Stat 432 Homework 7

Assigned: Mar 09, 2019; Due: 11:59pm Mar 15, 2019

Before starting this homework, you should read the rlab file of BayRegLinReg on the course website carefully.

Question 1 (Bayesian updates) [5 points]

Consider a simple normal model for the height y of students where $y|\theta \sim N(\theta, 16)$. Assume a conjugate prior $\theta \sim N(65, 36)$. We measure the height of two students and observe $y_1 = 68$ and $y_2 = 72$.

a. Find the posterior distribution of θ given $\{y_1, y_2\}$ when the data come sequentially (i.e., we first observe y_1, we update our prior, then observe y_2 and update our prior again).
b. Find the posterior distribution of θ given $\{y_1, y_2\}$ when data come simultaneously (i.e., we update our prior after observing both y_1 and y_2).
c. Plot the prior, the likelihood and all the different posterior distributions. Discuss your findings (just one paragraph).

Question 2 (Bayesian linear regression) [5 points]

Now we use the same Boston housig data to do a Bayesian linear regression.

```
data(Boston, package="MASS")
# head(Boston)

useLog = c(1,3,5,6,8,9,10,14)
Boston[,useLog] = log(Boston[,useLog])
Boston[,2] = Boston[,2] / 10
Boston[,7] = Boston[,7]^2.5 / 10^4
Boston[,11] = exp(0.4 * Boston[,11])/1000
Boston[,12] = Boston[,12] / 100
Boston[,13] = sqrt(Boston[,13])
```

Continuing with HW6 Q1 where we did ridge regression on a sequence of λ's, now we do the Bayesian linear regression with a sequence of priors for $\beta \sim N_{p+1}(0, \Lambda_0)$. The correspondence between Λ_0 and λ can be found on page 50 of lecture `BayesRegLinReg`. You can either use a fixed σ^2 estimated from the data or give it a prior similarly as the one on page 53. Compare the solutions between Frequentist (from HW6) and Bayesian. You might consider the similar plot as on page 61.

Extra-Credit Question [5 points]

Use the same Boston housig data to do a Bayesian lasso. Continuing with HW6 Q2 where we did lasso on a sequence of λ's, now we do the Bayesian lasso with a sequence priors that correspond to λ's specified on page 58. Compare the solutions between Frequentist (from HW6) and Bayesian. You might consider the similar plot as on page 61. Refer to the file on the repository included in rlab document `BayesRegLinReg` for help.