## STAT 431 — Applied Bayesian Analysis — Spring 2019

## Homework 2

Please submit your assignment on paper. Make sure your answers are completely justified and clear enough to read! Any computer code and output should be included.

- 1. Of the 70 respondents to the survey after the first lecture, 6 reported having played the card game Euchre. Assume the survey was a random sample from a large population of "people like us." Suppose we are interested in the proportion  $\pi$  of "people like us" who have played Euchre. Answer parts (a), (b), (c), and (d) under *each* of the following priors on  $\pi$ :
  - (I) a uniform ("flat") prior
  - (II) a (very informative) beta prior with parameters  $\alpha = 100$  and  $\beta = 100$  (see Cowles Table A.2)
  - (a) [4 pts] Find a (full) formula for the posterior density function for  $\pi$ . Also, accurately graph the posterior density function.
  - (b) [4 pts] Compute the posterior mean and posterior standard deviation of  $\pi$ .
  - (c) [2 pts] Compute a 95% equal-tailed credible interval for  $\pi$ .
  - (d) [2 pts] Compute the posterior probabilities of  $H_0: \pi \geq 0.2$  and  $H_1: \pi < 0.2$
- 2. [3 pts] Cowles, Problem 5.3 [ Note: "(5.2)" refers to equation (5.2) on p. 74. The equation that precedes it (the transformation-of-variables formula) should have  $g^{-1}$  in place of g.]

## 3. GRADUATE SECTION ONLY

Refer to Problem 1. The card game Euchre usually requires 4 players. Suppose 20 random "people like us" attend a party.

- (a) [2 pts] By using the usual frequentist estimate  $\hat{\pi}$  in place of  $\pi$ , compute the (approximate) probability that at least 4 of them have played Euchre.
- (b) [3 pts] Based on the posterior you computed from the <u>uniform</u> ("flat") prior in Problem 1, compute the *posterior predictive* probability that *at least* 4 of them have played Euchre.

Show your work and the final numerical answers, correct to at least three significant digits.