STAT431_HW2

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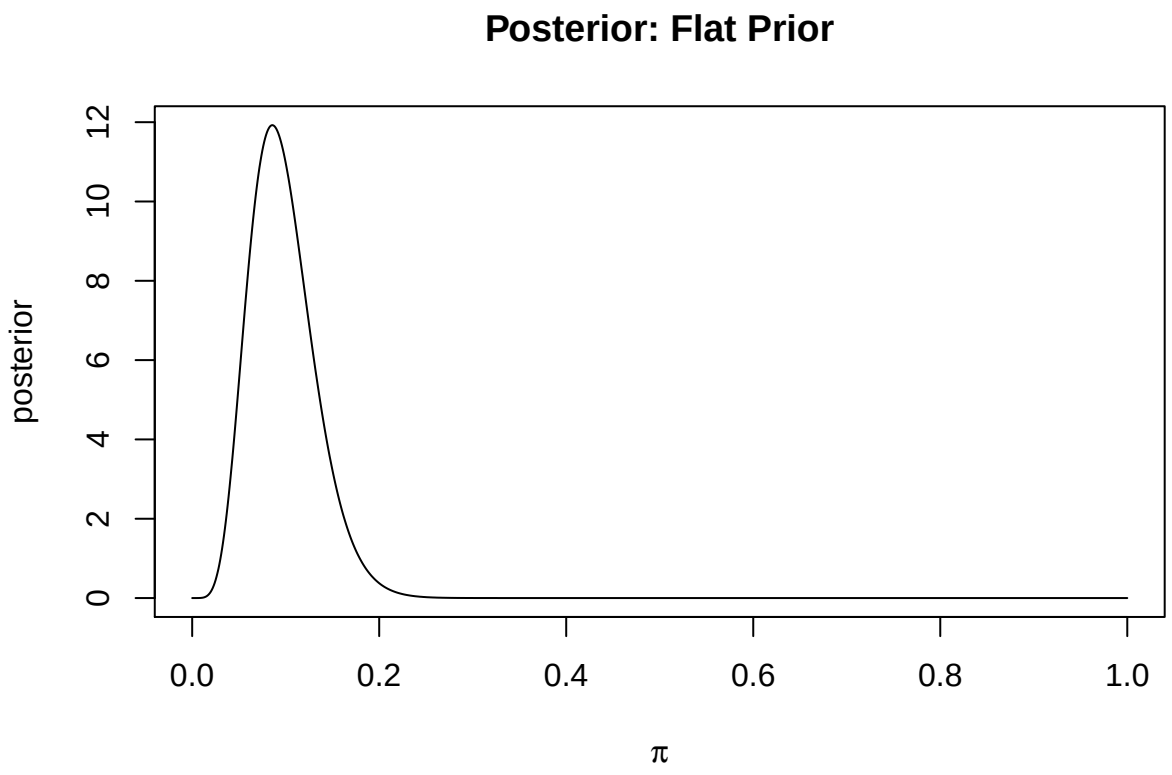
Question1

(a)

The posterior density function for uniform prior is

$\frac{\Gamma(72)}{\Gamma(7)\Gamma(65)}\pi^6(1-\pi)^{64}$.

```
n=70
y=6
dx <- 1/1000  # pi grid spacing
pi.grid <- seq(0, 1, dx)
flat.prior <- dunif(pi.grid,0,1)
post.unscaled <- flat.prior * dbinom(y,n,pi.grid)
posterior <- post.unscaled / sum(post.unscaled * dx)
plot(pi.grid, posterior, type="l",
     xlab=expression(pi), main="Posterior: Flat Prior")
```

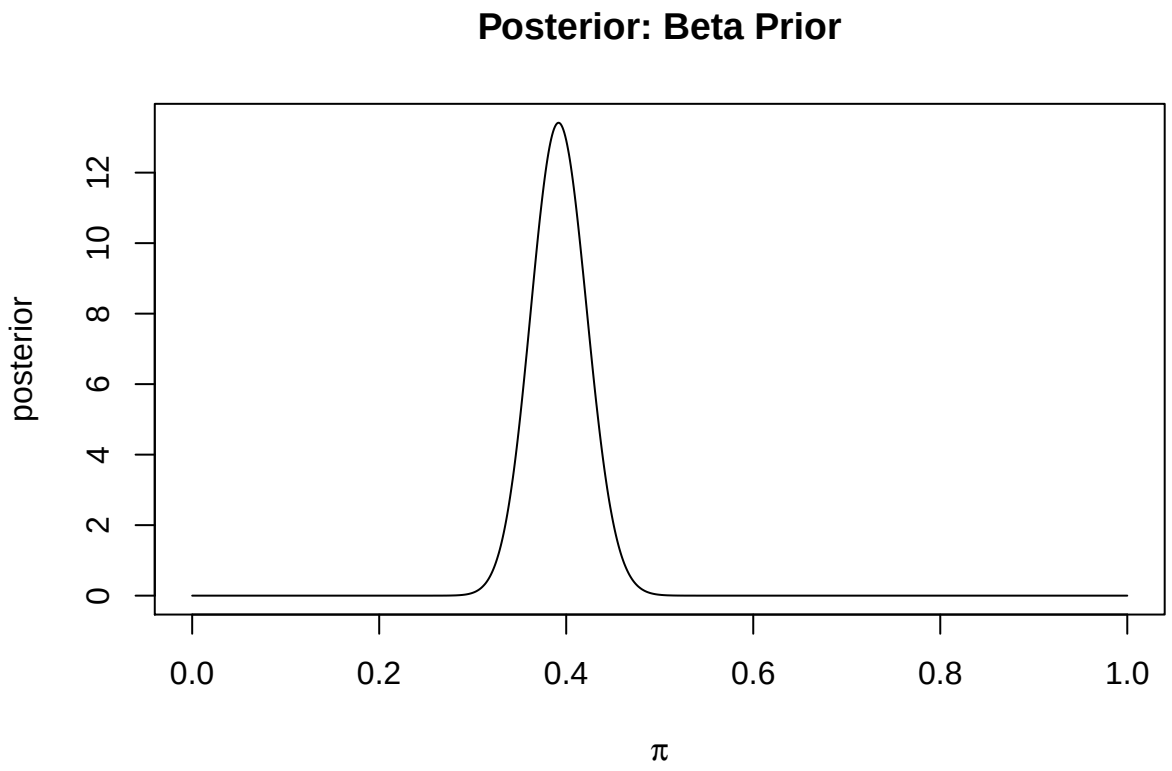


The posterior density function for beta prior is

$\frac{\Gamma(170)}{\Gamma(106)\Gamma(164)}\pi^{105}(1-\pi)^{163}$.

```
beta.prior <- dbeta(pi.grid,100,100)
post.unscaled <- beta.prior * dbinom(y,n,pi.grid)
posterior <- post.unscaled / sum(post.unscaled * dx)
```

```r
plot(pi.grid, posterior, type="l",
     xlab=expression(pi), main="Posterior: Beta Prior")
```



(b)

```r
#posterior mean when uniform prior
mean1=(6+1)/(70+1+1)
#postrior mean when beta prior
mean2=(6+100)/(70+100+100)
#posterior standard deviation when uniform prior
std1=sqrt((6+1)*(64+1)/(6+1+64+1)^2*(6+1+64+1+1))
#posterior standard deviation when beta prior
std2=sqrt((6+100)*(64+100)/(6+100+64+100)^2*(6+100+64+100+1))
```

When we used uniform prior, posterior mean was 0.0972222 and standard deviation was 2.5312476. When we used beta prior, posterior mean was 0.3925926 and standard deviation was 8.0388836.

(c)

```r
#95% equal-tailed credible interval of uniform prior
qbeta(c(0.025, 0.975), y+1, n-y+1)
```

```
## [1] 0.04056616 0.17491606
```

```r
#95% equal-tailed credible interval of beta prior
qbeta(c(0.025, 0.975), y+100, n-y+100)
```

```
## [1] 0.3352556 0.4514369
```

(d)

```r
#posterior probabilities of H0 of uniform prior
1 - pbeta(0.2, y+1, n-y+1)
```

```
## [1] 0.006916765
```

The null hypothesis was rejected. We are in favor of alternative.

```
#posterior probabilities of H0 of beta prior
1 - pbeta(0.2, y+100, n-y+100)
```

```
## [1] 1
```

The null hypothesis was not rejected .