, Barcelona, Spain, Apr. 2005, pp. 3113–3118.
[10] H. Fang, R. Fan, B. Thuilot, and P. Martinet, "Trajectory tracking control of farm vehicles in the presence of skidding," *Robot. Auton. Syst.*, vol. 54, pp. 828–839, Oct. 2006.

[11] B. d'Andréa Novel, G. Campion, and G. Bastin, "Control of wheeled mobile robots not satisfying ideal velocity constraints: A singular perturbation approach," *Int. J. Robust Nonlinear Control*, vol. 5, pp. 243–267, 1995.

[12] I. Motte and G. Campion, "A slow manifold approach for the control of mobile robots not satisfying the kinematic constraints," *IEEE Trans. Robot. Autom.*, vol. 16, no. 6, pp. 875–880, Dec. 2000.

[13] G. Reina, L. Ojeda, A. Milella, and J. Borenstein, "Wheel slippage and sinkage detection for planetary rovers," *IEEE/ASME Trans. Mechatronics*, vol. 11, no. 2, pp. 185–195, Apr. 2006.

[14] R. Lenain, B. Thuilot, C. Cariou, and P. Martinet, "A new nonlinear control for vehicle in sliding conditions: Application to automatic guidance of farm vehicles using RTK GPS," in *Proc. 2004 IEEE Int. Conf. Robot. Autom.*, pp. 4381–4386.

[15] R. Lenain, B. Thuilot, C. Cariou, and P. Martinet, "Adaptive and predictive nonlinear control for sliding vehicle guidance, application to trajectory tracking of farm vehicles relying on a single RTK GPS," in *Proc. 2004 IEEE Int. Conf. Intell. Robots Syst.*, pp. 455–460.

[16] D. Wang and C. B. Low, "Modeling and analysis of skidding and slipping in wheeled mobile robots: Control design perspective," IEEE Trans. Robot., vol. 24, no. 3, pp. 676–687, Jun. 2008.

[17] C. B. Low and D. Wang, "GPS-based path following control for a car-like wheeled mobile robot with skidding and slipping," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 2, pp. 340–347, Mar. 2008.

[18] H. K. Khalil, *Nonlinear Systems*, 3rd ed. ed. Englewood Cliffs, NJ: Prentice-Hall, 2002.

[19] C. B. Low and D. Wang, "Integrated estimation for wheeled mobile robot posture, velocities, and wheel skidding perturbations," in *Proc. 2007 IEEE Int. Conf. Robot. Autom.*, Rome, Italy, Apr., pp. 2355–2360.