# The Simple Genetic Algorithm Performance: A Comparative Study on the Operators Combination

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***Abstract*—This paper presents a comparative and experimental study about the performance of the Simple Genetic Algorithm (SGA) using five classic benchmarking functions. The performance analysis is accomplished on the combination of the operators of reproduction and crossover with the control parameters having been fixed. The overall behavior of the SGA is evaluated by the fitness of the best individual analyzed during the evolution and at the ending of the same one. The results that are presented show that the SGA can be effective and competitive to optimization on a test suite of benchmark functions.**

***Keywords*-Genetic Algorithm; Parameterization of GA; Generational Replacement Model; Single-Point Crossover; Uniform Crossover.**

## I. Introduction

The Simple Genetic Algorithm (SGA) presented by Goldberg [1] plays an important role in the use of the approaches based on the dynamics of natural genetics and still being a study target [2]. This proposal has been used in the implementation of many derived approaches, and many researchers have drawn the performance analysis of the Genetic Algorithms (GAs) basing its studies in the control parameters (populations size, crossover and mutation rates) and their potential adjustment [3]. Normally, this parameterization depends on the knowledge of the designer about the problem definition; of the values attributed to the parameters and of the adequate choice of the used methods to implement the operators. In this universe of choices, each designer can create a particular algorithm to a specific problem [4]–[7], being always a generic SGA as the worse one of the implementations.

In this paper, the SGA performance is examined into five benchmarking functions, considering a fixed size of the population and constant crossover and mutation rates. The performance test is carried out through for the combination of three strategies of reproduction and two kinds of crossover. No additional strategy was established, and the benchmarking functions were normalized to facilitate the comparisons.

The present paper is structured as follows: Section II presents the benchmarking functions and Section III describes the methodology used. In Section IV, results of experiments are reported. In Section V, some general conclusions are mentioned.

## II. Benchmarking Functions

Many benchmarking functions have been used to perform a stress test of various GA approaches. Digalakis [3] summarizes this set of benchmarking functions, which comes the set of characteristics required for benchmarking tests using GAs. In this set, five functions had been selected to perform the proposed study, which are listed below:

1) F1 function (Sphere): paraboloid function, smooth, unimodal, convex, symmetric, and whose convergence to the global optimum is easily achieved.

$$f_1(x) = \sum_{i=1}^{2} x_i^2 \tag{1}$$
$$-5.12 \leq x_i \leq 5.12$$

2) F2 function (Rosenbrock): considered of high difficulty level resembling a saddle function, imposing strong restrictions on the algorithms that are not suitable to search for directions.

$$f_2(x) = 100\left(x_1^2 - x_2\right)^2 + (1 - x_1)^2 \tag{2}$$
$$-2.048 \leq x_i \leq 2.048$$

3) F3 function (Step): function at representative levels of flat surfaces, which are obstacles for optimization algorithms, whereas the surfaces in the levels do not provide any information about which direction is favorable to the search.

$$f_3(x) = \sum_{i=1}^{5} integer(x_i) \tag{3}$$
$$-5.12 \leq x_i \leq 5.12$$

4) F4 function (Rastrigin's function): this function represents a surface performance of extreme complexity in the

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search for global optimal solution, given the existence of numerous local solutions.

$$f_4\left(x\right) = 10 \cdot n + \sum_{i=1}^{2} \left(x_i^2 - 10 \cdot \cos\left(2\pi \cdot x_i\right)\right) \qquad (4)$$
$$-5.12 \leq x_i \leq 5.12$$
$$n = dim(x_i)$$

where $n$ represents the numerical value of $x_i$ dimension.

5) F5 function (Foxholes): the main feature of this function is to produce local solutions in an independent environment with a high level of discontinuity.

$$f_5\left(x\right) = \left(0.002 + \sum_{j=1}^{25} \left(j + \sum_{i=1}^{2} \left(x_i - a_{ij}\right)^6\right)^{-1}\right)^{-1} \qquad (5)$$
$$-65.536 \leq x_i \leq 65.536$$
$$\left(a_{ij}\right)_{2\times25} = A$$

In this function, the matrix $A_{2\times25}$ is formed by constants and, in order to simplify the exibition, their values are grouped as follow:

$$C_0 = \begin{bmatrix}-32 & 16 & 0 & 16 & 32\end{bmatrix}$$
$$C_1 = \begin{bmatrix}-32 & -32 & -32 & -32 & -32\end{bmatrix}$$
$$C_2 = \begin{bmatrix}-16 & -16 & -16 & -16 & -16\end{bmatrix}$$
$$C_3 = \begin{bmatrix}0 & 0 & 0 & 0 & 0\end{bmatrix}$$
$$C_4 = \begin{bmatrix}32 & 32 & 32 & 32 & 32\end{bmatrix}$$
$$C_5 = \begin{bmatrix}16 & 16 & 16 & 16 & 16\end{bmatrix}$$

The groups C0-C5 can now be associated to $A$:

$$A = \begin{bmatrix}C_0 & C_0 & C_0 & C_0 & C_0 \\ C_1 & C_2 & C_3 & C_4 & C_5\end{bmatrix}$$

This subset contains important characteristics of many objective functions found in optimization problems, such as: smoothness, unimodality, multimodality, a very narrowness ridge, a flat surface, and too many local optima [3].

## III. Methodology

The objective of this paper is to present an analysis of the SGA performance, in its classic form [1], and to evaluate such performance using a combination of basic methods for reproduction and crossover. The populations size, crossover and mutation probability are used with constant values associated, to avoid the effect of these parameters in the overall analysis.

The stop condition of the evolution was established as a finite number of generations. The population replacement scheme adopted was the Generational Replacement Model (GRM), which replaces the entire population, in each generation, by its offspring. To guarantee the maintenance of the best solution gotten in each previous generation, the Elitist strategy is applied in each next generation [8].

The performance measure may be taken in two moments of the evolution: the first one can be since the initial steps, and is called ongoing analysis, and the other one at the evolution's end, called stopped analysis. The ongoing analysis gives an idea of the evolution until the present generation, and the stopped analysis supplies the best solution found until then. These criteria have been detailed in [1], [3]. In this work, the ongoing analysis was established having the measure being obtained at the 10$^{\text{th}}$ generation, and the stopped analysis at the 80$^{\text{th}}$ generation.

The selected methods of reproduction are:

- $R_1$ - Stochastic sampling with replacement (Roulette wheel selection);
- $R_2$ - Remainder stochastic sampling with replacement;
- $R_3$ - Stochastic tournament.

The selected crossover methods are:

- $X_1$ - Single-point crossover;
- $X_2$ - Uniform crossover.

The reproduction and crossover methods, above listed, were combined to assemble the set of tests. Such set was assigned as follows:

$C_{11}$ - Reproduction method $R_1$ with crossover $X_1$;
$C_{12}$ - Reproduction method $R_1$ with crossover $X_2$;
$C_{21}$ - Reproduction method $R_2$ with crossover $X_1$;
$C_{22}$ - Reproduction method $R_2$ with crossover $X_2$;
$C_{31}$ - Reproduction method $R_3$ with crossover $X_1$;
$C_{32}$ - Reproduction method $R_3$ with crossover $X_2$.

For each item in the set, 100 independent runs of the SGA were carried out. For each run, the best individual's evolution was obtained. For each combination, the mean values and variances were calculated. Figure 1, for example, illustrates the obtained results for F1 function optimization with the $C_{11}$ combination.

## IV. Results

To evaluate the SGA performance and to compare it with the results found in literature, the following values were used:

- population size: 50 individuals;
- number of generations: 80;
- number of runs: 100;
- chromosome or bit string length: 8 bits per solution;
- crossover mechanisms: single-point and uniform;
- crossover probability ($p_c$): 0.6;
- mutation probability ($p_m$): 0.001.

Considering the benchmarking function F1, after the entire tests, the obtained results to the combinations set are depicted in Figure 2. Table I summarizes the numerical values. The criteria adopted to measure the performance of best individuals were made by measuring the mean value

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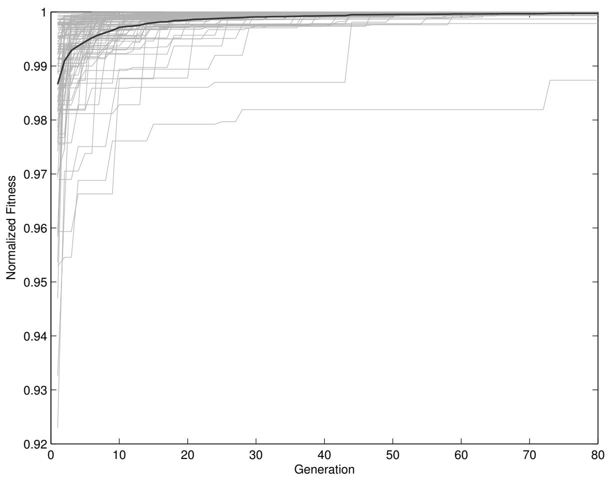


Figure 1. 100 Independent runs and the mean value to F1 function with combination $C_{11}$.

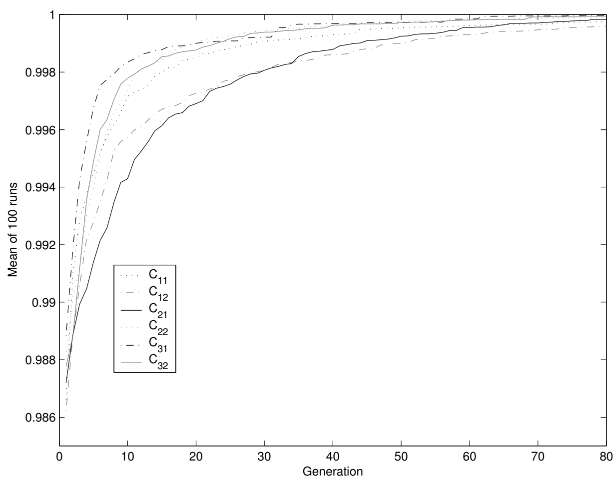


Figure 2. Comparative of the mean of the normalized fitness, to F1 function, in 100 runs.

Table I
MEAN VALUES AND VARIANCES TO F1 FUNCTION.

| | $10^{th}$ generation | | $80^{th}$ generation | |
|---|---|---|---|---|
| | $\mu$ | $\sigma^2 \times 10^{-5}$ | $\mu$ | $\sigma^2 \times 10^{-5}$ |
| $C_{11}$ | 0.9972 | 1.5339 | 0.9997 | 0.1644 |
| $C_{12}$ | 0.9957 | 2.3725 | 0.9996 | 0.0454 |
| $C_{21}$ | 0.9943 | 3.9221 | 0.9998 | 0.0167 |
| $C_{22}$ | 0.9974 | 1.7816 | 0.9999 | 0.0249 |
| $C_{31}$ | 0.9983 | 1.3449 | 1.0000 | 0.000119 |
| $C_{32}$ | 0.9978 | 0.8595 | 0.9999 | 0.005012 |

and variance over the runs, becoming a criteria for numerical comparisons.

The values in Table I show excellent results to any combination for ongoing and stopped analysis. The low values of variance demonstrate that any run can be effective in the search of the optimal solution.

The obtained results to the combinations for functions F2-F5 that are depicted in Figures 3-6 and Tables II-V show the summarization of the results respectively by the measure of mean and variance, respectively.

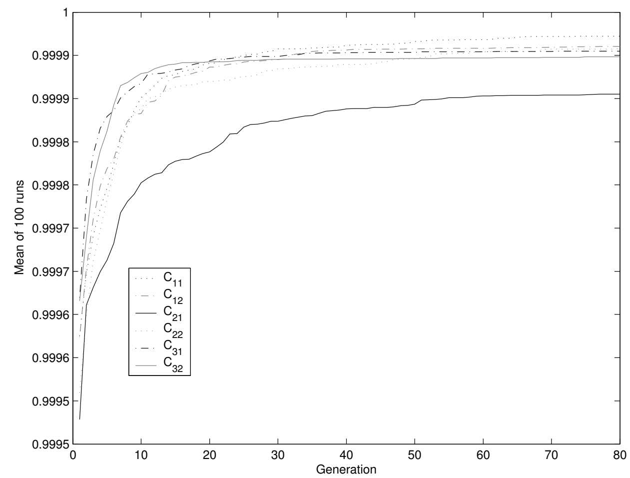


Figure 3. Comparative of the mean of the normalized fitness, to F2 function, in 100 runs.

Table II
MEAN VALUES AND VARIANCES TO F2 FUNCTION.

| | $10^{th}$ generation | | $80^{th}$ generation | |
|---|---|---|---|---|
| | $\mu$ | $\sigma^2 \times 10^{-8}$ | $\mu$ | $\sigma^2 \times 10^{-8}$ |
| $C_{11}$ | 0.9999 | 1.7021 | 1.0000 | 0.2462 |
| $C_{12}$ | 0.9999 | 3.5903 | 1.0000 | 0.5344 |
| $C_{21}$ | 0.9998 | 6.5557 | 0.9999 | 2.3654 |
| $C_{22}$ | 0.9999 | 2.5488 | 1.0000 | 0.9630 |
| $C_{31}$ | 0.9999 | 1.6755 | 1.0000 | 0.5718 |
| $C_{32}$ | 0.9999 | 1.2254 | 0,9999 | 0.7725 |

Table III
MEAN VALUES AND VARIANCE TO F3 FUNCTION.

| | $10^{th}$ generation | | $80^{th}$ generation | |
|---|---|---|---|---|
| | $\mu$ | $\sigma^2 \times 10^{-4}$ | $\mu$ | $\sigma^2 \times 10^{-4}$ |
| $C_{11}$ | 0.9978 | 2.3391 | 1.0000 | 0.0000 |
| $C_{12}$ | 0.9989 | 1.1815 | 1.0000 | 0.0000 |
| $C_{21}$ | 0.9967 | 3.4728 | 1.0000 | 0.0000 |
| $C_{22}$ | 1.0000 | 0.0000 | 1.0000 | 0.0000 |
| $C_{31}$ | 1.0000 | 0.0000 | 1.0000 | 0.0000 |
| $C_{32}$ | 1.0000 | 0.0000 | 1.0000 | 0.0000 |

In tables IV and V,the average decrease between rounds over the previous tables due to the nature of performance

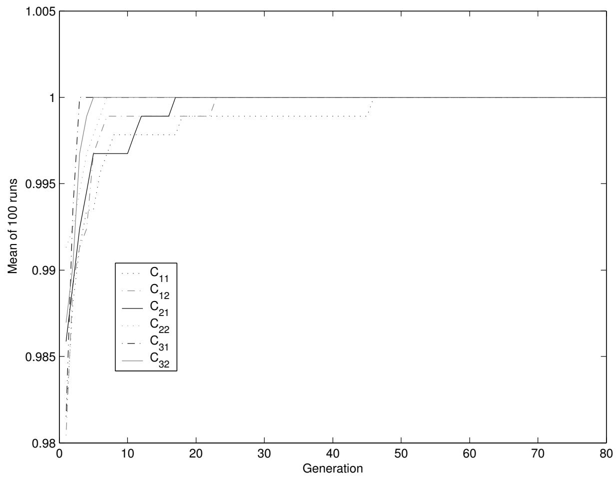


Figure 4. Comparative of the mean of the normalized fitness, to F3 function, in 100 runs.

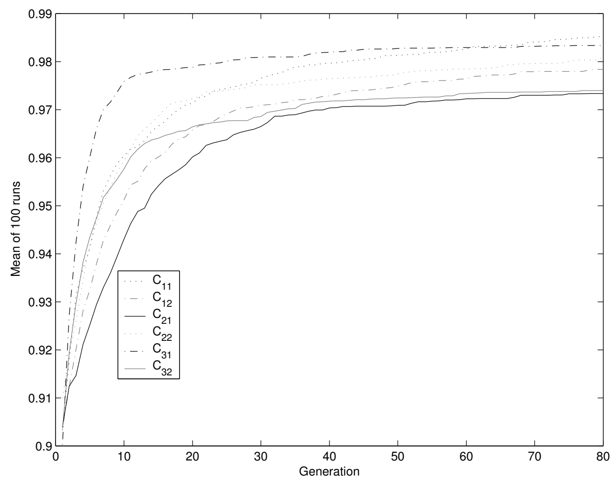


Figure 5. Comparative of the mean of the normalized fitness, to F4 function, in 100 runs.

Table IV
MEAN VALUES AND VARIANCES TO F4 FUNCTION.

| | $10^{th}$ generation | | $80^{th}$ generation | |
|---|---|---|---|---|
| | $\mu$ | $\sigma^2 \times 10^{-4}$ | $\mu$ | $\sigma^2 \times 10^{-4}$ |
| $C_{11}$ | 0.9604 | 7.1088 | 0.9853 | 2.2892 |
| $C_{12}$ | 0.9512 | 9.1016 | 0.9784 | 2.9781 |
| $C_{21}$ | 0.9431 | 13.123 | 0.9733 | 6.0680 |
| $C_{22}$ | 0.9594 | 6.5950 | 0.9809 | 2.5381 |
| $C_{31}$ | 0.9759 | 2.3428 | 0.9833 | 1.4531 |
| $C_{32}$ | 0.9576 | 5.6891 | 0.9743 | 4.5210 |

among the results is verified. To elect a winning combination we decided to analyze the one variance with the lowest since the average of the same order of magnitude. For the two moments of evolution, the combination $C_{31}$ is the one with the best performance among the results. The lower values obtained for the variance show that any round can be effective in finding the optimal solution. After analysis of these data, it appears that the solutions for all combinations of the set of operations can be considered as optimal solutions.

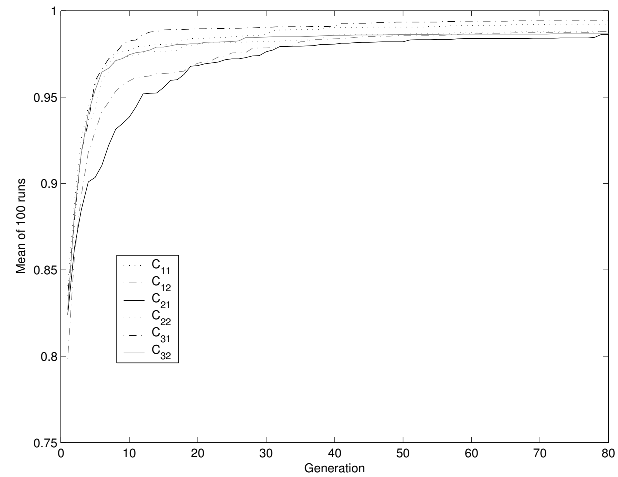


Figure 6. Comparative of the mean of the normalized fitness, to F5 function, in 100 runs.

Table V
MEAN VALUES AND VARIANCES TO F5 FUNCTION.

| | $10^{th}$ generation | | $80^{th}$ generation | |
|---|---|---|---|---|
| | $\mu$ | $\sigma^2 \times 10^{-3}$ | $\mu$ | $\sigma^2 \times 10^{-4}$ |
| $C_{11}$ | 0.9771 | 1.63 | 0.9923 | 0.8348 |
| $C_{12}$ | 0.9593 | 5.14 | 0.9882 | 1.4573 |
| $C_{21}$ | 0.9384 | 16.53 | 0.9863 | 1.6812 |
| $C_{22}$ | 0.9736 | 2.87 | 0.9884 | 1.3815 |
| $C_{31}$ | 0.9827 | 1,77 | 0.9941 | 0.5224 |
| $C_{32}$ | 0.9745 | 1,13 | 0.9866 | 1.2796 |

After examining these results, the good results gotten in all the considered set's combinations are verified. To compose a generic sketch with all benchmarking functions and the entire combinations set, these results were combined. The Figures 7 and 8 show the similarity in performance to the test functions and the combinations.

These results show that the benchmarking functions F1, F2 and F3 impose the same behavior to all operators combinations and, consequently, reliable results are obtained. The functions F4 and F5, due to their nature, impose a different behavior to the operators combinations. In this context, the $C_{31}$ combination preserves its good performance relatively to others combinations, and the differences in the values, verified according the variances values, are not significant.

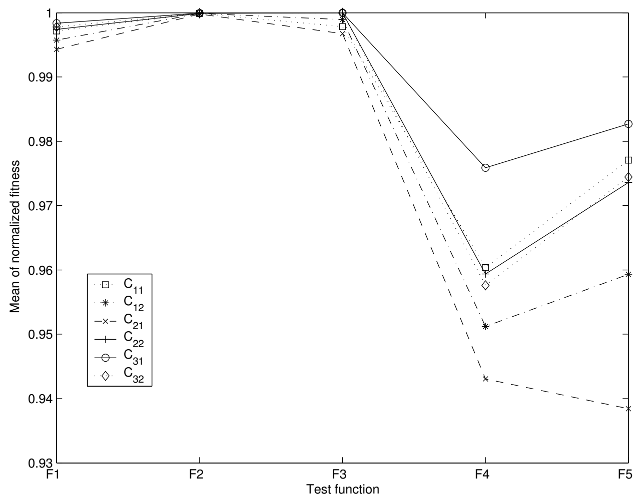


Figure 7. Performance analysis at 10th generation.

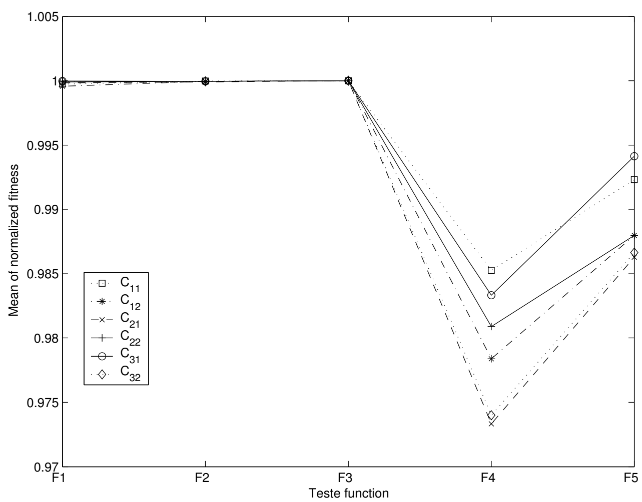


Figure 8. Performance analysis at 80th generation.

## V. Conclusion

This paper has accomplished a performance analysis for the SGA approach - a simple genetic algorithm. This analysis, using a subset of benchmarking functions and doing combinations of operators, show the effectiveness of the SGA algorithm. In other words, given a search space, a convergence region is provided; local optima, which are widespread in some objective functions, are overcomed; and a useful set of feasible solutions is reached. These experiments show that the combination of stochastic tournament with single-point crossover is the combination that provides better results. The results described in this paper are significant because they show that the basic formulation of SGA is competitive in the different contexts found in the objective functions.

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