STAT 429 HW01

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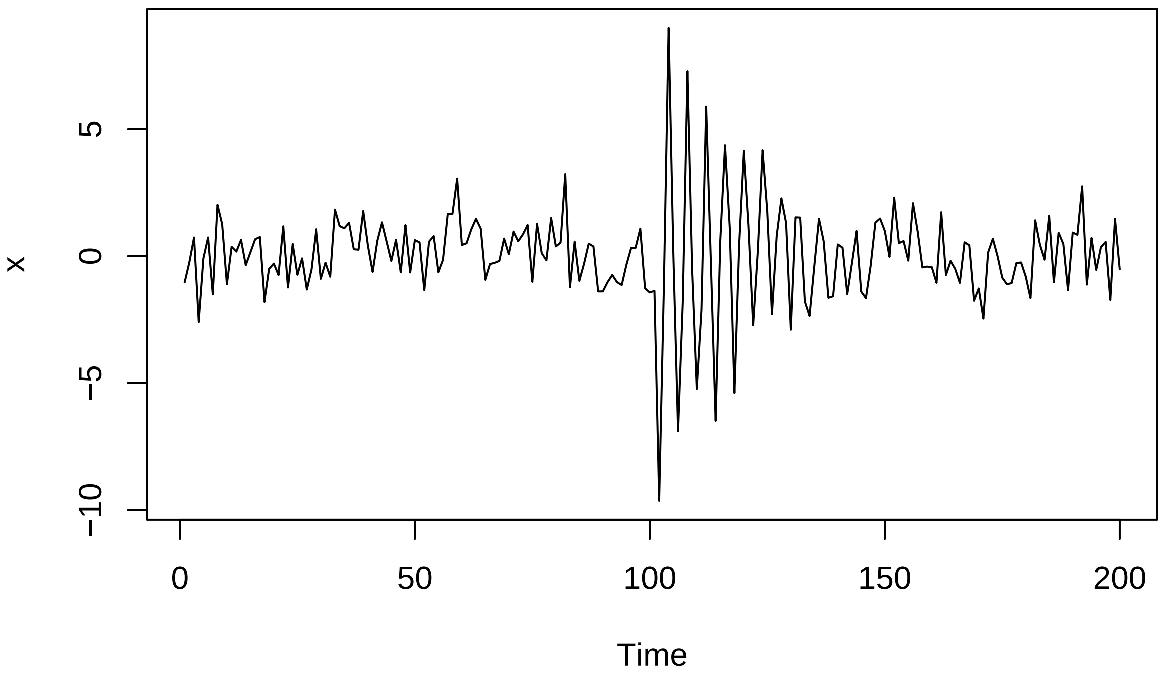
1.

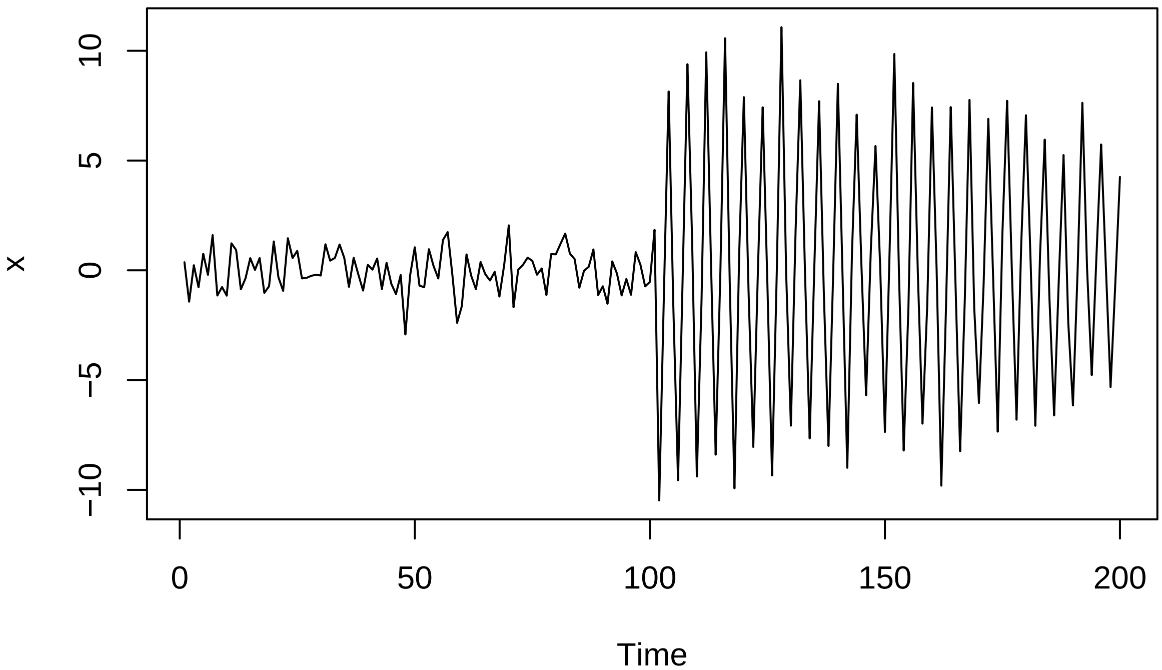
(a)

```r
s=c(rep(0,100),10*exp(-(1:100)/20)*cos(2*pi*1:100/4))
x=s+rnorm(200)
plot.ts(x)
```



(b)

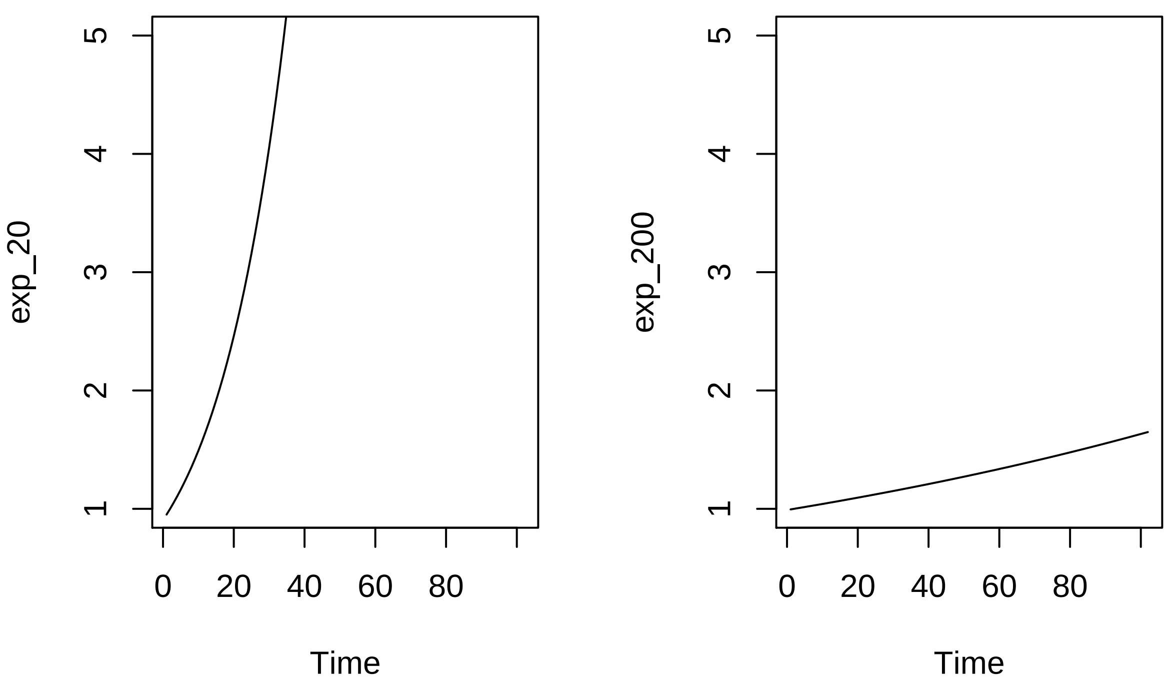
```r
s=c(rep(0,100),10*exp(-(1:100)/200)*cos(2*pi*1:100/4))
x=s+rnorm(200)
plot.ts(x)
```



(c)

series(a) looks like earthwake series in the point that oscillation is relateively smooth. On the other hand, series(b) looks like explosion series because their oscillation is violent.

```
exp_20=exp(-1:100/20)
exp_200=exp(-1:100/200)
par(mfrow=c(1,2))
plot.ts(exp_20,ylim=c(1,5))
plot.ts(exp_200,ylim=c(1,5))
```



As you can see from the graph above, $exp\{-t/20\}$ incresas more sharply than $exp\{-t/200\}$.

2

1.

Mean: $E[X_t] = a + bE[Z_t] + cE[Z_{t-2}] = a$

autocovariance:

$Cov(X_t, X_s) = b^2\sigma^2 + c^2\sigma^2$ (t=s)

$Cov(X_t, X_s) = E[(bZ_t + cZ_{t-2})(bZ_s + cZ_s - 2)] = 0$ (|t-s|=1)

$Cov(X_t, X_s) = bc\sigma^2$ (|t-s|=2)

$Cov(X_t, X_s) = 0$ (otherwise)

They are all independent of t. Hence, this is stationary.

2.

Mean: $E[X_t] = cos(ct)E[Z_1] + sin(ct)E[Z_2] = 0$

autocovariance: $Cov(X_t, X_s) = cos^2(ct)\sigma^2 + sin^2(cs)\sigma^2$

This autocovariance depends on t. Hence, this is not stationary.

3.

Mean: $E[X_t] = cos(ct)E[Z_t] + sin(ct)E[Z_{t-1}] = 0$

autocovariance:

$Cov(X_t, X_s) = cos^2(ct)\sigma^2 + sin^2(cs)\sigma^2$ (t=s)

$Cov(X_t, X_s) = cos(ct)sin(ct)\sigma^2$ (|t-s|=1)

$Cov(X_t, X_s) = 0$ (otherwise)

This depends on t. Hence, this is not stationary.

4.

Mean: $E[X_t] = a$

autocovariance: $Cov(X_t, X_s) = b^2\sigma^2$

Hecne, this is stationary.

5.

Mean: $E[X_t] = 0$

autocovariance: $Cov(X_t, X_s) = cos^2(ct)\sigma^2$

This autocovariance depends on t. Hence, this is not stationary.

6.

Mean: $E[X_t] = E[Z_t]E[Z_{t-1}] = 0$

autocovariance:

$Cov(X_t, X_s) = E[Z_t^2]E[Z_{t-1}^2] = \sigma^4$ (t=s)

$Cov(X_t, X_s) = 0$ (|t-s|=1)

$Cov(X_t, X_s) = 0$ (otherwise)

3

(a)

autocovariance:

$Cov(X_t, X_s) = 1 + 0.8^2 = 1.64$ (t=s)

$Cov(X_t, X_s) = 0.8$ (|t-s|=2)

$Cov(X_t, X_s) = 0$ (otherwise)

autocorrelation:

$\rho(h) = 1$ (t=s)

$\rho(h) = \frac{0.8}{1+0.8^2} = 0.48780$ (|t-s|=1)

$\rho(h) = 0$ (otherwise)

(b)

$Var[\frac{(X_1+X_2+X_3+X_4)}{4}] = Var[\frac{Z_1+Z_2+Z_3+0.8Z_1+Z_4+0.8Z_2}{4}] = Var[\frac{1.8Z_1+1.8Z_2+Z_3+Z_4}{4}] = 0.45^2 + 0.45^2 + 0.25^2 + 0.25^2 = 0.405 + 0.125 = 0.53$

(c)

$Var[\frac{(X_1+X_2+X_3+X_4)}{4}] = Var[\frac{Z_1+Z_2+Z_3-0.8Z_1+Z_4-0.8Z_2}{4}] = Var[\frac{0.2Z_1+0.2Z_2+Z_3+Z_4}{4}] = 0.05^2 + 0.05^2 + 0.25^2 + 0.25^2 = 0.005 + 0.125 = 0.13$

The variance of $\theta = 0.8$ is larger than that of $\theta = -0.8$.

4

(a)

mean: $E[Y_t] = E[\mu_t] = \mu_t constant$

covariance: $Cov(Y_t, Y_{t-k}) = cov(\mu_t+\sigma_t X_t, \mu_{t-k}+\sigma_{t-k}X_{t-k}) = cov(\sigma_t X_t, \sigma_{t-k}X_{t-k}) = \sigma_t\sigma_{t-k}Cov(X_t, X_{t-k}) = \sigma_t\sigma_{t-k}\frac{Corr(X_t,X_{t-k})}{\sqrt{Var(X_t)Var(X_{t-k})}} = \sigma_t\sigma_{t-k}\rho_k$

(b)

$Var(Y_t) = Var(\mu_t + \sigma_t X_t) = \sigma_t^2 Var(X_t) = \sigma_t^2$

$Corr(Y_t, Y_{t-k}) = \frac{Cov(Y_t,Y_{t-k})}{\sqrt{Var(Y_t)Var(Y_{t-k})}} = \frac{\sigma_t\sigma_{t-k}\rho_k}{\sigma_t\sigma_{t-k}} = \rho_k$

Hence the correlation depends only on time lag. However, Y_t is not stationary because the mean function μ_t is not constant over time.

(c)

If we change the μ_t into μ (that does not depend on time) and set Y_t as $Y_t = \mu + \sigma_t X_t$, the mean function of Y_t is μ and autocorrelation is ρ_k. However the autocovariance function is $\sigma_t\sigma_{t-k}\rho_k$. Y_t has a constant mean and correlation free of t but is not stationary.

5

mean: $E(x_t) = E(U_1 sin(2\pi\omega_0 t)) + E(U_2 cos(2\pi\omega_0 t)) = 0$

autocovariance: $\gamma(h) = Cov(t, t+h) = E([U_1 sin(2\pi\omega_0 t) + U_2 cos(2\pi\omega_0 t)][U_1 sin(2\pi\omega_0(t+h)) + U_2 cos(2\pi\omega_0(t+h))]) = E[U_1^2 sin(2\pi\omega_0 t) sin(2\pi\omega_0(t+h))] + E[U_2^2 cos(2\pi\omega_0 t) cos(2\pi\omega_0(t+h))] = \sigma^2 \frac{1}{2} \{cos(2\pi\omega_0 h) - cos(\alpha)\} + \sigma^2 \frac{1}{2} \{cos(\alpha) + cos(2\pi\omega_0 h)\} = \sigma^2 cos(2\pi\omega_0 h)$

Hence, this is stationary.

6

(a)

mean:

1. If t is even, $E[X_t] = E[Z_t] = 0$.
2. If t is odd, $E[X_t] = E[(Z_{t-1}^2 - 1)/\sqrt{2}] = \frac{1-1}{\sqrt{2}} = 0$.

autocovariance:

1. *When t is even,*

$Var(X_t) = 1$

if h=1

$Cov(X_t, X_{t+1}) = Cov(Z_t, (Z_t^2 - 1)/\sqrt{2}) = E[(Z_t^3 - Z_t)/\sqrt{2}] = 0$

if h 0,1

$Cov(X_t, X_{t+h}) = 0$

2. *When t is odd,*

$Var(X_t) = E[X_t^2] = E[(Z_{t-1}^4 - 2Z_{t-1}^2 + 1)/2] = (3 - 2 + 1)/2 = 1$

if h=1

$Cov(X_t, X_{t+1}) = E[\frac{Z_{t+1}(Z_{t-1}^2 - 1)}{\sqrt{2}}] = 0$

if h 0,1

$Cov(X_t, X_{t+h}) = 0$

That's why, X_t is WN(0,1). However, if t is odd, X_t is obiously correlated with X_{t-1} and so this is not iid noise.

(b)

if n is odd,

$E[X_{n+1}|X_1, X_2 \cdots X_n] = E[Z_{n+1}|Z_0, Z_2, \cdot\cdot Z_{n-1}] = 0$

if n is even,

$E[X_{n+1}|X_1, X_2 \cdots X_n] = E[(Z_n^2 - 1)/\sqrt{2}|Z_0, Z_2, \cdot\cdot, Z_n] = (Z_n^2 - 1)/\sqrt{2}$

7

MA(q)=$\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}$

if j=0 $\Rightarrow \psi_j = 1$

elif j=1,2,·· q $\Rightarrow \psi_j = \theta_j$

otherwise $\Rightarrow \psi_j = 0$

$Cov(X_t, X_{t+h}) = Cov(\sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}, \sum_{k=-\infty}^{\infty} \psi_k Z_{t+h-j}) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \psi_j \psi_k cov(Z_{t-j}, Z_{t+h-k}) = \sum_{j=-\infty}^{\infty} \psi_{j+h} \psi_j \sigma^2$

Hence,

if |h|≤ q; $\theta_h \sigma^2 + \sum_{j=1}^{q-|h|} \theta_{j+|h|} \theta_j \sigma^2$

if |h|>q; 0