STAT 431 — Applied Bayesian Analysis — Spring 2019

HW1 Solutions

February 2019

Contents

1

Let P denote the event that a randomly chosen student owns a pet.

Model	Prior	Likelihood for P	Prior $\times$ Likelihood	Posterior
Apartment	0.86	0.18	0.1548	0.72
Not Apartment	0.14	0.43	0.0602	0.28

If a randomly selected student owns a pet, there is a 72% chance they live in an apartment.

2 Cowles, Problem 2.4

$$P(\text{Stat}) = 0.35 \tag{1}$$
$$P(\text{Biostat}) = 0.25 \tag{2}$$
$$P(\text{Other}) = 0.40 \tag{3}$$
$$P(\text{Woman} \mid \text{Stat}) = 0.60 \tag{4}$$
$$P(\text{Woman} \mid \text{Biostat}) = 0.75 \tag{5}$$
$$P(\text{Woman} \mid \text{Other}) = 0.40 \tag{6}$$

2.1

Let W denote the event that a randomly drawn student is a woman.

$$P(\text{W}) = P(\text{W} \mid \text{Stat})P(\text{Stat}) + P(\text{W} \mid \text{Biostat})P(\text{Biostat}) + P(\text{W} \mid \text{Other})P(\text{Other}) \tag{7}$$
$$= 0.60 * 0.35 + 0.75 * 0.25 + 0.40 * 0.40 \tag{8}$$
$$= 0.5575 \tag{9}$$

2.2

$$P(\text{Other} \mid \text{W}) = \frac{P(\text{Other \& W})}{P(\text{W})} \tag{10}$$
$$= \frac{0.40 * 0.40}{0.5575} \tag{11}$$
$$= 0.2869 \tag{12}$$

3 GRADUATE SECTION ONLY

$$X \sim \text{Exp}(1) \tag{13}$$
$$Y \mid X = x \sim \text{Poisson}(x) \tag{14}$$

3.1

$$f(x) = e^{-x}, \quad x \geq 0 \tag{15}$$
$$P(Y = y \mid X = x) = \frac{x^y e^{-x}}{y!}, \quad y = 0, 1, 2, \ldots \tag{16}$$

3.2 Marginal density of Y

$$P(Y = y) = \int_0^\infty P(Y = y \mid X)P(X)dX \tag{17}$$
$$= \int_0^\infty \frac{X^y e^{-X}}{y!} e^{-X} dX \tag{18}$$
$$= \int_0^\infty \frac{X^y e^{-2X}}{y!} dX \tag{19}$$
$$= \left(\frac{1}{2}\right)^{y+1} \int_0^\infty \frac{2^{y+1} X^y e^{-2X}}{y!} dX \tag{20}$$
$$= \left(\frac{1}{2}\right)^{y+1}, \qquad y = 0, 1, 2, \ldots \tag{21}$$

The integral in equation 20 is the integral over the pdf of a Gamma$(y+1, 2)$ distribution (following the parameterization of Cowles), which integrates to 1. From the parameter bounds for the Gamma distribution, this expression holds for $y > -1$.

3.3

The marginal distribution for Y is the Geometric$(p = \frac{1}{2})$ distribution, where Y represents the number of trials up to, but *not* including, the first success.