hep-th/0002145

DeSitter Entropy, Quantum Entanglement and AdS/CFT

Stephen Hawking*, Juan Maldacena† and Andrew Strominger†

*Department of Applied Mathematics and Theoretical Physics,
Centre for Mathematical Sciences
Wilberforce Road, Cambridge CB3 OWA, UK

†Department of Physics
Harvard University
Cambridge, MA 02138, USA

Abstract

A deSitter brane-world bounding regions of anti-deSitter space has a macroscopic entropy given by one-quarter the area of the observer horizon. A proposed variant of the AdS/CFT correspondence gives a dual description of this cosmology as conformal field theory coupled to gravity in deSitter space. In the case of two-dimensional deSitter space this provides a microscopic derivation of the entropy, including the one-quarter, as quantum entanglement of the conformal field theory across the horizon.

Contents

1. Introduction

Despite advances in the understanding of black hole entropy, a satisfactory microscopic derivation of the entropy of deSitter space [1] remains to be found. In this paper we address this issue in the context of a deSitter space arising as a brane-world of the type discussed by Randall and Sundrum [2] [1] bounding two regions of anti-deSitter space. It is natural to suppose that such theories are dual, in the spirit of AdS/CFT [3], to a conformal theory on the brane-world coupled to gravity with a cutoff. The cutoff scales with the deSitter radius in such a way that the usual AdS/CFT correspondence is recovered when the cutoff is taken to infinity. This duality provides an alternate description of the deSitter cosmology which can be used, in the case of two dimensions, for a microscopic derivation of the deSitter entropy. We find that the entropy can be ascribed to the quantum entanglement of the CFT vacuum across the deSitter horizon. Quantum entanglement entropy can also be viewed as the entropy of the thermal Rindler particles near the horizon, thereby avoiding reference to the unobservable region behind the horizon.

Our derivation is closely related to the observation of reference [4] that in two dimensions black hole entropy can be ascribed to quantum entanglement if Newton's constant is wholly induced by quantum fluctuations of ordinary matter fields (see also [5,6,7,8]). In the context of [4] this seemed to be a rather artificial and unmotivated assumption. However the AdS brane-world scenarios do appear to have this feature. The basic reason is that, in a semiclassical expansion, the Einstein action on the brane arises mainly from bulk degrees of freedom[2] which correspond, in the dual picture, to ordinary matter fields

[1] Unlike [2] we include a nonzero cosmological constant on the brane.

[2] This follows when the AdS radius is large compared to the Planck length.

on the brane. The semiclassical expansion in the bulk corresponds to a large N expansion in the brane, in which the leading term in Newton's constant is induced by matter fields.

Our derivation of two-dimensional deSitter entropy is similar to the derivation of black hole entropy in [9] in that it uses a brane field theory dual to the spacetime gravity theory to compute the entropy. However it differs in that in [9] the black hole entropy was given by the logarithm of the number of unobserved microstates of the black hole, whereas here the deSitter entropy arises from entanglement with the unobserved states behind the horizon.[3] Alternatively, it can be viewed as the number of microstates of the thermal gas of Rindler particles near the horizon. This latter viewpoint is closer to that of [9]. This issue is explored in the final section by throwing a black hole in the bulk of AdS at the brane. When the bulk black hole reaches the brane, the brane state collapses to a brane black hole. At all stages the entropy is accounted for by a thermal gas on the brane.

Formally, the derivation can be generalized to higher-dimensional deSitter spaces which bound higher-dimensional anti-deSitter spaces. It was conjectured by Susskind and Uglum [6] that there is a general a precise relationship between entanglement entropy and the one loop correction to Newton's constant. Based on this, Jacobson [5] argued that black hole entropy can be ascribed to quantum entanglement if Newton's constant is wholly induced. However, while we are sympathetic to the conjecture of [6], and it fits well with the discussion herein, its status remains unclear [10,11]. The basic problem is that in greater than two dimensions the corrections have power law divergences and hence are regulator dependent. This makes precise statements difficult above two dimensions.

A further significant fly in the ointment - even in two dimensions- is that there is no known example of the type of brane-world scenario considered in [2] embedded in a fully consistent manner into string theory [4]. The observations of the present paper are relevant only if such examples exist. For the time being however they provide intriguing connections along the circle of ideas pursued in [1-9].

2. Classical Geometry

[3] In general the entanglement entropy is less than the logarithm of the number of possible microstates of the unobserved sector of the Hilbert space. If the total system is in a pure state, the entanglement entropy is bounded from above by the logarithm of the number of possibly entangled states in the unobserved sector of the Hilbert space.

[4] See however [12] for related scenarios in string theory.

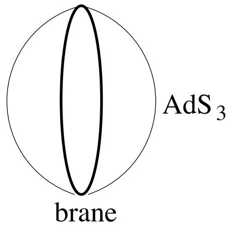

Fig. 1: Euclidean instanton geometry. The brane is an S^2 which bounds two patches of euclidean $AdS_3 = H^3$.

The euclidean action for a onebrane coupled to gravity with a negative cosmological constant ($\Lambda = -\frac{1}{L^2}$) is

$$S_{tot} = -\frac{M_p}{16\pi}\int d^3x\sqrt{g}(R+\frac{2}{L^2}) + T\int d^2\sigma\sqrt{h}. \tag{2.1}$$

M_p here is the three dimensional Planck mass, T is the brane tension and h is the induced metric on the brane. We have assumed that there is no boundary. We wish to consider a spherically symmetric brane at radius r_B which bounds two regions of AdS_3 with metrics

$$ds_3^2 = L^2dr^2 + L^2\sinh^2 r d\Omega_2^2, \tag{2.2}$$

where $0 \leq r \leq r_B$, as shown in fig. 1. The two copies of AdS_3 are glued together along the S^2 at $r = r_B$ where the bulk curvature has a delta function. The topology of the spacetime is S^3. The induced metric on the brane is

$$ds_2^2 = \ell^2 d\Omega_2^2, \tag{2.3}$$

with

$$\ell \equiv L\sinh r_B. \tag{2.4}$$

The action for such a configuration is

$$S_{tot} = \frac{M_p}{4\pi L^2}V_3 - \frac{M_p\coth r_B}{2\pi L}V_2 + TV_2, \tag{2.5}$$

where V_3 is the bulk volume and V_2 is the brane volume. The second term arises from a delta function in the bulk curvature at $r = r_B$. One finds using (2.2) that

$$S_{tot} = -\frac{LM_p}{2}(\sinh 2r_B + 2r_B) + 4\pi TL^2\sinh^2 r_B. \tag{2.6}$$

The action (2.6) has an extremum at

$$\tanh r_B = \frac{M_p}{4\pi T L} \tag{2.7}$$

for which

$$S_{tot} = -L M_p r_B. \tag{2.8}$$

We are interested in the case that the right hand side of (2.7) is close to (but less than) one so that r_B and ℓ are large. We may then approximate

$$S_{tot} = -L M_p \ln \ell + \dots \tag{2.9}$$

The subleading corrections are suppressed for large ℓ.

The induced brane metric (2.3) is the two-dimensional euclidean deSitter (*i.e.* round S^2) metric with a large deSitter radius ℓ. A lorentzian deSitter solution can be obtained by analytic continuation of the periodic angle ϕ on S^2 to it. One finds

$$\begin{aligned} ds_3^2 &= L^2 dr^2 + L^2 \sinh^2 r (d\theta^2 - sin^2\theta dt^2), \\ ds_2^2 &= \ell^2 d\theta^2 - \ell^2 sin^2\theta dt^2. \end{aligned} \tag{2.10}$$

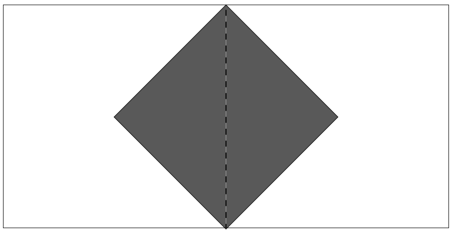

Fig. 2: Penrose diagram of Lorentzian two dimensional de-Sitter space. The dotted line indicates the trajectory of a geodesic observer. We have also indicated the past and future horizons for that observer, and the shaded region indicates the patch covered by the coordinates (2.10).

These coordinates cover the diamond-shaped region of DS_2 illustrated in fig. 2. It is the region outside both the future and past horizons of any timelike observer at constant $\theta \neq 0, \pi$.

3. Dual Representations

Let us assume there is a unitary quantum theory whose semiclassical gravitational dynamics is described by (2.1). Such a theory should have two dual descriptions.[5] The first is, as described above, a three-dimensional bulk theory containing gravity and a brane.

The second description is as a two-dimensional effective theory of the light fields on the brane worldvolume. These light fields include holographic matter living on the brane. To see this we first consider a single copy of AdS_3 in coordinates (2.2) with a fixed boundary at $r = r_B$, for large r_B. If we keep the metric on the boundary fixed, but integrate over the bulk metric, the resulting theory has a holographic description as a $1+1$ conformal field theory on a sphere of radius ℓ with central charge $c = \frac{3LM_p}{2}$ [15] and a cutoff at L[3,16,17].[6] To recover the geometry under consideration, we must take two copies of such bounded AdS_3 spacetimes, identify them along their boundaries, and then integrate over boundary metrics. Because there are two copies, one has two copies of the matter action on the boundary, with a total central charge

$$c = 3LM_p. \tag{3.1}$$

The second brane tension term in the action (2.1) corresponds to a counterterm which renormalizes the cosmological constant.

We are not fixing the boundary by hand so we have two-dimensional gravity because, as shown in [2], there is a graviton zero mode trapped on the brane, as well as the radion field representing the radial position of the brane. The radion has a small mass in the case of a large deSitter boundary. For a D-dimensional boundary brane, gravity plus the radion comprise $\frac{1}{2}(D^2 - 3D + 2)$ local degrees of freedom (after implementing the constraints). Hence for the present case of $D = 2$ the radion-gravity system has no local degrees of freedom.

For our purposes we need only the gravity part of the effective action, with the radion field set at the minimum of its potential. The gravity effective action is most easily represented in conformal gauge

$$ds_2^2 = e^{2\rho} d\hat{s}_2^2, \tag{3.2}$$

[5] Related discussions appear in [13,14].

[6] Alternately one may take a sphere of unit radius and a cutoff at L/ℓ.

where $d\hat{s}^2$ is the unit metric on S^2 obeying

$$\hat{R}_{z\bar{z}} = \hat{g}_{z\bar{z}}, \qquad \int d^2 z \hat{g}_{z\bar{z}} = 4\pi \tag{3.3}$$

in complex coordinates. One then finds[7]

$$S_g = -\frac{LM_p}{4\pi} \int d^2 z \big(\partial_z \rho \partial_{\bar{z}} \rho + \hat{R}_{z\bar{z}} \rho - \frac{1}{2\ell^2} \hat{g}_{z\bar{z}} e^{2\rho}\big). \tag{3.4}$$

The equations of motion for constant fields give

$$\rho = \ln \ell. \tag{3.5}$$

The action (3.4) evaluated at this solution agrees with (2.9) We note also that the total gravity plus matter central charge vanishes, as required for general covariance. These considerations determine (3.4).

4. DeSitter Entropy

In this section we give macroscopic and microscopic derivations of the entropy.

4.1. Semiclassical Macroscopic Entropy

The macroscopic entropy can be computed directly in three dimensions from the area entropy-law

$$S_{dS} = \frac{Area}{4G_3}. \tag{4.1}$$

In this expression $G_3 = 1/M_p$ and the horizon area is the area of the fixed point of a $U(1)$ isometry [20] of the instanton geometry (2.2). This consists of a geodesic circle intersecting the S^2 brane at the north and south poles. The area (length) of this circle is $4Lr_B \sim 4L \ln \ell$. Hence we obtain

$$S_{dS} = LM_p \ln \ell. \tag{4.2}$$

An alternate derivation can be given from the two dimensional deSitter space using

$$S_{dS} = \frac{Area}{4G_2}. \tag{4.3}$$

The area in this formula is just the area of the observer horizon ($\theta = 0, \pi$ in (2.10)) which consists of two points and is therefore equal to 2. G_2 is determined as the (field-dependent) coefficient of the the scalar curvature $R = \frac{1}{2} g^{z\bar{z}} R_{z\bar{z}}$. From (3.4) this is

$$\frac{1}{G_2} = 2LM_p \rho = 2LM_p \ln \ell. \tag{4.4}$$

Inserting (4.4) into (4.3) reproduces (4.2).

---

7 This is equivalent to the computation in [18,19].

4.2. Microscopic Entropy

Let us now consider the entropy from the point of view of the brane CFT with $c = 3M_pL$. An $SO(2,1)$ invariant vacuum for quantum field theory in lorentzian deSitter space $|0\rangle$ can be defined as the state annihilated by positive frequency modes in the metric

$$ds_2^2 = \ell^2 \frac{-dt^2 + dx^2}{cos^2 t}. \tag{4.5}$$

The proper time τ of an observer moving along a geodesic at $x = 0$ is related to the time t in (4.5) by time $e^{\tau/\ell} = \tan(\frac{t}{2} + \frac{\pi}{4})$. Green functions in this vacuum are single valued functions of t. Therefore they are periodic in imaginary τ with period $2\pi i\ell$, and the observer accordingly detects a thermal bath of particles with temperature $\frac{1}{2\pi\ell}$.

The vacuum $|0\rangle$ is a pure state of this CFT. However a single observer can probe features of this state only within the observer horizons, i.e. in the diamond region covered by the coordinates (2.10). The results of all such measurements are described by an observable density matrix $\rho_{\rm obs}$. $\rho_{\rm obs}$ is constructed from the pure density matrix $|0\rangle\langle 0|$ by tracing over the unobservable sector of the Hilbert space supported behind the horizon. The entropy

$$S_{ent} = -tr\rho_{\rm obs} \ln \rho_{\rm obs} \tag{4.6}$$

is nonzero because of correlations between the quantum states inside and outside of the horizon. S_{ent} is called the entanglement entropy because it measures the extent to which the observable and unobservable Hilbert spaces are entangled. Note that the entanglement entropy, defined this way, agrees with the entropy of the gas of particles at the local Rindler temperature. A general formula for S_{ent} was derived in [21,4]:

$$S_{ent} = \frac{c}{6}\rho|_{boundary} - \frac{c\Delta}{6}, \tag{4.7}$$

where Δ is the short distance cutoff and ρ is the conformal factor of the metric in the coordinates (in our case (4.5)) used to define the vacuum evaluated at the boundary (consisting of two points) of the unobserved region. From (4.5) we see that the boundary is at $t = 0$, so $\rho = \ln \ell$. Putting this all together and using $c = 3M_pL$ we get

$$S_{ent} = LM_p \ln \ell, \tag{4.8}$$

in agreement with (2.8).

This result is a generalization to the two-dimensional deSitter case of the observation of [4] that, in a two-dimensional theory in which the entirety of Newton's constant is induced from matter, the Bekenstein-Hawking black hole entropy can be microscopically derived as entanglement entropy. The missing ingredient in both of these previous discussions was a motivation for the assumption that Newton's constant is induced. Here we see it is natural - or at least equivalent to other assumptions - in the brane-world context.

5. Four Dimensions

We can also consider a four dimensional brane world model. We have a four dimensional brane bounding two AdS_5 regions. If we consider perturbations of the four dimensional metric we can analyze the system by first finding a five dimensional solution which has a given four dimensional metric at the brane. The solution will look like

$$ds^2 = L^2 \left(\frac{g_{\mu\nu}(z,x)dx^\mu dx^\nu + dz^2}{z^2} \right) + ... \qquad z \geq \epsilon \tag{5.1}$$

where $g_{\mu\nu}(z=\epsilon, x) = g_{4\ \mu\nu}$ is the four dimensional metric. We can then insert the solution with a given four dimensional metric back into the action and get an effective action for the four dimensional metric. This effective action will contain a leading term going like $1/\epsilon^4$ which will be canceled by the brane tension so that the next nontrivial term will be

$$S(g_4) = \frac{2}{16\pi G_5} \int_{z\geq\epsilon} d^5x \sqrt{g_5} R_5 = \frac{L^3}{16\pi G_5 \epsilon^2} \int d^4x \sqrt{g_4} R_4, \tag{5.2}$$

where to this order of approximation we can assume that the five dimensional metric is independent of z (the z dependent parts give terms going like lower powers of $1/\epsilon$) and we took into account the two copies of AdS_5. This of course the way that the four dimensional Newton's constant is computed in [2].[8] We have phrased the calculation in this way to make connection with AdS/CFT [3,16,17], so that the integral over five dimensional metrics with the boundary metric at $z = \epsilon$ held fixed can be interpreted as a field theory with a cutoff ϵ on that particular four dimensional space. So the physics of [2] is the same as the physics of a conformal field theory with a cutoff coupled to four dimensional gravity (as considered in [22]), where the four dimensional conformal field theory as an AdS dual, (see also [13,23]). Notice that the four dimensional Newton constant can be written as

$$\frac{1}{G_4} = \frac{8N_{dof}}{\pi\epsilon^2} , \qquad N_{dof} \equiv \frac{\pi L^3}{8G_5} \tag{5.3}$$

where N_{dof} is the quantity that appears in all AdS/CFT calculations involving the stress tensor, calculations such as the two point function of the stress tensor or the free energy at finite temperature, etc. It can be viewed as the effective number of degrees of freedom of the CFT, ($N_{dof} = N^2/4$ for $\mathcal{N} = 4$ SYM). This form for the four dimensional Newton constant is very suggestive. It is of the general form expected for induced gravity in four

[8] In [2] ϵ does not appear since the four dimensional metric is rescaled by $\frac{L^2}{\epsilon^2}$.

dimensions. If we start in four dimensions with a theory with infinite or very large Newton constant and we integrate out the matter fields we expect to get a four dimensional value for the Newton constant which is rougly as in (5.3)[24,25,26,6,27,10,28,29]. The precise value that we would get seems to depend on the cutoff procedure. Indeed, if we use heat kernel regularization we would get that for $N = 4$ Yang Mills this cuadratic divergence cancels. The gravity procedure of fixing the boundary at some finite distance must correspond to a suitable cutoff for the field theory and it is not obvious that we should get the same results for divergent terms. Indeed, the supergravity regularization procedure would also give a divergent value for the vacuum energy (which is being cancelled by the brane). Again in theories where the four dimensional Newton constant is induced one can interpret black hole entropy as entanglement entropy [5]. If we consider a four dimensional metric with a horizon, like a black hole or de-Sitter space we indeed find that the entropy is given by

$$S = \frac{A_4}{4G_4} = \frac{2N_{dof}A_4}{\epsilon^2 \pi} \tag{5.4}$$

where we just used the relation of the 4d Newton constant and the four dimensional parameters. The right hand side can be interpreted as entanglement entropy. In other words, we can compute the entanglement entropy in the field theory as entropy of the gas of particles in thermal Rindler space and we would obtain precisely the right hand side of (5.4). We can do the entropy calculation at weak coupling in weakly coupled $\mathcal{N} = 4$ SYM and we would obtain agreement up to a numerical factor, which could be be related to the ignorance of the cutoff procedure, but more fundamentally can also be related to strong coupling effects like the 3/4 appearing in the relation between the weakly coupled and the strongly coupled expressions for the free energy.

6. Black Hole Formation on the Brane

The entropy of a bulk black hole in the interior of AdS can be accounted for by representing it as a thermal state in the brane theory on its boundary. At first this may seem to be at odds with the accounting given here of the entropy of a black hole on the brane in terms of quantum entanglement. In this section we will attempt to reconcile the accounts by throwing a bulk black hole at the brane and watching it turn into a brane black hole.[9]

[9] Similar ideas are being pursued by H. Verlinde.

Consider a bulk black hole at the origin of AdS at temperature T_H whose size is large compared to the AdS radius L but small compared to the brane radius ℓ. This has a stable ground state in which there is a cloud of thermal radiation surrounding the black hole. In the brane theory, this is represented as homogeneous thermal state at temperature T_H. The statistical entropy of this state agrees with Bekenstein-Hawking entropy of the black hole.[10]

The center of mass of the black hole can be given momentum by the action of an AdS_{D+1} $SO(D,2)$ isometry. These isometries are broken by the presence of the brane, but if black hole is not too near the brane this should not matter. The $SO(D,2)$ action will impart momenta to the black hole and make it oscillate about the origin. The brane version of such a state can be found by applying an $SO(D,2)$ conformal transformation to the thermal brane state. The resulting state will carry conformal charges and have energy densities with bipolar oscillations. The statistical entropy of this oscillating state will of course still agree with Bekenstein-Hawking entropy of the oscillating black hole.

In the above discussion we implicitly assumed that the field theory was defined on the cylinder ($S^3 \times R$). When we think of the field theory defined in flat space or de-Sitter space we are looking only at some coordinate patch of the AdS cylinder. In this case we only see half a period of oscillation which can be interpreted as a gas of particles in the field theory that contracts and expands again.

If the oscillation is made large enough, the black hole actually reaches the brane where it will stick, at least temporarily. The brane picture of this process is that the oscillations in the energy density have become so large that the thermal radiation collapses to from a black hole.[11]

Before collapse, the entropy is accounted for on the brane as the entropy of thermal radiation. After collapse, it is accounted for by the thermal gas of Rindler particles near the horizon. (In general the black hole formation is not adiabatic.) This latter entropy is localized within a distance of the order of the cutoff from the horizon. This could be described by saying that all stages the entropy is stored in thermal radiation, and this radiation hovers outside the horizon when the black hole is formed. In this description

[10] The equality is precise for AdS_3. In higher dimensional cases such as AdS_5 it follows if one accepts the factor of $4/3$ as a feature of strongly coupled gauge theory, which we shall for the purposes of this discussion.

[11] Note that the coupling to gravity breaks conformal invariance so the conformal charges are not conserved when the collapse occurs.

the statistical origin of the entropy of bulk and brane black holes appears to be similar. Eventually the black hole will evaporate and the final state will be just outgoing thermal radiation on the brane theory.

It is interesting that there is a "correspondence principle" in the sense that when the AdS black hole has a radius of the order of the five dimensional anti-de-Sitter space and it makes a grazing collision with the brane, then the entropies calculated as a thermal gas and as entanglement entropy are the same up to a numerical constant. Such a black hole would have a Schwarschild radius in the boundary theory equal to the field theory cutoff ϵ.

In closing, it remains to find a fully consistent quantum realization of such a brane-world scenario to which our observations can be applied. Alternately, perhaps it is applicable in a more general setting. One of the important lessons of string duality is that something which is classical from one point of view can be quantum from another. What is needed here is a point of view from which Newton's constant - usually regarded as a largely classical quantity - is a purely quantum effect. It is notable in this regard that closed string poles arise as a loop effect in open string theory, indicating that there might be a way in which the full closed string dynamics is contained in open string field theory. In that case Newton's constant could be induced.

Acknowledgements

This work was supported in part by DOE grant DE-FG02-91ER40654. J.M. was also supported in part by the Sloan and Packard fellowships. We would like to thank T. Jacobson, A. Tseytlin and H. Verlinde for discussions.

References

[1] G. W. Gibbons and S. W. Hawking, *Action integrals and partition functions in quantum gravity*, Phys. Rev. **D15** (1977) 2752.

[2] L. Randall and R. Sundrum, *An alternative to compactification*, Phys. Rev. Lett. **83** (1999) 4690, hep-th/9906064.

[3] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231, hep-th/9711200.

[4] T. Fiola, J. Preskill, A. Strominger and S. P. Trivedi, *Black hole thermodynamics and information loss in two-dimensions*, Phys. Rev. **D50** (1994) 3987, hep-th/9403137.

[5] T. Jacobson, *Black hole entropy and induced gravity*, gr-qc/9404039.

[6] L. Susskind and J. Uglum, *Black hole entropy in canonical quantum gravity and superstring theory*, Phys. Rev. **D50** (1994) 2700, hep-th/9401070.

[7] V. Frolov, D. Fursaev and A. I. Zelnikov, *Statistical origin of black hole entropy in induced gravity*, Nucl. Phys. **B486** (1997) 339, hep-th/9607104.

[8] V. Frolov and D. Fursaev, *Mechanism of generation of black hole entropy in Sakharov's induced gravity*, Phys. Rev. **D56** (1997) 2212, hep-th/9703178.

[9] A. Strominger and C. Vafa, *Microscopic origin of the Beckenstein-Hawking entropy*, Phys. Lett. **B379** (1996) 99, hep-th/9601029.

[10] D. Kabat, *Black hole entropy and entropy of entanglement*, Nucl. Phys. **B453** (1995), hep-th/9503016.

[11] T. Jacobson, *On the nature of black hole entropy*, gr-qc/9908031.

[12] H. Verlinde, *Holography and compactification*, hep-th/9906182.

[13] S. S. Gubser, *AdS/CFT and gravity*, hep-th/9912001.

[14] J. DeBoer, E. Verlinde. and H. Verlinde, *On the holographic renormalization group*, hep-th/9912012.

[15] J. D. Brown and M. Henneaux, *Central charges in the canonical realization of asymptotic symmetries: An example from three dimensional gravity*, Commun. Math. Phys. (1986), 104.

[16] L. Susskind and E. Witten, *The holographic bound in Anti-de-Sitter space*, hep-th/9805114.

[17] A. Peet and J. Polchinski, *UV/IR relations in AdS dynamics*, Phys. Rev. **D59** (1999) 065011, hep-th/9809022.

[18] M. Henningson and K. Skenderis, **JHEP** 9807 (1998) 023, hep-th/9806087.

[19] V. Balasubramanian and P. Kraus, *A stress tensor for anti de Sitter gravity*, Commun. Math. Phys. (1999) 208, hep-th/9902121.

[20] S. Hawking and C. Hunter, *Gravitational entropy and global structure*, Phys. Rev. **D59** (1999)044025, hep-th/9808085.

[21] C. Holzhey, F. Larsen and F. Wilczek, *Geometric and renormalized entropy in conformal field theory*, Nucl. Phys. **B424** (1994) 443, hep-th/9403108.

[22] P. Frampton and C. Vafa, *Conformal approach to particle phenomenology*, hep-th/9903226.

[23] E. Witten, *New dimensions in field theory and string theory*, http://www.itp.ucsb.edu/online/susyc9

[24] L. Bombelli, R. Koul, J. Lee and R. Sorkin, *A quantum source of entropy for black holes*, Phys. Rev. **D34** (1986) 373.

[25] M. Srednicki, Phys. Rev. Lett. **71** (1993) 666, *Entropy and area*, hep-th/9303048.

[26] S. Carlip and C. Teitelboim, *The off shell black hole* gr-qc/9312002.

[27] C. Callan and F. Wilczek, *On geometric entropy*, Phys. Lett. **B333** (1994) 55; hep-th/9401072.

[28] J-G. Demers, R. LaFrance and R. Myers, *Black hole entropy and renormalization*, gr-qc/9507042.

[29] F. Larsen and F. Wilczek, *Renormalization of black hole entropy and of the gravitational coupling constant*, Nucl. Phys. **B58** (1996) 249, hep-th/9506066.