CHEM 116 January 29, 2015

Unit 2, Extra Notes

Numerical Methods and Statistics

The purpose of this document is to show you how English sentences represent probability equations
Consider the probability distribution

$$\mathrm{P}(X = x, Y = y, Z = z) \tag{1}$$

with no assumptions of independence, the following is true

	Sentence	Equation
1.	What is the probability of $X = 2$, $Y = 2$, and $Z = 0$?	$\mathrm{P}(X = 2, Y = 2, Z = 0)$
2.	What is the probability of $X = 1$, $Y = 5$?	$\sum_z \mathrm{P}(X = 1, Y = 2, Z = z)$
3.	What is the probability of $X = 1$, $Y = 5$?	$\sum_z \mathrm{P}\left(X = 1, Y = 2 \mid Z = z\right) \mathrm{P}(Z = z)$
4.	If $Z = 4$, what is the probability of $X = 2$, $Y = 4$?	$\mathrm{P}\left(X = 2, Y = 4 \mid Z = 4\right)$
5.	If $Z = 4$, what is the probability of $X = 2$, $Y = 4$?	$\mathrm{P}(X = 2, Y = 4)/\mathrm{P}(Z = 4)$
6.	What is the probability of $X = 2$, $Y = 4$, given Z is 4?	$\mathrm{P}\left(X = 2, Y = 4 \mid Z = 4\right)$
7.	If $Y = 2$ and $Z = 0$, what is the probability of X?	$\mathrm{P}\left(X = x \mid Y = 2, Z = 0\right)$
8.	If $Y = 2$ and $Z = 0$, what is the probability of X?	$\mathrm{P}(X = x, Y = 2, Z = 0)/\mathrm{P}(Y = 2, Z = 0)$

If we assume that X and Y are conditionally independent on Z, the following is true:

$$\mathrm{P}\left(X = x, Y = y \mid Z = z\right) = \mathrm{P}\left(X = x \mid Z = z\right) \mathrm{P}\left(Y = y \mid Z = z\right) \tag{2}$$

The equations above are always true, but the following can be simplified

	Sentence	Equation
3.	What is the probability of $X = 1$, $Y = 5$?	$\sum_z \mathrm{P}\left(X = 1 \mid Z = z\right) \mathrm{P}\left(Y = 2 \mid Z = z\right) \mathrm{P}(Z = z)$
4.	If $Z = 4$, what is the probability of $X = 2$, $Y = 4$?	$\mathrm{P}\left(X = 2 \mid Z = 4\right) \mathrm{P}\left(Y = 4 \mid Z = 4\right)$
6.	What is the probability of $X = 2$, $Y = 4$, given Z is 4?	$\mathrm{P}\left(X = 2 \mid Z = 4\right) \mathrm{P}\left(Y = 4 \mid Z = 4\right)$
7.	If $Y = 2$ and $Z = 0$, what is the probability of X?	$\mathrm{P}\left(X = x \mid Z = 0\right)$