## STAT431 HW3

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## Question1

```
(a)  x=c(481,144,93,446,69,170,383,63,79,181) \\ logx=log(x) \\ ybar=mean(logx) \\ s=sd(logx) \\ n=length(logx) \\ mu0=0 \\ kappa=1 \\ sigma.2.prior.mean=1 \\ sigma.2.prior.var=1 \\ alpha=sigma.2.prior.mean^2 / sigma.2.prior.var + 2 \\ beta=(alpha-1) * sigma.2.prior.mean \\ The prior distribution is normal-inverse gamma distribution. The mean of normal distribution is 0 and variance is 1 and <math>\alpha is 3 and \beta is 2
```

(b)

```
mu1=(kappa*mu0+n*ybar)/(kappa+n)
tau1square=(alpha + n/2) / (beta + 0.5 * (n-1)*s^2 +0.5 * kappa*n*(mu0-ybar)^2/(kappa + n))
mu.posterior.location=mu1
mu.posterior.scale=1/(tau1square*(kappa+n))
mu.posterior.df=2*alpha+n
```

The posterior marginal distribution of  $\mu$  is t distirbution. The location is 4.6188744, the scale is 0.1868746 and the degree of freedom is 16.

(c)

```
mu.posterior.mean=mu.posterior.location
mu.posterior.var=(mu.posterior.df/ (mu.posterior.df - 2)) * mu.posterior.scale
mu.posterior.location+qt(c(0.025, 1-0.025), mu.posterior.df) * sqrt(mu.posterior.scale)

## [1] 3.702461 5.535288

(d)
alpha.posterior=alpha + n/2
beta.posterior=beta + 0.5 * (n-1)*s^2 +0.5 * kappa*n*(mu0-ybar)^2/(kappa + n)
```

The posterior marginal distribution of  $\sigma^2$  is inverse gamma distribution with  $\alpha$  8 and  $\beta$  16.4449679.

(e)

```
1 / qgamma(c(1-0.025, 1-0.975), alpha.posterior, beta.posterior)
```

```
## [1] 1.140216 4.761369
```

## Question2

(a)

```
mu.location=ybar
mu.scale=s^2/n
mu.df=n-1
```

The posterior marginal distribution of  $\mu$  is t distribution. The location is 5.0807619 and scale is 0.0602504 and degree of freedom is 9.

(b)

```
mu.posterior.mean=ybar
mu.posterior.var=(n-1) * mu.scale / (n-3)
ybar + qt(c(0.025, 1-0.025), n-1) * sqrt(mu.scale)

## [1] 4.525494 5.636030
  (c)
alpha.posterior=(n-1) / 2
beta.posterior=(n-1)*s^2/2
```

The posterior marginal distribution of  $\sigma^2$  is inverse gamma distribution with  $\alpha$  4.5 and  $\beta$  2.7112675.

(d)

```
(n-1) * s^2 / qchisq(c(1-0.025, 1-0.975), n-1)
```

```
## [1] 0.285055 2.008057
```