Networks: An Introduction (Newman)
Chapter 8.1 - 8.4 discussion

Kwame Okrah

April 3, 2014

The large-scale structure of real world networks

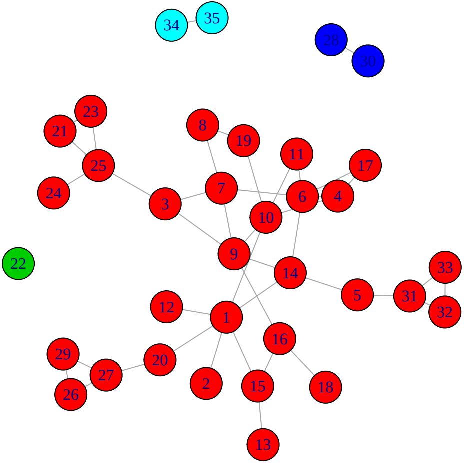
We will apply the ideas of chapter 6 and 7 to real world networks to get a sense of how they behave in general (mostly).

For today we will look at:

1. **Components**
   - Undirected and
   - Directed networks
2. **Shortest Path and the Small-World Effect**
3. **Degree Distribution**
4. **Power Laws and Scale-Free Networks**

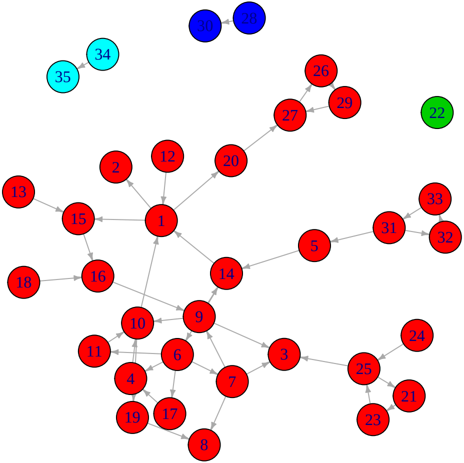
Components (Undirected Networks)

Any two vertices in a component must be connected.



Weakly Connected Components (Directed Networks)

Any two vertices in a component must be connected without regards to



Some Examples of Large Components and Large Weakly Connected Components

We typically find that large components are usually more than half of vertex size (n) and not infrequently over 90% .

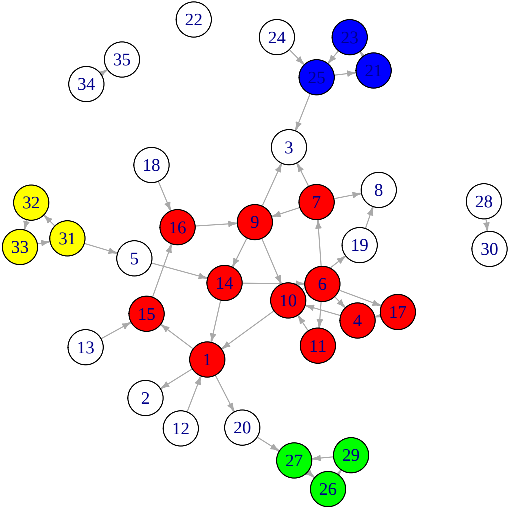
Table : (Table 18.1) Basic stats for some selected networks. Where n=no. of vertices, m=no. of edges, c=mean degree, S=% of largest component.

Network	Type	n	m	c	S
Marine food web	Directed	134	598	4.46	1.000
Software pack.	Directed	1,439	1,723	1.20	0.998
Metabolic	Undirected	765	3,686	9.64	0.996
Bio. co-auth.	Undirected	1,520,251	11,803,064	15.53	0.918
Phy. co-auth.	Undirected	52, 909	245,300	9.27	0.838
Protein int.	Undirected	2,115	2,240	2.12	0.689
Student dating	Undirected	573	477	1.66	0.503

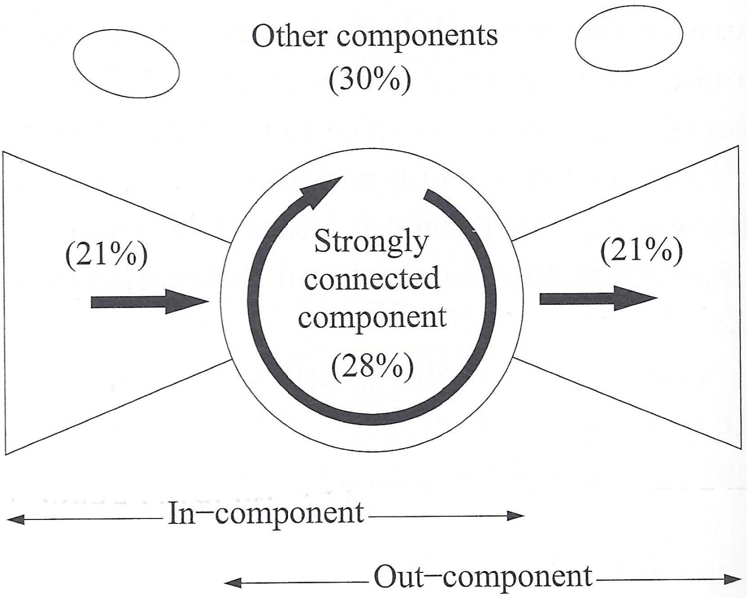
Strongly Connected Components (Directed Networks)

- Any two vertices in a component must be connected (directed).
- Every strongly connected component generate an **in-component** and an **out-component**.

Example: Strongly Connected Components



The Bowtie Effect



The Shortest Path and the Small-World Effect

- **Small-world effect**: the observation that in most real world networks the distances between vertices are small.
- Recall: $l_i = \frac{1}{n}\sum_j d_{ij}$, where d_{ij} is the shortest path between i and j; is the mean geodesic distance for vertex i.

  The mean geodesic distance of the network is defined as:
  $l = \frac{1}{n}\sum_i l_i$.
- For random graphs (Erdos-Reyni graphs): $l \propto log(n)$
- For scale free networks: $l \propto log(log(n))$

  (The definition of a scale-free network is below)

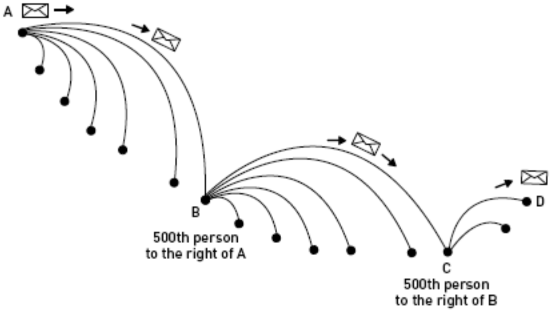
The Shortest Path and the Small-World Effect

Table : (Table 18.1) Basic stats for some selected networks. Where l=mean geodesic distance connected vertex pairs.

Network	Type	n	m	c	l
Metabolic	Undirected	765	3,686	9.64	2.56
Bio. co-auth.	Undirected	1,520,251	11,803,064	15.53	4.92
Phy. co-auth.	Undirected	52, 909	245,300	9.27	6.19
Protein int.	Undirected	2,115	2,240	2.12	6.80
Student dating	Undirected	573	477	1.66	16.01

Funneling and the Sociometric Superstars

- (Suggested by Milgram) **Funneling**: The idea that a given vertex has one or two connecting vertices that are well connected (i.e. have a high degree).



Degree Distributions

- The **distribution of degrees** of a given network is a *KEY* fundamental property.
- Consider a simple undirected network with vertices, $\{1, 2, \ldots, n\}$.
- If we pick a vertex $i \in \{1, 2, \ldots, n\}$ at random, can we guess what it's degree k_i will be ?

  No: But we know that it can be any counting number between 0 and $n - 1$.

  Also depending on what natural phenomena the network is describing, we can say something sensible (on average).

Ways to Characterize A Degree Distribution

Table : Degree Distribution in table form

degree:	0	1	2	...	n-1
probability:	p_0	p_1	p_2	...	p_{n-1}

Or

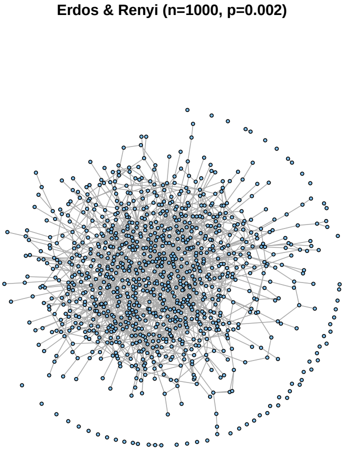
- Mathematical Function (aka. probability mass function, pmf):

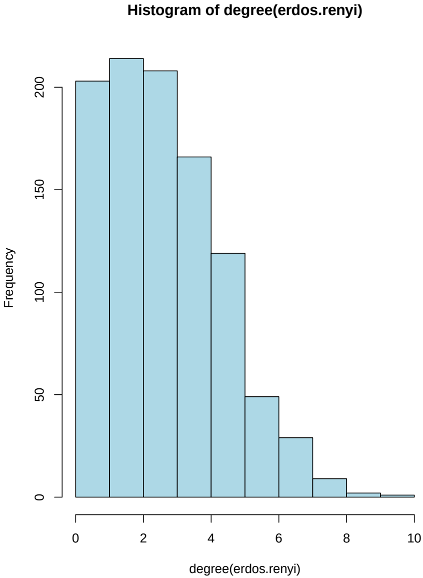
  $$p(k), \quad k = 0, 1, 2, \ldots, n-1.$$

When $p(k)$ follows the Power Law we say that we have a **scale-network**.

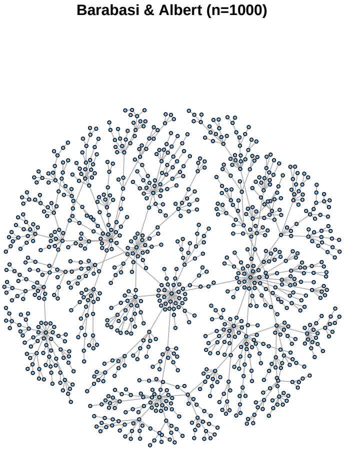
When a network has random connections (with a fixed probability of connection) then we have the **Erdos-Renyi network**. In this case $p(k)$ will follow a Binomial Law (usually approximated by a Poisson Law).

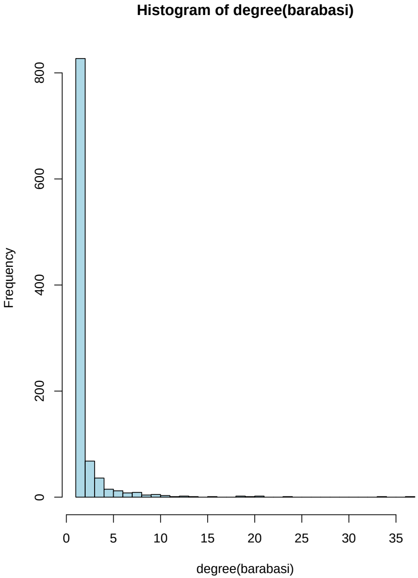
Example: Erdos-Renyi (random)





Example: Barabasi and Albert (scale free)





Directed Networks

For directed networks we can talk about the in-degree distribution and the out-degree distribution, separately.

Ideally we would like to know the joint distribution instead.
This is an on-going research topic.

Next time we will look more closely at the Power Law and some of its consequences.