INFLUENCE OF TRAVEL DIRECTION ON GPS ACCURACY FOR VEHICLE TRACKING

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ABSTRACT. *The influence of travel direction on GPS dynamic accuracy for vehicle tracking is discussed in two sections. The first section investigates the influence of travel direction on GPS accuracy due to the GPS satellite sky distribution. GPS dilution of precision (DOP) was calculated based on GPS satellite geometry at a variety of locations and different mask angle settings. Results show a significant difference between north DOP and east DOP in a mid-latitude area. A clear trend of the 24 h average ratio of the north DOP to the east DOP was found related to latitudes and mask angle settings. Cross-track dilution of precision (XDOP) is defined as the GPS DOP perpendicular to the travel direction. The influence of the GPS satellite geometry on GPS accuracy was mapped into the vehicle platform frame to derive the XDOP, and accordingly to derive the influence of travel direction on the GPS dynamic accuracy. Results showed that the XDOP increased as the course over ground (COG) changed from 0° to 90°. Considering that a regression line fitting through GPS data may be referenced as the true path for calculating GPS errors, the second section reviews methods for fitting linear models. The most commonly used approach for linear fitting is least-square regression that minimizes the sum square of vertical offsets, rather than perpendicular offsets. This approach can result in a potential model fitting error, which was found to be dependent on the direction of travel and the dynamic accuracy of the tested GPS receiver when this approach was used to generate the referenced true path for calculating GPS cross-track errors. Our results showed that the fitting error reached its maximum when the tested vehicle was traveling in the N-S (or S-N) direction and decreased when the travel direction moved away from the N-S direction.*

Keywords. *Cross-track error, DOP, GPS dynamic accuracy, XDOP.*

The Global Positioning System (GPS) has been widely used for vehicle parallel tracking (Han et al., 2004), animal movement monitoring (Turner et al., 2001; Agouridis et al. 2003), yield mapping (Shannon et al., 2002), automated guidance (Bell, 2000; Guo and Zhang, 2004), vehicle performance monitoring (Keong, 1999), and military vehicle movement analysis (Haugen et al., 2003). GPS has also been used in aerial platforms for obtaining aircraft attitude (Hayward et al., 1998) and for georeferencing video-based remote sensing images (Thomson et al., 2002). Quantification of and standards for GPS accuracy depend on individual applications: some require high absolute accuracy, while others need high relative accuracy. Buick (2002) provided some guidelines for choosing different classes of GPS for different applications.

As dynamic applications of GPS increase, measuring and evaluating GPS dynamic accuracy becomes more important. The Institute of Navigation (ION, 1997) recommended test procedures to quantify GPS accuracy. Several issues for testing and evaluating GPS dynamic accuracy have recently been brought forward and discussed (Buick, 2002; Ehsani et al., 2002; Han et al., 2004; Stombaugh et al., 2002; Taylor et al., 2004). A new standard for testing GPS dynamic accuracy in agricultural applications is under development. Most protocols used for testing GPS dynamic accuracy can be classified as on-vehicle comparison testing (Stombaugh et al., 2002), in which the tested GPS receivers are mounted on a vehicle driven along a pre-defined path. The cross-track error is the GPS relative error perpendicular to the travel direction and is considered the critical component of GPS dynamic accuracy.

Measurement of GPS dynamic accuracy is difficult. Buick (2002) mentioned that measuring GPS dynamic accuracy is more difficult than measuring static accuracy, because more variables affect GPS dynamic performance and are difficult to control. Haugen et al. (2000) found that the time element of GPS accuracy can produce large GPS errors, and that time lags of 1.6 s from GPS receivers were measured. Smith and Thomson (2005) found relatively small longitudinal position latencies and negligible cross-track error with an aircraft-based SATLOC M3 GPS receiver. Low-cost GPS receivers used in aircraft were shown to have larger GPS time differences (Thomson et al., 2004). Stombaugh et al. (2002) found that the static performance of GPS receivers was not necessarily indicative of dynamic performance. Han et al. (2004) also found that the dynamic performance of a receiver was extremely variable from test to test. Different testing time periods and tested locations have different GPS satellite constellations, signal levels, atmospheric errors, and correction accuracy, all of which influence accuracy.

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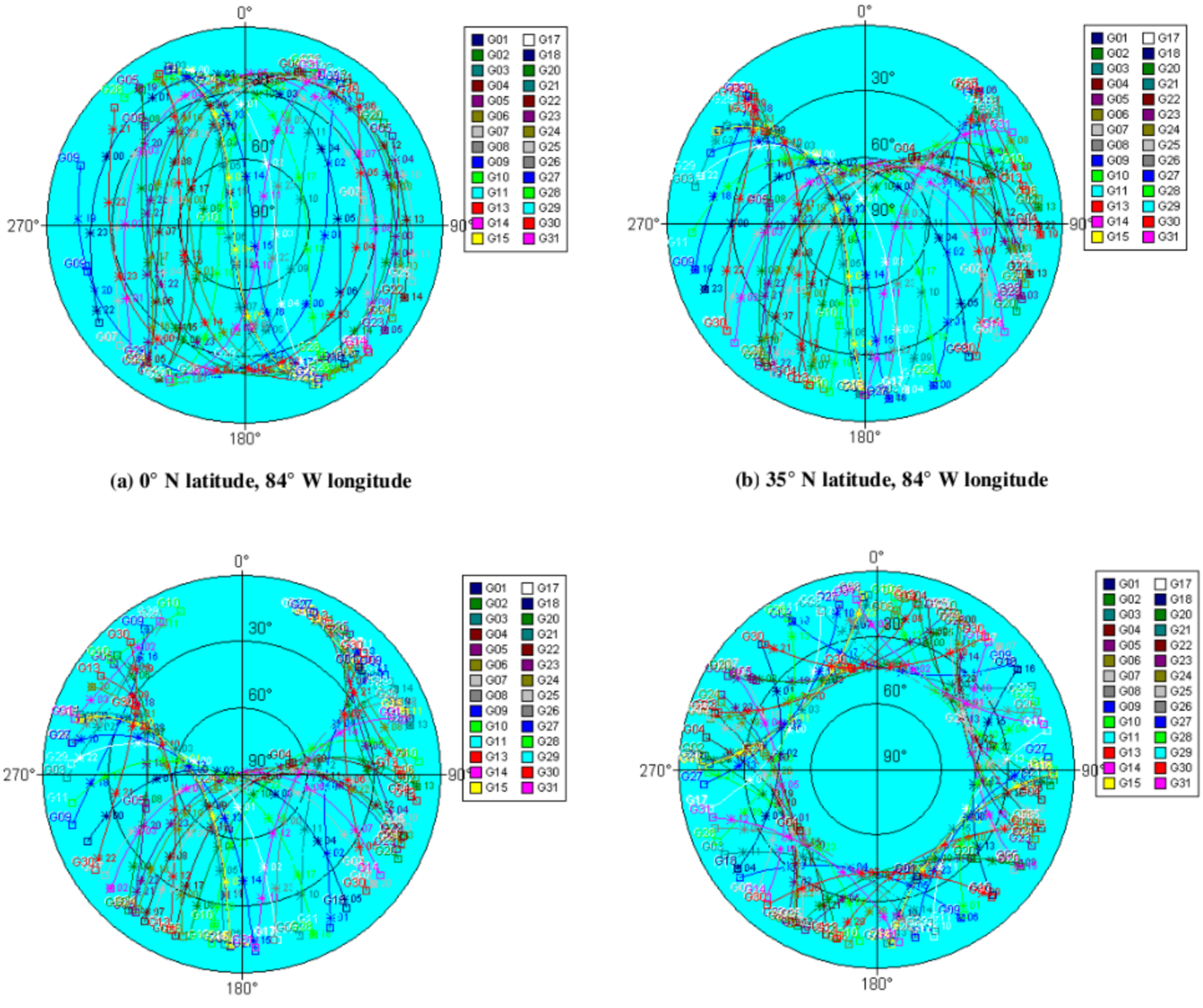

(a) 0° N latitude, 84° W longitude

(b) 35° N latitude, 84° W longitude

(c) 55° N latitude, 84° W longitude

(d) 90° N latitude, 84° W longitude

Figure 1. GPS satellite sky plots (from Trimble's Planning software, ver. 2.7) on 9 January 2003; elevation cutoff (mask angle) = 10°.

The objective of this study was to address the influence of the travel direction on measuring GPS dynamic accuracy. First, the influence of travel direction on GPS accuracy due to the GPS satellite sky distribution was investigated. The influence of the GPS satellite geometry on GPS accuracy was mapped into the vehicle platform frame to derive the cross-track dilution of precision (XDOP), and accordingly to derive the influence of travel direction on GPS dynamic accuracy. Secondly, the influence of travel direction on GPS dynamic accuracy was explored as a regression line was used in calculating the GPS cross-track error. Results showed that a fitting error may be introduced into GPS cross-track error due to regression methods, especially when a vehicle traveled in the north-south (N-S) direction. The least-square regression minimizing perpendicular offsets is suggested to generate the baseline for calculating GPS cross-track errors.

Directional GPS Errors Due to Satellite Geometry

GPS Satellite Geometry

The GPS constellation is comprised of 24 satellites on six orbital planes with a 55° inclination relative to the equatorial plane. The orbit is approximately 26,560 km above the center of the earth, with an orbital period of 12 h on asymmetrically repeating ground tracks. Standing on the earth, the GPS satellites' orbital period appears to be 24 h, since the satellites are orbiting the earth in the same direction as the earth's rotation.

Due to the 55° inclination of the GPS satellite orbits, the satellite distribution across the sky in mid-latitude areas is uneven. GPS satellite availability depends on the latitude of the observation site and its surrounding obstructions. Trimble's Planning software is a powerful stand-alone tool that allows users to determine visibility for GPS, GLONASS, IGSO, and geostationary satellites for a given time interval (Trimble, 2002). Figure 1 shows changes in the sky plots (from Trimble's Planning software) of GPS satellite distribution at latitudes ranging from 0° to 90° N.

The GPS satellite sky distributions in mid-latitude areas show an apparent lack of satellites near the northern horizon, as previously reported by Buick (2002). Previous studies found that GPS satellite sky distribution influenced not only the static relative accuracy of GPS (Ayers et al., 2004; Charrois, 1999) but also the dynamic cross-track accuracy (Ehsani et al., 2003). Ayers et al. (2004) found that the standard deviation of GPS errors in the north-south (N-S)

Table 1. Standard deviations (m) of GPS point spread in four directions.

GPS Receiver	0°	90°	45°	135°
T01	2.48	3.35	3.00	2.90
T02	2.83	3.36	3.13	3.09
T03	2.65	7.52	5.16	6.08
T04	2.66	3.34	2.80	3.23
T05	2.37	2.96	2.70	2.66
T06	2.51	3.07	2.84	2.77
T07	2.29	3.44	2.86	2.98
T08	2.25	2.56	2.27	2.54
T09	2.92	4.47	3.94	3.61
T10	2.16	3.37	2.72	2.94
T11	2.40	3.79	3.30	3.04
T12	2.42	4.16	3.55	3.25
T13	2.85	4.19	3.73	3.43
T14	2.15	3.16	2.56	2.83
T15	2.46	2.91	2.72	2.67
T16	2.51	3.27	2.79	3.03
T17	2.41	3.98	3.45	3.13
Average	2.49	3.70	3.15	3.19

direction is higher than that in the east-west (E-W) direction. Ehsani et al. (2003) observed a higher cross-track error in the N-S direction when vehicles traveled along the E-W direction. Buick (2002) suggested an optimal figure-8 pattern for dynamic GPS testing in order to span the range of directions and prevent bias due to differences in east DOP (EDOP) and north DOP (NDOP).

Difference of GPS Data Spread in N-S and W-E Directions

GPS data were collected for 24 h on 9 January 2003 from 17 Garmin GPS 35 receivers to estimate their static accuracies (Ayers et al., 2004). The standard deviations of the GPS points spread were calculated relative to four axes. The four axes chosen were 0°, 45°, 90°, and 135°. Table 1 shows the results for the 17 tested Garmin GPS 35 receivers.

Analysis of the GPS data distributions in the northing and easting directions revealed an interesting trend in the standard deviations (fig. 2). Figure 2 shows that the standard deviation of the GPS error relative to the northing direction (axis = 0°) is always less than that relative to the easting direction (axis = 90°). This is believed to be a result of the lack of satellites in the northern skies at mid-latitude. The ratio of northing standard deviation to easting standard deviation averaged 1.48.

As stated earlier, Ehsani et al. (2003) found that cross-track errors were higher when the vehicle traveled along the E-W direction (90° or 270° course over ground (COG)), compared to the N-S direction (0° or 180° COG). The difference in the GPS errors between the N-S direction and the E-W direction is possibly due to the difference in EDOP and NDOP (Buick, 2002). NDOP and EDOP can be determined using the method described by Beutel (1999). The following two sections of this article show calculations of EDOP and NDOP based on satellite geometry. The calculation result supports Buick's (2002) point that the difference in the GPS errors between the N-S direction and the E-W direction wa s proba bly due to the difference in EDOP and NDOP and explains the directional differences observed by Ehsani et al. (2003).

GDOP Calculation

The GPS error can be estimated by multiplying the pseudorange measurement error by a scaling factor: satellite geometry dilution of precision (GDOP). GDOP consists of horizontal DOP (HDOP), vertical DOP (VDOP), and time DOP (TDOP) (Kennedy, 1996). HDOP can be further resolved into its X and Y components. If the X axis is oriented in the easting direction, and the Y axis is oriented in the northing direction, then HDOP can be written as: $\text{HDOP} = \sqrt{\text{EDOP}^2 + \text{NDOP}^2}$. When the position coordinates are the ordered right hand set (east, north, and vertical), the GDOP could also be written as:

$$\text{GDOP} = \begin{bmatrix} \text{EDOP} & & & \\ & \text{NDOP} & & \\ & & \text{VDOP} & \\ & & & \text{TDOP} \end{bmatrix} \quad (1)$$

GDOP can be derived from the user-satellite geometry (G, called the geometry matrix): $\text{GDOP}^2 = [G^T G]^{-1}$. EDOP is the estimate in the upper left of the $[G^T G]^{-1}$ matrix and so forth for NDOP, VDOP, and TDOP (Beutel, 1999).

The geometry matrix (G) is:

$$G = \begin{bmatrix} \cos el_1 \sin az_1 & \cos el_1 \cos az_1 & \sin el_1 & 1 \\ \cos el_2 \sin az_2 & \cos el_2 \cos az_2 & \sin el_2 & 1 \\ \ldots\ldots & \ldots & \ldots & \ldots \\ \cos el_i \sin az_i & \cos el_i \cos az_i & \sin el_i & 1 \end{bmatrix} \quad (2)$$

where the satellite azimuth (az_i) is measured clockwise from true north, and the elevation (el_i) is measured up from local horizontal.

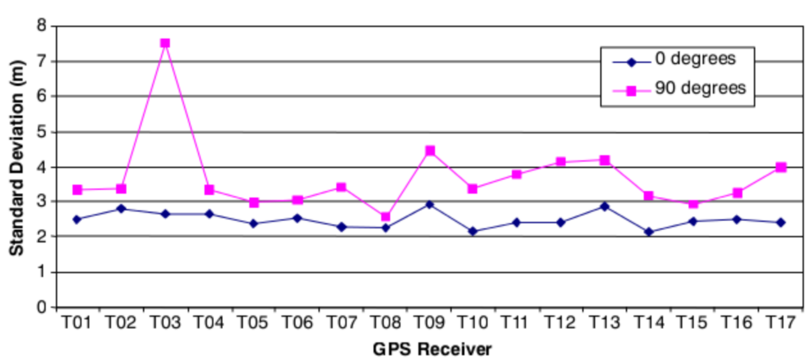

Figure 2. Standard deviations of 24 h GPS data spread relative to N-S (0°) and E-W (90°) directions.

DOP versus Latitude

Utilizing the GDOP calculation procedure, 24 h average NDOP and EDOP were determined using the satellite almanac data for 9 January 2003 when the 24 h GPS static accuracy test was conducted. The satellite almanac data was acquired from the U.S. Coast Guard website (U.S. Coast Guard Navigation Center, 2005) and imported to Trimble's Planning software to extract satellite geometry information. DOP values were determined at various latitudes at a fixed longitude (84° W). Mask angles were set as 0°, 10°, 15°, and 20°. The ratios of NDOP to EDOP are shown in figure 3. As seen in the figure, the ratio of NDOP to EDOP increases at the mid-latitudes, and an increasing mask angle increases the NDOP/EDOP ratio. This trend held true when evaluating the NDOP to EDOP ratios at other days (fig. 4). The trend is also true as the longitude varies from 0° W to 180° W (fig. 5).

According to the manufacturers, the Garmin GPS 35 receiver utilizes a dynamic mask angle, meaning it will lower its mask angle in order to acquire enough satellites to generate an optimum location solution. Since the ratio of NDOP to EDOP changes depending on mask angles, as indicated in figure 3, the exact value of the ratio for the tested

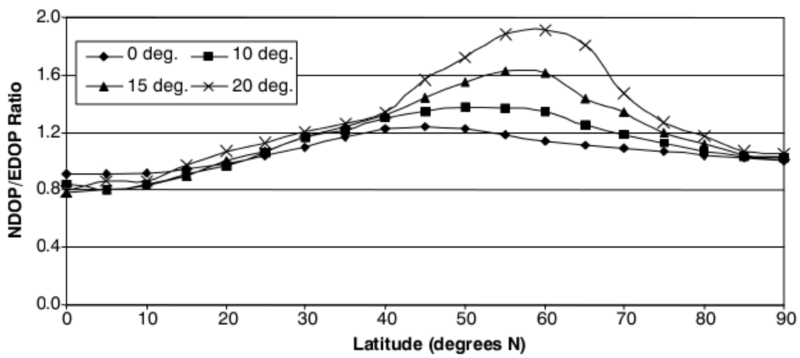

Figure 3. Ratio of NDOP to EDOP at different mask angle settings (degrees) at 84° W longitude.

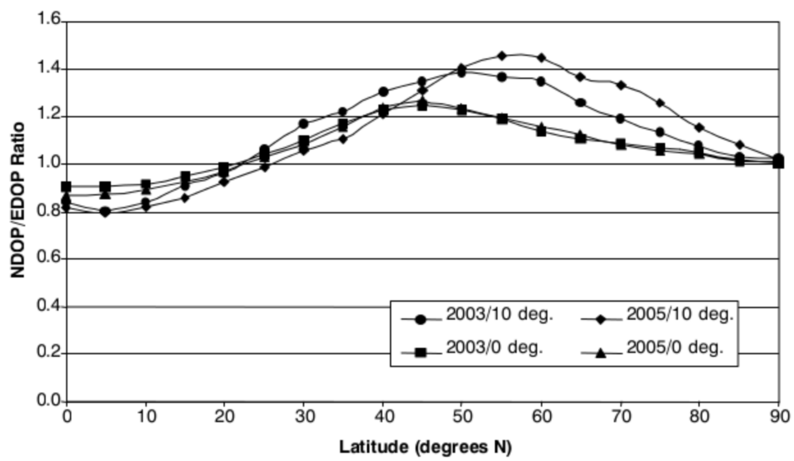

Figure 4. NDOP/EDOP ratio at different time periods and mask angles.

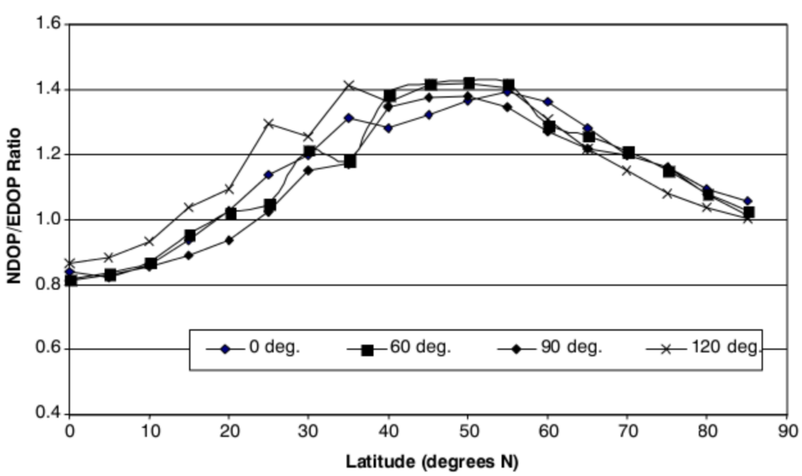

Figure 5. NDOP/EDOP ratio at different longitudes and a 10° mask angle on 14 February 2005.

Garmin GPS 35 at any given time was unknown. However, figure 3 also shows that the ratios of NDOP to EDOP at the test location 84° W longitude and 36° N latitude were constantly above 1.0 for different mask angle settings. It is believed that this ratio of NDOP to EDOP was directly responsible for the higher standard deviations of GPS static data in the northing direction (relative to 90° axis) compared to the easting direction (relative to 0° axis) as seen in table 1. The authors recognize that the influences of EDOP and NDOP on GPS accuracy were supported by GPS static testing data. However, as noted by Buick (2002), the EDOP and NDOP differences still may be responsible for the differences of dynamic cross-track errors found by Ehsani et al. (2003).

Many vehicles travel in directions other than N-S and E-W. The influence of DOP-induced cross-track error can be determined at any direction of travel (COG). The cross-track DOP (XDOP) can be defined as the DOP in the direction perpendicular to the direction of travel (U.S. Coast Guard Navigation Center, 1996). This XDOP may influence the GPS cross-track error.

XDOP Calculation

GPS errors can be propagated from the local coordinate system (easting, northing) into the vehicle platform frame by multiplying a rotation matrix (R). Meng et al. (2004) used this method to map the GPS errors into a bridge coordinate system (BCS) to monitor the deformation of the bridge along lateral, longitudinal, and vertical directions. Based on the Euler's rotation theorem, the rotation matrix R would be (Weisstein, 1999a):

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where the angle γ is the vehicle's course over ground. Defining the matrix:

$$\begin{bmatrix} \mathrm{EDOP}^2 & & (\text{other} & \text{terms}) \\ & \mathrm{NDOP}^2 & & \\ & & \mathrm{VDOP}^2 & \\ (\text{other} & \text{terms}) & & \mathrm{TDOP}^2 \end{bmatrix} \tag{4}$$

as DOPs, the GPS position error due to satellite geometry could be propagated to the vehicle's coordinate system:

$$DOPs\_vehicle = R_z * \mathrm{DOPs} * R_z^T$$

$$= \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$* \begin{bmatrix} \mathrm{EDOP}^2 & & (\text{other} & \text{terms}) \\ & \mathrm{NDOP}^2 & & \\ & & \mathrm{VDOP}^2 & \\ (\text{other} & \text{terms}) & & \mathrm{TDOP}^2 \end{bmatrix}$$

$$* \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} \mathrm{XDOP}^2 & & (\text{other} & \text{terms}) \\ & \mathrm{ADOP}^2 & & \\ & & \mathrm{VDOP}^2 & \\ (\text{other} & \text{terms}) & & \mathrm{TDOP}^2 \end{bmatrix} \tag{5}$$

The square root of the upper left diagonal element of the matrix *DOPs_vehicle* is the XDOP, so forth for ADOP, VDOP, and TDOP. Then XDOP is the DOP perpendicular to the direction of travel, and ADOP is the component along the direction of travel. Because the cross-track error of GPS dynamic accuracy is more important in vehicle tracking and agricultural operations, only XDOP was discussed in this study.

The XDOP for 9 January 2003 can be calculated based on GPS satellite information, which was extracted from Trimble's Planning software (Trimble, 2002) using YUMA satellite almanac data from U.S. Coast Guard website. Figure 6 shows the relationship between the XDOP and the relevant axis at 36° N latitude, 84° W longitude, and 10° mask angle. An interesting situation in figure 6 is that the DOP in the northing direction calculated based on 24 h satel–

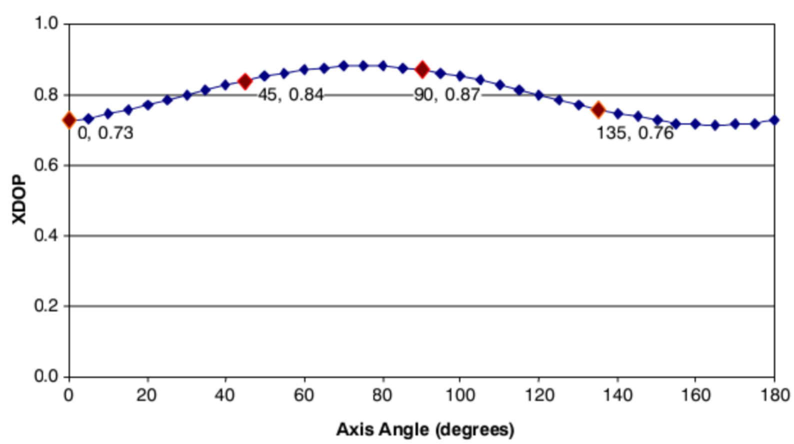

Figure 6. XDOP vs. axis angle (0°, 45°, 90°, and 135° angles highlighted) with a 10° mask angle.

Table 2. Relationship between standard deviations SD and XDOP (24 h average of all 17 Garmin GPS receivers).

Reference Axis (°)	SD	XDOP
0	2.49	0.73
10	2.53	0.75
20	2.65	0.77
30	2.84	0.80
45	3.15	0.84
40	3.04	0.83
50	3.25	0.85
60	3.43	0.87
70	3.57	0.88
80	3.67	0.88
90	3.70	0.87
100	3.68	0.85
110	3.59	0.83
120	3.46	0.80
130	3.28	0.77
135	3.19	0.76
140	3.08	0.75
150	2.88	0.73
160	2.69	0.72
170	2.55	0.72
180	2.49	0.73

lite geometry on 9 January 2003 was not the maximum. The maximum XDOP occurs at a 75° axis instead of 90°.

Expected GPS position error is typically reported as a product of DOP times the satellite pseudorange errors (Charrois, 1999; Hayward et al., 1998). Thus, for constant satellite pseudorange errors, the GPS position error is expected to be proportional to DOP. Table 2 shows the average standard deviation of the GPS static test data (Ayers et al., 2004) relative to different directions, and the calculated XDOPs for the relevant directions. The relationships are also shown in figure 7. There appears to be a general relationship of increasing standard deviation (SD) with increasing XDOP. While these findings are the result of static testing, the trends may also hold true for dynamic testing. Additional dynamic testing along different directions is needed to investigate the relationship between the cross-track error and XDOP.

The analysis described above was conducted for DOP and standard deviations determined as 24 h averages. Instantaneous DOPs are much different and vary significantly throughout the day. A plot of DOP determined at 10 min intervals is shown in figure 8.

Conclusions

GPS dilution of precision (DOP) was calculated based on GPS satellite geometry at a variety of locations and different mask angle settings. A substantial difference was found between the NDOP and the EDOP in mid-latitude areas due to the lack of satellites near the northern horizon. A clear

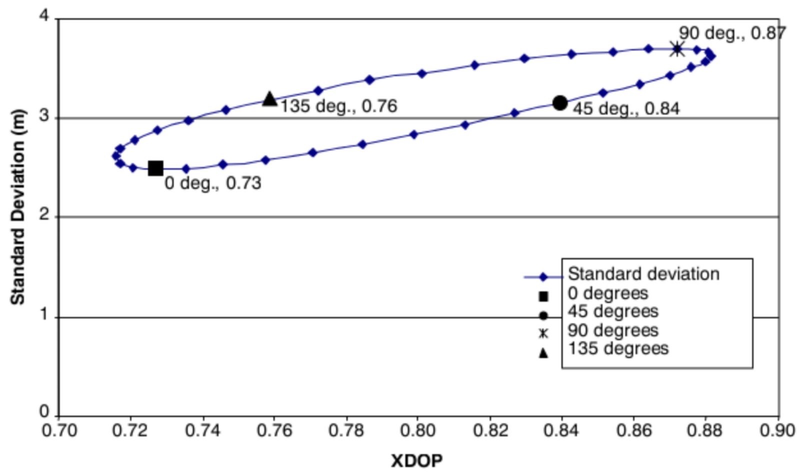

Figure 7. Relationship between the standard deviation of static data distribution and XDOP (0°, 45°, 90°, and 135° angles highlighted).

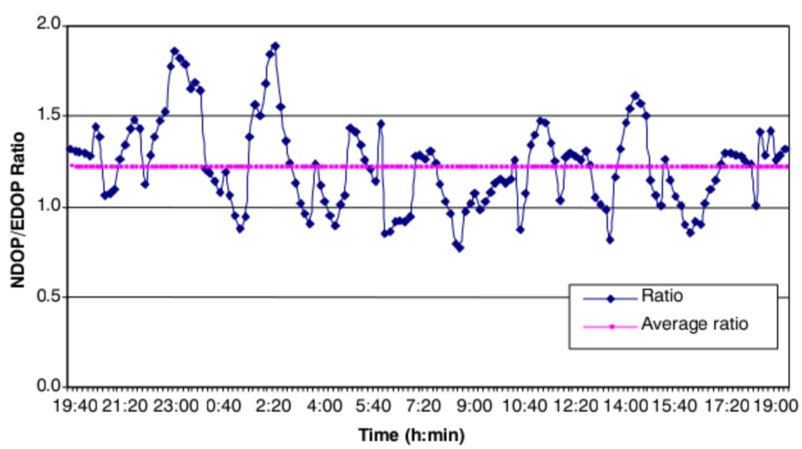

Figure 8. NDOP/EDOP changes over time on 9 January 2003 (36° N latitude, 84° W longitude, 10° mask angle).

trend of the 24 h average ratio of the NDOP to the EDOP was found to be related to latitude and mask angle. Cross-track DOP (XDOP) is defined as the GPS DOP perpendicular to the travel direction. The influence of the GPS satellite geometry on GPS accuracy was mapped into the vehicle platform frame to derive the XDOP, and accordingly to derive the influence of travel direction on GPS dynamic accuracy. Results indicate that the XDOP increases as the reference axis changes from 0° to 90°, which explains the differences of cross-track errors found by Ehsani et al. (2003). For static GPS data analysis, the standard deviations were influenced by XDOP. But to draw a precise conclusion on the relationship between XDOP and cross-track error requires additional dynamic accuracy testing in different travel directions.

Directional GPS Error Associated with Regression Techniques

When measuring GPS dynamic accuracy, the cross-track error was usually calculated by measuring the perpendicular distance from the GPS points to a referenced true path. A regression line fitted from higher accurate GPS data points can be referenced as the true path in real-world operations. Ehsani et al. (2002) generated regression lines for each pass from GPS data from different guide systems, and compared the slopes of regression lines to determine if tracks were parallel. Taylor et al. (2004) utilized a localized linear regression line, generated by 21 local GPS points, as the baseline to calculate GPS cross-track errors. A regression line also was used by Han et al. (2004) to analyze the GPS pass accuracy. None of these publications provides details on the methods used to generate the regression lines.

Fitting a Linear Model

The linear model is assumed to have the form: $Y_i = aX_i + \varepsilon_i$ (i = 1, 2, ..., n), where α is the unknown parameter, ε_i the unknown errors, and the data comprise the coordinates (X_i, Y_i) at each measured position i. The problem fitting the linear model here is to find the parameter values α that most closely match the data. An important decision to be made in fitting a linear model is the choice of norm used to define the parameter estimates (or approximation parameters). Rice and White (1964) gave the details of the definition and description of the norms for L_p smoothing and estimation:

The number a_p is the L_p estimate or L_p smoothed value of the data ($y_i = a^* + e_i$, $i = 1, 2, ..., n$.) if a_p minimizes the L_p norm:

$$\left\| y_i - a_p \right\|_p = \left[\sum_{i=1}^{n} \left| y_i - a_p \right|^p \right]^{1/p} \tag{6}$$

Clearly, L_1 norm can be written as:

$$L_1 : \left\| y_i - a_1 \right\|_1 = \left[\sum_{i=1}^{n} \left| y_i - a_1 \right|^1 \right]^{1/1} = \sum_{i=1}^{n} \left| y_i - a_1 \right| \tag{7}$$

(least absolute value criterion)

and L_2 norm can be written as:

$$L_2 : \left\| y_i - a_2 \right\|_2 = \left[\sum_{i=1}^{n} \left| y_i - a_2 \right|^2 \right]^{1/2} = \sqrt{\sum_{i=1}^{n} \left| y_i - a_2 \right|^2} \tag{8}$$

(least squares criterion).

The form used for smoothing or estimation depends on the distribution of the errors. Rice and White (1964) investigated the effect of L_p smoothing for five different types of distributions of e_i, which included uniform, triangle, normal, Laplace, and Cauchy. Conclusions drawn by Rice and White (1964) included: (1) L_∞ norm smoothing should be used for error distributions with sharply defined extremes (typified by the uniform distribution), (2) L_1 smoothing should be used for error distributions with long tails ("wild point" data), and (3) between the extremes of (1) and (2), L_2 (least squares) smoothing appeared to work best. Narula et al. (1999) also pointed out that least absolute values (LAV) regression is a more robust alternative to the popular least-square regression under certain conditions: (1) when there are outliers in the values of the response variable, (2) the errors follow a long-tailed distribution, and (3) the loss function is proportional to the absolute errors rather than their squared values.

Least-square regression is the most appropriate method for fitting the GPS data points for the following reasons: (1) the distribution of GPS errors is usually assumed to be normal, and (2) least-square regression allows the residuals to be treated as a continuous differentiable quantity.

In addition, solving the least-square regression problem is simpler than solving the least absolute value regression problem, because the absolute value function is not continuous derivatives and is not amenable to analytic solution. It has been noted that under the L_1 criterion, minimizing the least absolute values, a regression problem could be formulated as a linear programming problem (Wagner, 1959; Barrodale and Young, 1966; Davies, 1967; Robers and Ben-Israel, 1969; Kiountouzis, 1973; Sposito et al., 1978). Computation of the solution of the least absolute value regression has been difficult, and the existence of an optimum solution is not guaranteed (Sposito et al., 1978). At present, computer programs for least absolute value regression are only available in some statistical packages such as S-plus (L_1 fit function) (Insightful, 2001) and SAS (proIML) (SAS, 1983).

Vertical and Perpendicular Offsets Least-Square Fitting

A commonly used mathematical procedure for least-square regression is to find the parameter to minimize the sum of the squares of the vertical offsets of the points from the line, instead of the perpendicular offsets. Since both of the components of GPS measured position data (easting, northing) are subject to error, it is more reasonable to use perpendicular offsets least-square fitting (Madansky, 1959).

If one assumes the equation of the fitting line is $y = a + bx$, using least-square vertical offsets fitting, parameters a and b can be estimated as (Weisstein, 1999b):

$$a = \bar{y} - b\bar{x} \tag{9}$$

$$b = \frac{\text{cov}(x, y)}{\sigma_x^2} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{10}$$

Utilizing least-square perpendicular offsets fitting, parameters a and b can be estimated as (Weisstein, 1999b):

$$a = \bar{y} - b\bar{x} \tag{11}$$

$$b = -B \pm \sqrt{B^2 + 1} \quad (12)$$

where

$$B \equiv \frac{1}{2} \frac{(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2) - (\sum_{i=1}^{n} x_i^2 - n\bar{x}^2)}{n\bar{x}\bar{y} - \sum_{i=1}^{n} x_i y_i} \quad (13)$$

Notice that b has two possible values for each data set when using the perpendicular offsets fitting; the one providing the smaller value of the sum square of the perpendicular offsets $\left(\sum_{i=1}^{n} \frac{[y_i - (a + bx_i)]^2}{1 + b^2} \right)$ will be chosen.

In most the cases, for a reasonable number of noisy data points, the difference between vertical and perpendicular offsets is quite small (Weisstein, 1999b). Ledvij (2003) suggested investigating the standard errors of parameters to assess the certainty of the best-fit parameter values. When using least-square vertical fitting, the standard errors (SE) for a and b are (Weisstein, 1999b):

$$\mathrm{SE}(a) = s\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{ss_{xx}}} \quad (14)$$

$$\mathrm{SE}(b) = \frac{s}{\sqrt{ss_{xx}}} \quad (15)$$

where

$$s_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad (16)$$

and

$$s = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2 - \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}{n - 2}} \quad (17)$$

Based on the form of the standard errors of the estimated parameters, a case that $s_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$ is very small may result in large fitting errors. In GPS dynamic testing, that is the case when the vehicle is traveling in the N-S (or S-N) direction. Thus, the least-square vertical offset fitting method may not be appropriate for determining the path of travel in the N-S direction.

A Case Study

In July 2001, two GPS receivers, a Trimble AgGPS 132 and a Garmin GPS 35, were mounted in a vehicle conducting repeated tracking activities along a road at the Yakima Training Center. This road was oriented in the N-S direction. The vehicle was traveling in two opposite directions: north-south and south-north. The GPS data set selected from the vehicle tracking data is shown in figure 9.

Cross-track errors were calculated by measuring the distance from GPS points to the reference lines generated by using the two regression approaches. The differences in the calculated errors caused by the two regression methods are shown in table 3 (V_lsq represents the least-square fitting method using the vertical offsets, and P_lsq represents perpendicular offsets). Note the mean error for the V_lsq fitting is much higher than the P_lsq mean error, indicating a poorer fit. Clearly, in this case when the vehicle was traveling north-south, the commonly used regression approach, i.e., vertical offsets fitting, resulted in a model fitting error. Figure 10 shows the V_lsq and P_lsq fitting lines for the Trimble and Garmin field data. The line determined using the P_lsq fitting method shows a better representation of the vehicle path for the given travel direction. Results also show a larger fitting error for the Garmin GPS 35 data than for the Trimble AgGPS 132 data, which indicates that the fitting error depends on the accuracy of the tested GPS receiver.

To investigate the influence of travel direction on the fitting error, GPS data were programmed to rotate at different angles. Assuming the GPS data spread is constant, then the difference in the calculated errors shows the influence of travel direction on the fitting error. Figure 11 shows the comparison of the regression lines generated based on the two least-square regression approaches at data rotations of

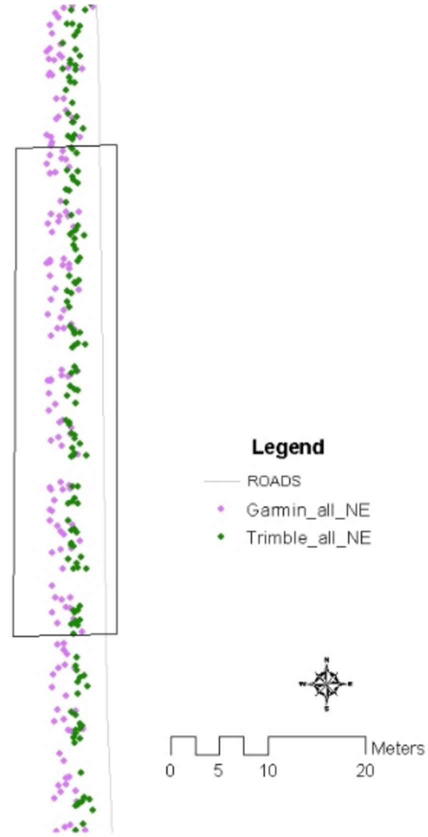

Figure 9. GPS data from the Yakima Training Center road dust study.

Table 3. Potential model fitting error for Trimble and Garmin GPS data.

	Trimble AgGPS 132 Error (m)		Garmin 35 Error (m)	
	Mean	SD	Mean	SD
V_lsq	2.38	1.34	5.46	3.13
P_Lsq	0.35	0.28	0.81	0.50
Fitting error	2.03	1.06	4.65	2.63

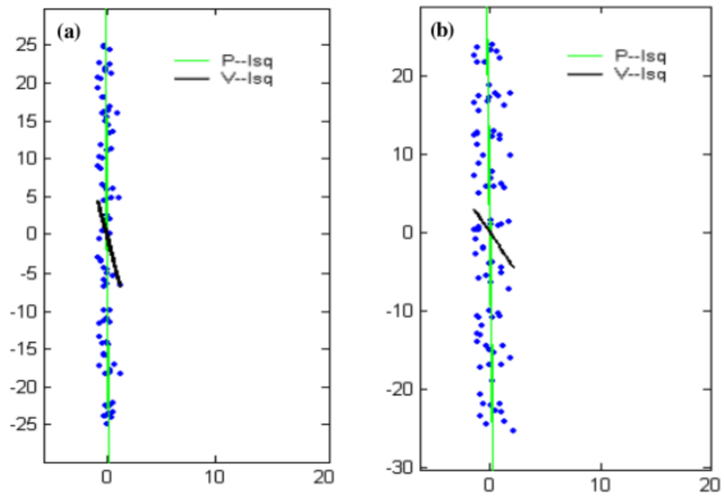

Figure 10. Regression lines for (a) Trimble AgGPS 132 and (b) Garmin 35 GPS field test data.

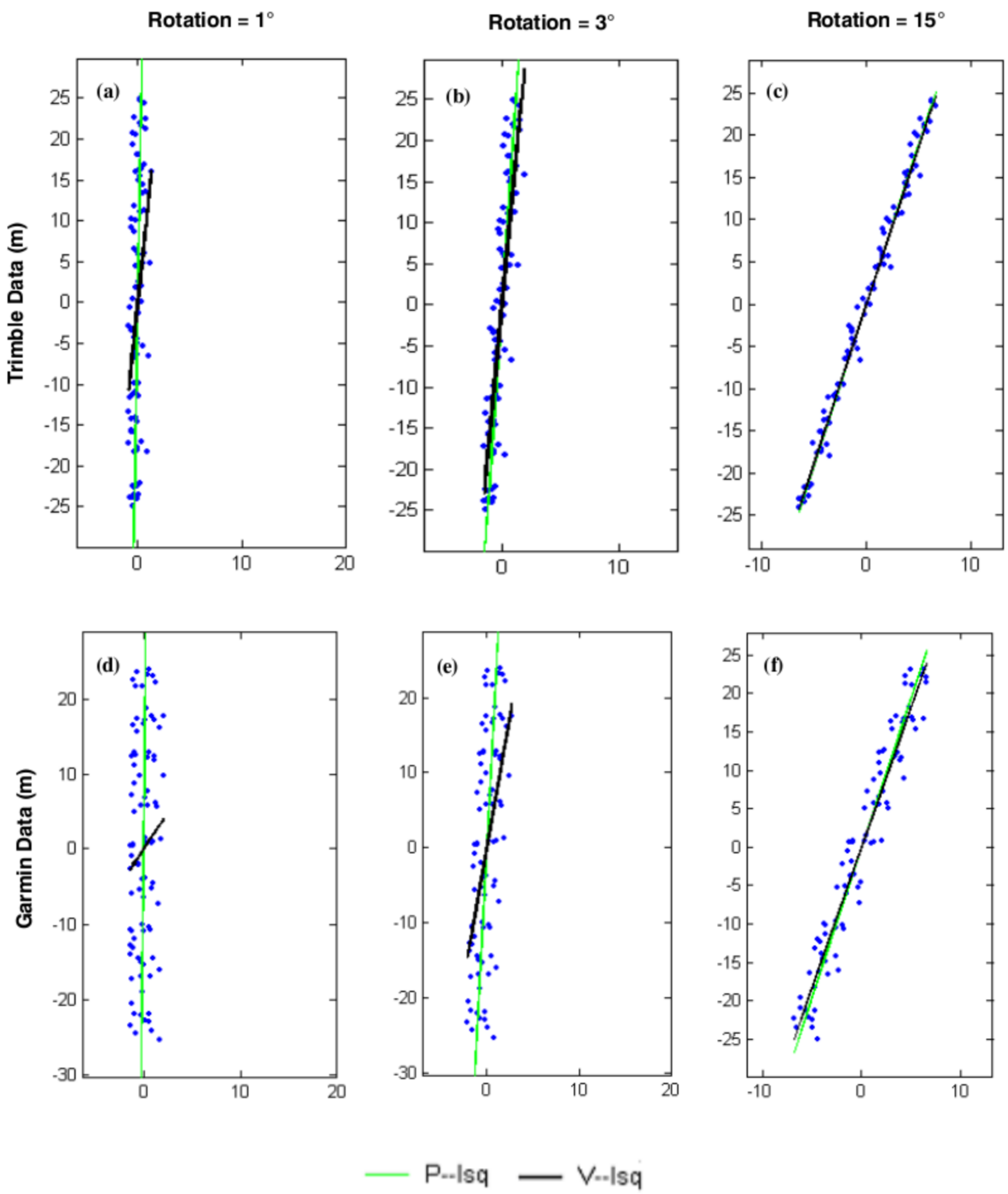

Figure 11. Regression lines from two least-square regression approaches.

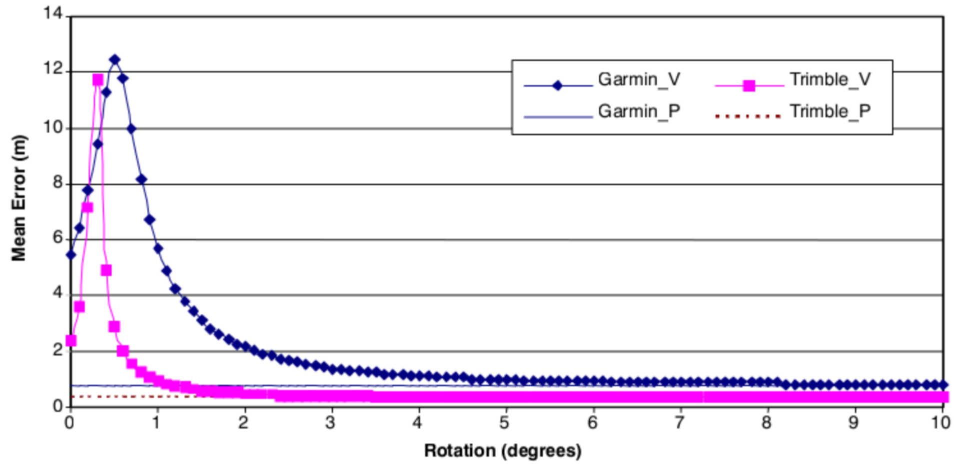

Figure 12. Influence of travel direction on the fitting error for Garmin and Trimble GPS receivers.

1°, 3°, and 15° for the Trimble AgGPS 132 and Garmin GPS 35 data.

Fitting errors at different rotation angles were calculated and are shown in figure 12. Results demonstrate that the regression fitting error reached a maximum when the vehicle was traveling in the N-S direction. The fitting error decreased as the direction of travel moved away from the true N-S direction, even though the same GPS point spreads were used. The fitting error converges to 0 at a point that is dependent on the distribution of GPS receiver's cross-track error. From figure 12, the fitting error for the Trimble data set converged to 0 when the rotation angle increased to 3°. A rotation angle of 6° was required for the convergence of the Garmin 35 GPS data. The influence of travel direction on the fitting error depends on the GPS accuracy.

The least-square regression method that minimizes the sum of squares of perpendicular offsets is recommended for estimating the baseline to be referenced as the true path for calculating GPS cross-track errors. An alternative method is to switch easting and northing (i.e., using easting as the dependent variable y and northing as the independent variable x) to apply the commonly used least-square regression approach when the travel direction is oriented nearly in a north-south or a south-north direction.

Conclusions

This section has reviewed two basic methods for fitting a line through GPS data: least-square regression, and least absolute value regression. The least-square regression was found to be appropriate for finding the best line to fit the GPS data. However, the commonly used procedure for finding the best-fit line is to find parameters minimizing the sum of the squares of the vertical offsets from points to the fitting line. Compared to the regression method that minimizes the sum of the squares of the perpendicular offsets, the commonly used regression method may cause a potential fitting error. This fitting error can be estimated by comparing the difference in errors calculated using the two regression approaches. Data were collected and analyzed from Trimble AgGPS 132 and Garmin GPS 35 receivers while the GPS-mounted vehicle was traveling along a road oriented north-south. The two regression approaches were found to yield substantial differences in the calculated cross-track errors. The GPS data were rotated at different angles to investigate the influence of travel direction on the regression fitting error. Results show that the regression fitting error reached a maximum when the vehicle was traveling in the N-S direction and decreased as the direction of travel moved away from the N-S direction, even though the same GPS point spread was used. The fitting error will converge to 0 at a certain point that is dependent on the distribution of the GPS receiver's cross-track error.

Summary

GPS dynamic accuracy testing is more difficult and complicated compared to static accuracy testing. The measured GPS error depends on the test time, the test site location, and the travel direction. The influence of the travel direction on GPS dynamic accuracy was discussed in two sections in this article.

The first section investigated the influence of travel direction on GPS accuracy caused by GPS satellite geometry. GPS error is determined by the pseudorange measurement error and the GDOP. Previous studies found significant differences in measured GPS errors for two perpendicular directions (N-S and E-W) for both dynamic (Ehsani et al., 2003) and static accuracy testing (Ayers et al., 2004; Charrois, 1999). GDOPs were calculated based on GPS satellite geometry for a variety of locations and different mask angle settings on 9 January 2003 when the 24 h GPS static testing was conducted (Ayers et al., 2004). NDOP was found to be higher than EDOP in mid-latitude areas, and was believed to be responsible for the differences in the measured GPS errors in the N-S and E-W directions. A clear trend of the 24 h averaged NDOP to EDOP ratio was found to be related to latitude and mask angle. XDOP, defined as the component of the GDOP perpendicular to the travel direction, was derived by propagating the GDOP into the vehicle platform frame to investigate the influence of travel direction on GPS dynamic accuracy. Results showed that the XDOP increased as the reference axis changed from 0° to 90°. This trend was consistent with the different standard deviations of the static GPS errors relative to different reference axes (table 2 and fig. 7). The XDOP was believed to be responsible for GPS cross-track error. More dynamic testing along different directions should be conducted to test this hypothesis and quantify the influence of XDOP on the cross-track error. When conducting GPS dynamic accuracy testing, analysis of satellite information for the test location and the tested time period is suggested to help understand and estimate the overall accuracy of the tested GPS receivers.

The second section explored the influence of the travel direction on estimates of GPS cross-track error when a regression line fitted through GPS points was referenced as the true path for calculating errors. Techniques used for determining the vehicle track influence the calculated cross-track errors. This section reviewed two basic methods for fitting linear models: least-square regression, and least absolute value regression. The least-square regression method was found to be appropriate for finding the best line to fit the GPS data. The commonly used procedure of least-square regression is to estimate model parameters that minimize the sum of the squares of the vertical offsets from data points to the fitting line, instead of the perpendicular offsets. A regression approach that minimizes the vertical offsets can cause errors in the regression line and cause additional errors in the calculated cross-track errors.

The fitting error can be estimated by comparing the calculated cross-track errors using the two regression approaches. Data were collected and analyzed from a Trimble AgGPS 132 and a Garmin GPS 35, while the GPS-mounted vehicle was traveling along a road oriented in the N-S direction. Significant differences were seen in the calculated cross-track errors when using the different regression methods; the regression approach that minimized the vertical offsets resulted in a large model fitting error. The same GPS data were rotated at different angles to investigate the influence of travel direction on the fitting error. Results found that the fitting error reached the maximum when the vehicle was traveling in the N-S direction and decreased as the direction of travel moved away from the true N-S direction. The fitting error will converge to 0 at a certain point that is dependent on the accuracy of the tested GPS receiver. The fitting error converged to 0 at a lower deviation from the N-S direction for the Trimble AgGPS 132 receiver than for the Garmin GPS 35 receiver.

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