Sisukord

1 Hamiltoniaan

$$
\begin{aligned}
\hat{H} =& \sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}+\\
&+\sum_{\alpha}\sum_{\mathbf{k},\mathbf{k}'}V_{\alpha\alpha}(\mathbf{k},\mathbf{k}')a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha\mathbf{k}'\uparrow}+\\
&+\sum_{\alpha}\sum_{\mathbf{k},\mathbf{k}'}U_{\alpha\alpha}(\mathbf{k},\mathbf{k}')a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha\mathbf{k}'\uparrow}+\\
&+\sum_{\alpha}\sum_{\mathbf{k},\mathbf{k}'}V_{\alpha\alpha'}(\mathbf{k},\mathbf{k}')a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}a_{\alpha'-\mathbf{k}'\downarrow}a_{\alpha'\mathbf{k}'\uparrow}+\\
&+\sum_{\alpha}\sum_{\mathbf{k},\mathbf{k}'}U_{\alpha\alpha'}(\mathbf{k},\mathbf{k}')a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}a_{\alpha'-\mathbf{k}'\downarrow}a_{\alpha'\mathbf{k}'\uparrow}
\end{aligned}
\tag{1.1}
$$

Siin $a^{\dagger}$ ja a on ülijuhtivus elektroni tekke ja kadumise operaatorid, $\alpha = 1,2$ on tsooni indeks, $\mathbf{k}$ on elektroni lainevektor, $s =\uparrow,\downarrow$ on spinni indeks, $\tilde{\varepsilon}_{\alpha}(\mathbf{k}) = \varepsilon_{\alpha}(\mathbf{k}) - \mu$ on elektroni energia tsoonis α, μ on keemiline potenstiaal, $V_{\alpha\alpha'}(\mathbf{k},\mathbf{k}')$ on tsooniseesmise $(\alpha = \alpha')$ või tsoonidevahelise $(\alpha \neq \alpha')$ efektiivse tõmbeinderaktsiooni konstant ja $U_{\alpha\alpha'}(\mathbf{k},\mathbf{k}')$ on tsooniseesmise $(\alpha = \alpha')$ või tsoonidevahelise $(\alpha \neq \alpha')$ tõukeinteraktsiooni konstant. Efektiivne elektronidevaheline tõmbeinterkatsioon on indutseeritud elekton-foonon interaktsiooni poolt.

2 Keskmise välja lähendus

Kasutame nüüd keskmise välja lähendust, mille käigus defineerime ka ülijuhtivuspilud, mis kirjeldavad ülijuhtivat faasi ja faasisiiret. Selleks teeme järgmise asenduse

$$
a^{\dagger}a^{\dagger}aa \rightarrow \left\langle a^{\dagger}a^{\dagger}\right\rangle aa + a^{\dagger}a^{\dagger}\left\langle aa\right\rangle - \left\langle a^{\dagger}a^{\dagger}\right\rangle\left\langle aa\right\rangle \tag{2.1}
$$

$$
\begin{aligned}
\hat{H} = & \sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}V_{11}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}V_{22}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}U_{11}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}U_{22}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}V_{12}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}U_{12}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}V_{21}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}U_{21}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}
\end{aligned}
\tag{2.2}
$$

$$
\begin{aligned}
\hat{H}_{mf} = & \sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}+\\
& +\sum_{\mathbf{k},\mathbf{k}'}[V_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+V_{11}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle-\\
& -V_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+\\
& +U_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+U_{11}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle-\\
& -U_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+\\
& +V_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+V_{22}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle-\\
& -V_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+\\
& +U_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+U_{22}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle-\\
& -U_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+\\
& +V_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+V_{12}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle-\\
& -V_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+\\
& +U_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}+U_{12}(\mathbf{k},\mathbf{k}')a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle-\\
& -U_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+\\
& +V_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+V_{21}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle-\\
& -V_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+\\
& +U_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+U_{21}(\mathbf{k},\mathbf{k}')a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle-\\
& -U_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle]
\end{aligned}
\tag{2.3}
$$

Grupeerime liikmed

$$
\begin{aligned}
\hat{H}_{mf} = & \sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}+ \\
& +\sum_{\mathbf{k},\mathbf{k}'}\{[V_{11}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+U_{11}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+ \\
& +V_{12}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+U_{12}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle]\,a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}+ \\
& +[V_{22}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+U_{22}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+ \\
& +V_{21}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+U_{21}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle]\,a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}+ \\
& +[V_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle+U_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle+ \\
& +V_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle+U_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle]\,a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+ \\
& +[V_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle+U_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle+ \\
& +V_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle+U_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle]\,a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}- \\
& -[V_{11}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+U_{11}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+ \\
& +V_{12}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+U_{12}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle]\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle- \\
& -[V_{22}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+U_{22}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+ \\
& +V_{21}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+U_{21}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle]\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle\}
\end{aligned}
\tag{2.4}
$$

Defineerime ülijuhtivuspilud

$$
\begin{aligned}
\Delta_{1\mathbf{k}} = & \sum_{\mathbf{k}'}[V_{11}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+U_{11}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+ \\
& +V_{12}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+U_{12}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle] \\
\Delta_{2\mathbf{k}} = & \sum_{\mathbf{k}'}[V_{22}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+U_{22}(\mathbf{k},\mathbf{k}')\left\langle a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}\right\rangle+ \\
& +V_{21}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle+U_{21}(\mathbf{k},\mathbf{k}')\left\langle a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}\right\rangle] \\
\Delta^{*}_{1\mathbf{k}'} = & \sum_{\mathbf{k}}[V_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle+U_{11}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle+ \\
& +V_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle+U_{21}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle] \\
\Delta^{*}_{2\mathbf{k}'} = & \sum_{\mathbf{k}}[V_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle+U_{22}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle+ \\
& +V_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle+U_{12}(\mathbf{k},\mathbf{k}')\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle]
\end{aligned}
\tag{2.5}
$$

Seega

$$
\begin{aligned}
\hat{H}_{mf} =& \sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}+\\
&+\sum_{\mathbf{k},\mathbf{k}'}\{\Delta_{1\mathbf{k}}a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}+\Delta_{2\mathbf{k}}a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}+\\
&+\Delta^{*}_{1\mathbf{k}}a_{1-\mathbf{k}'\downarrow}a_{1\mathbf{k}'\uparrow}+\Delta^{*}_{2\mathbf{k}}a_{2-\mathbf{k}'\downarrow}a_{2\mathbf{k}'\uparrow}-\\
&-\Delta_{1\mathbf{k}}\left\langle a^{\dagger}_{1\mathbf{k}\uparrow}a^{\dagger}_{1-\mathbf{k}\downarrow}\right\rangle-\Delta_{2\mathbf{k}}\left\langle a^{\dagger}_{2\mathbf{k}\uparrow}a^{\dagger}_{2-\mathbf{k}\downarrow}\right\rangle\}=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}+\\
&+\sum_{\alpha}\{\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}+\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha\mathbf{k}'\uparrow}-\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}\left\langle a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}\right\rangle\}
\end{aligned}
\tag{2.6}
$$

3 Bogoljubovi-Valantini teisendus

$$
\begin{cases}
a_{\alpha\mathbf{k}\uparrow}=u_{\alpha\mathbf{k}}\alpha_{\alpha\mathbf{k}\uparrow}+v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\\
a_{\alpha-\mathbf{k}\downarrow}=u_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow}-v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}
\end{cases}
\tag{3.1}
$$

$$
|u_{\mathbf{k}}|^2+|v_{\mathbf{k}}|^2=1;\quad u_{-\mathbf{k}}=u_{\mathbf{k}};\quad v_{-\mathbf{k}}=-v_{\mathbf{k}}
\tag{3.2}
$$

Fermionide statistikast tulenevad seosed, kus $[A,B]_{+}$ on operaatorite A ja B antikommutatsioon.

$$
\left[\alpha_{\mathbf{k}s},\alpha^{\dagger}_{\mathbf{k}'s'}\right]_{+}=\delta_{\mathbf{k}\mathbf{k}'}\delta_{ss'},\quad [\alpha_{\mathbf{k}s},\alpha_{\mathbf{k}'s'}]_{+}=0,\quad \left[\alpha^{\dagger}_{\mathbf{k}s},\alpha^{\dagger}_{\mathbf{k}'s'}\right]_{+}=0
\tag{3.3}
$$

Teisendame liikmed

I liige

$$
\begin{aligned}
&\sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}\tilde{\varepsilon}_{\alpha}(\mathbf{k})a^{\dagger}_{\alpha\mathbf{k}s}a_{\alpha\mathbf{k}s}=\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})\left[a^{\dagger}_{\alpha\mathbf{k}\uparrow}a_{\alpha\mathbf{k}\uparrow}+a^{\dagger}_{\alpha-\mathbf{k}\downarrow}a_{\alpha-\mathbf{k}\downarrow}\right]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})[\left(u^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}+v_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow}\right)\left(u_{\alpha\mathbf{k}}\alpha_{\alpha\mathbf{k}\uparrow}+v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\right)+\\
&+\left(u^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-v_{\alpha\mathbf{k}}\alpha_{\alpha\mathbf{k}\uparrow}\right)\left(u_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow}-v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\right)]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})[\underbrace{u^{*}_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}}_{|u_{\alpha\mathbf{k}}|^{2}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}+\\
&+v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}+\underbrace{v_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}}_{|v_{\alpha\mathbf{k}}|^{2}}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}+\\
&+\underbrace{u_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}}_{|u_{\alpha\mathbf{k}}|^{2}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}-u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}-\\
&-v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}\alpha_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha-\mathbf{k}\downarrow}+\underbrace{v_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}}_{|v_{\alpha\mathbf{k}}|^{2}}\alpha_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})[|u_{\alpha\mathbf{k}}|^{2}\left(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}\right)+\\
&+|v_{\alpha\mathbf{k}}|^{2}(\underbrace{\alpha_{\alpha-\mathbf{k}\downarrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}}_{1-\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}}+\underbrace{\alpha_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}}_{1-\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}})+\\
&+u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-\underbrace{\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}}_{-\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}})+\\
&+v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}(\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}-\underbrace{\alpha_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha-\mathbf{k}\downarrow}}_{-\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}})]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})[|u_{\alpha\mathbf{k}}|^{2}\left(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}\right)+\\
&+|v_{\alpha\mathbf{k}}|^{2}(1-\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}+1-\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow})+\\
&+u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}+\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow})+\\
&+v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}(\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}+\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow})]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})[2|v_{\alpha\mathbf{k}}|^{2}+(|u_{\alpha\mathbf{k}}|^{2}-|v_{\alpha\mathbf{k}}|^{2})\left(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}\right)+\\
&+2u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}+2v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}]
\end{aligned}
\tag{3.4}
$$

II liige

$$
\begin{aligned}
&\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}=\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}[(u^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}+v_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow})(u^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-v_{\alpha\mathbf{k}}\alpha_{\alpha\mathbf{k}\uparrow})]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}[\underbrace{u^{*}_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}}_{(u_{\alpha\mathbf{k}^{*}})^{2}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-u^{*}_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\\
&+v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}\underbrace{\alpha_{\alpha-\mathbf{k}\downarrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}}_{1-\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}}-\underbrace{v_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}}_{(v_{\alpha\mathbf{k}})^{2}}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}[(u^{*}_{\alpha\mathbf{k}})^{2}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-u^{*}_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\\
&+v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}-v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}-(v_{\alpha\mathbf{k}})^{2}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}[(u^{*}_{\alpha\mathbf{k}})^{2}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-(v_{\alpha\mathbf{k}})^{2}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}-\\
&-v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}\left(\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}+\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}\right)+v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}]
\end{aligned}
\tag{3.5}
$$

III liige

$$
\begin{aligned}
&\sum_{\alpha}\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha\mathbf{k}'\uparrow}=\\
&=\sum_{\alpha}\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}\left(u_{\alpha\mathbf{k}'}\alpha_{\alpha-\mathbf{k}'\downarrow}-v^{*}_{\alpha\mathbf{k}'}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\right)\left(u_{\alpha\mathbf{k}'}\alpha_{\alpha\mathbf{k}'\uparrow}+v^{*}_{\alpha\mathbf{k}'}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}\right)=\\
&=\sum_{\alpha}\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}[u_{\alpha\mathbf{k}'}u_{\alpha\mathbf{k}'}\alpha_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha\mathbf{k}'\uparrow}+u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}\underbrace{\alpha_{\alpha-\mathbf{k}'\downarrow}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}}_{1-\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha-\mathbf{k}'\downarrow}}-\\
&-v^{*}_{\alpha\mathbf{k}'}u_{\alpha\mathbf{k}'}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha_{\alpha\mathbf{k}'\uparrow}-v^{*}_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}[(u_{\alpha\mathbf{k}'})^{2}\alpha_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha\mathbf{k}'\uparrow}-(v^{*}_{\alpha\mathbf{k}'})^{2}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}+\\
&+u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}-u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha-\mathbf{k}'\downarrow}-v^{*}_{\alpha\mathbf{k}'}u_{\alpha\mathbf{k}'}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha_{\alpha\mathbf{k}'\uparrow}]=\\
&=\sum_{\alpha}\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}[(u_{\alpha\mathbf{k}'})^{2}\alpha_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha\mathbf{k}'\uparrow}-(v^{*}_{\alpha\mathbf{k}'})^{2}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}-\\
&-u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}\left(\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha-\mathbf{k}'\downarrow}+\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha_{\alpha\mathbf{k}'\uparrow}\right)+u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}]
\end{aligned}
\tag{3.6}
$$

IV liige jääb hetkel samaks, hiljem arvutame keskväärtuse.

Kokku saame

$$
\begin{aligned}
\hat{H}_{mf} =&\sum_{\alpha}\sum_{\mathbf{k}}\tilde{\varepsilon}_{\alpha}(\mathbf{k})[2\left|v_{\alpha\mathbf{k}}\right|^{2}+(\left|u_{\alpha\mathbf{k}}\right|^{2}-\left|v_{\alpha\mathbf{k}}\right|^{2})\left(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}\right)+\\
&+2u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}+2v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}]+\\
&+\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}[(u^{*}_{\alpha\mathbf{k}})^{2}\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}-(v_{\alpha\mathbf{k}})^{2}\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}-\\
&-v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}\left(\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}+\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}\right)+v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}]+\\
&+\sum_{\alpha}\sum_{\mathbf{k}'}\Delta^{*}_{\alpha\mathbf{k}'}[(u_{\alpha\mathbf{k}'})^{2}\alpha_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha\mathbf{k}'\uparrow}-(v^{*}_{\alpha\mathbf{k}'})^{2}\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}-\\
&-u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}\left(\alpha^{\dagger}_{\alpha-\mathbf{k}'\downarrow}\alpha_{\alpha-\mathbf{k}'\downarrow}+\alpha^{\dagger}_{\alpha\mathbf{k}'\uparrow}\alpha_{\alpha\mathbf{k}'\uparrow}\right)+u_{\alpha\mathbf{k}'}v^{*}_{\alpha\mathbf{k}'}]\\
&-\sum_{\alpha}\sum_{\mathbf{k}}\Delta_{\alpha\mathbf{k}}\left\langle a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}\right\rangle=\\
\overset{\mathbf{k}'=\mathbf{k}}{=}&\sum_{\alpha}\sum_{\mathbf{k}}\{2\tilde{\varepsilon}_{\alpha}(\mathbf{k})\left|v_{\alpha\mathbf{k}}\right|^{2}+\Delta_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}+\Delta^{*}_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}-\Delta_{\alpha\mathbf{k}}\left\langle a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}\right\rangle+\\
&+\left[\tilde{\varepsilon}_{\alpha}(\mathbf{k})\left(\left|u_{\alpha\mathbf{k}}\right|^{2}-\left|v_{\alpha\mathbf{k}}\right|^{2}\right)-\Delta_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}-\Delta^{*}_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}\right]\times\\
&\times\left(\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}+\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha-\mathbf{k}\downarrow}\right)+\\
&+\left(2\tilde{\varepsilon}_{\alpha}(\mathbf{k})u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}+\Delta_{\alpha\mathbf{k}}\left(u^{*}_{\alpha\mathbf{k}}\right)^{2}-\Delta^{*}_{\alpha\mathbf{k}}\left(v^{*}_{\alpha\mathbf{k}}\right)^{2}\right)\alpha^{\dagger}_{\alpha\mathbf{k}\uparrow}\alpha^{\dagger}_{\alpha-\mathbf{k}\downarrow}+\\
&+\left(2\tilde{\varepsilon}_{\alpha}(\mathbf{k})v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}-\Delta_{\alpha\mathbf{k}}\left(v_{\alpha\mathbf{k}}\right)^{2}+\Delta^{*}_{\alpha\mathbf{k}}\left(u_{\alpha\mathbf{k}}\right)^{2}\right)\alpha_{\alpha-\mathbf{k}\downarrow}\alpha_{\alpha\mathbf{k}\uparrow}\}
\end{aligned}
\tag{3.7}
$$

Liikmed operaatoritega $\alpha\alpha$ ja $\alpha^{\dagger}\alpha^{\dagger}$ ei ole diagonaalil, nõuame, et kehtiks tingimused

$$
\begin{aligned}
2\tilde{\varepsilon}_{\alpha}(\mathbf{k})v_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}-\Delta_{\alpha\mathbf{k}}\left(v_{\alpha\mathbf{k}}\right)^{2}+\Delta^{*}_{\alpha\mathbf{k}}\left(u_{\alpha\mathbf{k}}\right)^{2}=&0\\
2\tilde{\varepsilon}_{\alpha}(\mathbf{k})u^{*}_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}+\Delta_{\alpha\mathbf{k}}\left(u^{*}_{\alpha\mathbf{k}}\right)^{2}-\Delta^{*}_{\alpha\mathbf{k}}\left(v^{*}_{\alpha\mathbf{k}}\right)^{2}=&0
\end{aligned}
\tag{3.8}
$$

Tähistame

$$
\begin{aligned}
E_{0}=&\sum_{\alpha}\sum_{\mathbf{k}}\left\{2\tilde{\varepsilon}_{\alpha}(\mathbf{k})\left|v_{\alpha\mathbf{k}}\right|^{2}+\Delta_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}+\Delta^{*}_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}-\Delta_{\alpha\mathbf{k}}\left\langle a^{\dagger}_{\alpha\mathbf{k}\uparrow}a^{\dagger}_{\alpha-\mathbf{k}\downarrow}\right\rangle\right\}\\
E_{\alpha}(\mathbf{k})=&\tilde{\varepsilon}_{\alpha}(\mathbf{k})\left(\left|u_{\alpha\mathbf{k}}\right|^{2}-\left|v_{\alpha\mathbf{k}}\right|^{2}\right)-\Delta_{\alpha\mathbf{k}}v_{\alpha\mathbf{k}}u^{*}_{\alpha\mathbf{k}}-\Delta^{*}_{\alpha\mathbf{k}}u_{\alpha\mathbf{k}}v^{*}_{\alpha\mathbf{k}}
\end{aligned}
\tag{3.9}
$$

Seega

$$
H_{mf}=E_{0}+\sum_{\alpha}\sum_{\mathbf{k}}\sum_{s}E_{\alpha}(\mathbf{k})\alpha^{\dagger}_{\alpha\mathbf{k}s}\alpha_{\alpha\mathbf{k}s}
\tag{3.10}
$$

3.1 Bogoljubov-Valatini teisenduse kordajad

Kkasutame seoseid (3.8), lihtsuse mõttes jätame indeksid ära

$$\begin{aligned}2\varepsilon vu-\Delta v^2+\Delta^*u^2 =&0 \quad |\cdot\frac{\Delta}{u^2}\\ \frac{2\varepsilon vu\Delta}{u^2}-\frac{\Delta v^2\Delta}{u_2}+\frac{\Delta^*u^2\Delta}{u^2} =&0|\cdot(-1)\\ \left(\frac{\Delta v}{u}\right)^2-2\varepsilon\frac{\Delta v}{u}-|\Delta|^2 =&0\\ \Rightarrow\Delta\frac{v}{u}=\varepsilon\pm\sqrt{\varepsilon^2+|\Delta|^2} \equiv& E\pm\varepsilon\Rightarrow\frac{v}{u}=\frac{\varepsilon-\sqrt{\varepsilon^2+|\Delta|^2}}{\Delta}\end{aligned}\tag{3.11}$$

$$\begin{aligned}2\varepsilon u^*v^*+\Delta(u^*)^2-\Delta^*(v^*)^2 =&0 \quad |\cdot\frac{\Delta^*}{(u^*)^2}\\ 2\varepsilon u^*v^*\frac{\Delta^*}{(u^*)^2}+\Delta(u^*)^2\frac{\Delta^*}{(u^*)^2}-\Delta^*(v^*)^2\frac{\Delta^*}{(u^*)^2} =&0 \quad |\cdot(-1)\\ \left(\frac{\Delta^*v^*}{u^*}\right)^2-2\varepsilon\frac{\Delta^*v^*}{u^*}-|\Delta|^2 =&0\\ \Rightarrow\frac{\Delta^*v^*}{u^*}=\varepsilon\pm\sqrt{\varepsilon^2+|\Delta|^2} \equiv&\varepsilon\pm E\Rightarrow\frac{v^*}{u^*}=\frac{\varepsilon-\sqrt{\varepsilon^2+|\Delta|^2}}{\Delta^*}\end{aligned}\tag{3.12}$$

Leiame $|u|^2$ ja $|v|^2$, kasutame seostest (3.2) esimest

$$|u|^2+|v|^2=1\Rightarrow|u|^2=1-|v|^2\tag{3.13}$$

$$\frac{1}{1+\left|\frac{v}{u}\right|^2}=\frac{1}{\frac{|u|^2+|v|^2}{|u|^2}}=\frac{|u|^2}{\underbrace{|u|^2+|v|^2}_{=1}}=|u|^2\tag{3.14}$$

Asendame seosed (3.11) (3.12) võrrandisse (3.14)

$$\frac{v}{u}\frac{v^*}{u^*}=\frac{\varepsilon-E}{\Delta}\frac{\varepsilon-E}{\Delta^*}=\frac{(\varepsilon-E)^2}{|\Delta|^2}=\left|\frac{v}{u}\right|^2\tag{3.15}$$

$$\begin{aligned}|u|^2=&\left(1+\left|\frac{v}{u}\right|^2\right)^{-1}=\left(1+\left|\frac{\sqrt{\varepsilon^2+|\Delta|^2}-\varepsilon}{\Delta}\right|^2\right)^{-1}=\\ =&\left(1+\frac{(\varepsilon-E)^2}{|\Delta|^2}\right)^{-1}=\left(\frac{|\Delta|^2+\varepsilon^2-2\varepsilon E+E^2}{E^2-\varepsilon^2}\right)^{-1}=\left(\frac{2E^2-2\varepsilon E}{E^2-\varepsilon^2}\right)^{-1}=\\ =&\left(\frac{2E^2(1-\varepsilon/E)}{E^2(1-(\varepsilon^2/E)^2)}\right)^{-1}=\left(\frac{2}{1+\varepsilon/E}\right)^{-1}=\frac{1}{2}\left(1+\frac{\varepsilon}{E}\right)\end{aligned}\tag{3.16}$$

$$|u|^2+|v|^2=1 \Rightarrow |v|^2=1-|u|^2=1-\frac{1}{2}\left(1+\frac{\varepsilon}{E}\right)=\frac{1}{2}\left(1-\frac{\varepsilon}{E}\right) \tag{3.17}$$

Leiame vu^* ja uv^*. Seosest (3.11) ja (3.12) saame

$$\begin{aligned} \frac{v}{u} =& \frac{v}{u}\frac{u^*}{u^*}=\frac{vu^*}{|u|^2}=\frac{\varepsilon-E}{\Delta} \Rightarrow vu^*=|u|^2\frac{\varepsilon-E}{\Delta} \overset{(3.16)}{=} \frac{1}{2}\left(1+\frac{\varepsilon}{E}\right)\frac{\varepsilon-E}{\Delta}= \\ =&\frac{1}{2}\frac{E+\varepsilon}{E}\frac{\varepsilon-E}{\Delta}=\frac{1}{2}\frac{\varepsilon^2-E^2}{E\Delta}=\frac{1}{2}\frac{\varepsilon^2-\varepsilon^2-|\Delta|^2}{E\Delta}=-\frac{\Delta^*}{2E} \end{aligned} \tag{3.18}$$

$$\begin{aligned} \frac{v^*}{u^*} =& \frac{v^*}{u^*}\frac{u}{u}=\frac{v^*u}{|u|^2}=\frac{\varepsilon-E}{\Delta^*} \\ \Rightarrow uv^* =& \frac{\varepsilon-E}{\Delta^*}|u|^2=\frac{\varepsilon-E}{\Delta^*}\frac{1}{2}\left(1+\frac{\varepsilon}{E}\right)=\frac{1}{2E\Delta^*}(\varepsilon^2-E^2)=\frac{-|\Delta|^2}{2E\Delta^*}=-\frac{\Delta}{2E} \end{aligned} \tag{3.19}$$

Kokku saime

$$|u|^2=\frac{1}{2}\left(1+\frac{\varepsilon}{E}\right), \quad |v|^2=\frac{1}{2}\left(1-\frac{\varepsilon}{E}\right), \quad vu^*=-\frac{\Delta^*}{2E}, \quad uv^*=-\frac{\Delta}{2E} \tag{3.20}$$

3.2 Ergastusenergia ja põhiseisundi energia

Saadud seostega (3.20) saame arvutada E_0-s oleva keskväärtuse, lihtsuse mõttes jätame alles ainult spinni indeksi, pidades meeles, et $\downarrow$ spinnile vastab $-\mathbf{k}$ ja $\uparrow$ spinnile vastab $\mathbf{k}$

$$\begin{aligned} \langle a_\uparrow a_\downarrow\rangle =& \langle (u\alpha_\uparrow+v^*\alpha_\downarrow^\dagger)(u\alpha_\downarrow-v^*\alpha_\uparrow^\dagger)\rangle = \\ =& \langle u\alpha_\uparrow u\alpha_\downarrow - u\alpha_\uparrow v^*\alpha_\uparrow^\dagger + v^*\alpha_\downarrow^\dagger u\alpha_\downarrow - v^*\alpha_\downarrow^\dagger v^*\alpha_\uparrow^\dagger\rangle = \\ =& u^2\underbrace{\langle\alpha_\uparrow\alpha_\downarrow\rangle}_{0} - uv^*\underbrace{\langle\alpha_\uparrow\alpha_\uparrow^\dagger\rangle}_{1-\langle\alpha_\uparrow^\dagger\alpha_\uparrow\rangle} + v^*u\langle\alpha_\downarrow^\dagger\alpha_\downarrow\rangle - (v^*)^2\underbrace{\langle\alpha_\downarrow^\dagger\alpha_\uparrow^\dagger\rangle}_{0} = \\ =& uv^*\left(-1+\langle\alpha_\uparrow^\dagger\alpha_\uparrow\rangle+\langle\alpha_\downarrow^\dagger\alpha_\downarrow\rangle\right)=uv^*\left(2f(E)-1\right)= \\ =& -\frac{\Delta}{2E}\left(\frac{2}{\exp(\beta E)+1}-1\right)=-\frac{\Delta}{2E}\left(\frac{1-\exp(\beta E)}{\exp(\beta E)+1}\right)= \\ =& \frac{\Delta}{2E}\frac{\exp(\beta E)-1}{\exp(\beta E)+1}=\frac{\Delta}{2E}\tanh\left(\frac{\beta E}{2}\right) \end{aligned} \tag{3.21}$$

ehk

$$\left\langle a_{\alpha\mathbf{k}\uparrow}a_{\alpha-\mathbf{k}\downarrow}\right\rangle=\frac{1}{2}\Delta_{\alpha\mathbf{k}}\xi_{\alpha\mathbf{k}} \Rightarrow \left\langle a_{\alpha\mathbf{k}\uparrow}^\dagger a_{\alpha-\mathbf{k}\downarrow}^\dagger\right\rangle=\frac{1}{2}\Delta_{\alpha\mathbf{k}}^*\xi_{\alpha\mathbf{k}}, \tag{3.22}$$

kus

$$\xi_{\alpha\mathbf{k}}=E_\alpha^{-1}(\mathbf{k})\tanh\frac{E_\alpha(\mathbf{k})}{2k_BT} \tag{3.23}$$

Analoogselt

$$
\begin{aligned}
\langle a_\downarrow a_\uparrow\rangle =&\langle(u\alpha_\downarrow - v^*\alpha_\uparrow^\dagger)(u\alpha_\uparrow + v^*\alpha_\downarrow^\dagger)\rangle = \\
=&\langle u\alpha_\downarrow u\alpha_\uparrow + u\alpha_\downarrow v^*\alpha_\downarrow^\dagger - v^*\alpha_\uparrow^\dagger u\alpha_\uparrow - v^*\alpha_\uparrow^\dagger v^*\alpha_\downarrow^\dagger\rangle = \\
=&u^2\underbrace{\langle\alpha_\downarrow\alpha_\downarrow\rangle}_{0} + uv^*\underbrace{\langle\alpha_{\alpha\mathbf{k}\uparrow}\alpha_{\alpha\mathbf{k}\uparrow}^\dagger\rangle}_{1-\langle\alpha_{\alpha\mathbf{k}\uparrow}^\dagger\alpha_{\alpha\mathbf{k}\uparrow}\rangle} - v^*u\langle\alpha_\uparrow^\dagger\alpha_\uparrow\rangle - (v^*)^2\underbrace{\langle\alpha_\uparrow^\dagger\alpha_\downarrow^\dagger\rangle}_{0} = \\
=&uv^*(1-2\langle\alpha_{\alpha\mathbf{k}\uparrow}^\dagger\alpha_{\alpha\mathbf{k}\uparrow}\rangle) = uv^*(1-2f(E)) = \\
=&-\frac{\Delta}{2E}\left(1-\frac{2}{\exp(\beta E)-1}\right) = -\frac{\Delta}{2E}\left(\frac{\exp(\beta E)-1+2}{\exp(\beta E)-1}\right) = \\
=&-\frac{\Delta}{2E}\left(\frac{\exp(\beta E)+1}{\exp(\beta E)-1}\right) = -\frac{\Delta}{2E}\tanh\left(\frac{\beta E}{2}\right)
\end{aligned}
\tag{3.24}
$$

ehk

$$
\left\langle a_{\alpha-\mathbf{k}\downarrow}a_{\alpha\mathbf{k}\uparrow}\right\rangle = -\frac{1}{2}\Delta_{\alpha\mathbf{k}}\xi_{\alpha\mathbf{k}}
\tag{3.25}
$$

Arvutame E_0 ja $E_\alpha(\mathbf{k})$

$$
\begin{aligned}
E_0 =&2\varepsilon|v|^2 + \Delta vu^* + \Delta^* uv^* + \frac{1}{2}|\Delta|^2\xi = \\
=&2\varepsilon\frac{1}{2}\left(1-\frac{\varepsilon}{E}\right) + \Delta\left(-\frac{1}{2}\frac{\Delta^*}{E}\right) + \Delta^*\left(-\frac{1}{2}\frac{\Delta}{E}\right) + \frac{1}{2}|\Delta|^2\xi = \\
=&\varepsilon - \frac{\varepsilon^2}{E} - \frac{1}{2}\frac{|\Delta|^2}{E} - \frac{1}{2}\frac{|\Delta|^2}{E} + \frac{1}{2}|\Delta|^2\xi \\
=&\varepsilon - \frac{\varepsilon^2+|\Delta|^2}{E} + \frac{1}{2}|\Delta|^2\xi = \varepsilon - E + \frac{1}{2}|\Delta|^2\xi
\end{aligned}
\tag{3.26}
$$

$$
\begin{aligned}
E =&\varepsilon(|u|^2-|v|^2) - \Delta vu^* - \Delta^* uv^* = \\
=&\varepsilon(2|u|^2-1) - \Delta vu^* - \Delta^* uv^* = \\
=&\varepsilon 2\frac{1}{2}\left(1+\frac{\varepsilon}{E}\right) - \varepsilon - \Delta\left(-\frac{1}{2}\frac{\Delta^*}{E}\right) - \Delta^*\frac{1}{2}\frac{\Delta}{E} = \\
=&\varepsilon + \frac{\varepsilon^2}{E} - \varepsilon + \frac{|\Delta|^2}{2E} - \frac{|\Delta|^2}{2E} = \frac{\varepsilon^2}{E} + \frac{|\Delta|^2}{E} = E \Rightarrow \varepsilon^2 + |\Delta|^2 = E^2
\end{aligned}
\tag{3.27}
$$

4 Interkatsioonikonstantide lähendused

Lähendame interaktsioonikonstandid energeetilistes piirkondades Heaviside'i funktsioonile

$$
\begin{aligned}
V_{\alpha\alpha}(\mathbf{k},\mathbf{k}') &= V_{\alpha\alpha}\Theta\left(\hbar\omega_D - |\tilde{\varepsilon}_\alpha(\mathbf{k})|\right)\Theta\left(\hbar\omega_D - \left|\tilde{\varepsilon}_\alpha(\mathbf{k}')\right|\right)\\
U_{\alpha\alpha}(\mathbf{k},\mathbf{k}') &= U_{\alpha\alpha}\Theta\left(\hbar\omega_C - |\tilde{\varepsilon}_\alpha(\mathbf{k})|\right)\Theta\left(\hbar\omega_C - \left|\tilde{\varepsilon}_\alpha(\mathbf{k}')\right|\right)\\
V_{12}(\mathbf{k},\mathbf{k}') &= V_{12}\Theta\left(\hbar\omega_D - |\tilde{\varepsilon}_1(\mathbf{k})|\right)\Theta\left(\hbar\omega_D - \left|\tilde{\varepsilon}_2(\mathbf{k}')\right|\right)\\
V_{21}(\mathbf{k},\mathbf{k}') &= V_{21}\Theta\left(\hbar\omega_D - |\tilde{\varepsilon}_2(\mathbf{k})|\right)\Theta\left(\hbar\omega_D - \left|\tilde{\varepsilon}_1(\mathbf{k}')\right|\right)\\
U_{12}(\mathbf{k},\mathbf{k}') &= U_{12}\Theta\left(\hbar\omega_C - |\tilde{\varepsilon}_1(\mathbf{k})|\right)\Theta\left(\hbar\omega_C - \left|\tilde{\varepsilon}_2(\mathbf{k}')\right|\right)\\
U_{21}(\mathbf{k},\mathbf{k}') &= U_{21}\Theta\left(\hbar\omega_C - |\tilde{\varepsilon}_2(\mathbf{k})|\right)\Theta\left(\hbar\omega_C - \left|\tilde{\varepsilon}_1(\mathbf{k}')\right|\right),
\end{aligned}
\tag{4.1}
$$

$\hbar\omega_C \geqslant \hbar\omega_D$, $V_{\alpha\alpha} < 0$, $V_{12} = V_{21} < 0$, $U_{\alpha\alpha} > 0$ ja $U_{12} = U_{21} > 0$.

5 Võrrandisüsteemi teisendamine

$$
\begin{aligned}
\Delta_{\alpha\mathbf{k}} = \sum_{\mathbf{k}'}[&V_{\alpha\alpha}(\mathbf{k},\mathbf{k}')\left\langle a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha\mathbf{k}'\uparrow}\right\rangle + U_{\alpha\alpha}(\mathbf{k},\mathbf{k}')\left\langle a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha\mathbf{k}'\uparrow}\right\rangle +\\
&+V_{\alpha\alpha'}(\mathbf{k},\mathbf{k}')\left\langle a_{\alpha'-\mathbf{k}'\downarrow}a_{\alpha'\mathbf{k}'\uparrow}\right\rangle + U_{\alpha\alpha'}(\mathbf{k},\mathbf{k}')\left\langle a_{\alpha'-\mathbf{k}'\downarrow}a_{\alpha'\mathbf{k}'\uparrow}\right\rangle]
\end{aligned}
\tag{5.1}
$$

Lähme summeerimiselt üle integreerimisele

$$
\sum_{\mathbf{x}} f(y(\mathbf{x})) = \int f(y)\rho(y)dy, \tag{5.2}
$$

kus suurus ρ on olekute tihedus.
Tähistame

$$
\begin{aligned}
\eta_\alpha(T,\Delta_\alpha) &= \int_0^{\hbar\omega_D} E^{-1}(\Delta_\alpha)\tanh\left(\frac{E(\Delta_\alpha)}{2k_BT}\right)d\tilde{\varepsilon}_\alpha \equiv \eta_\alpha\\
\tilde{\eta}_\alpha(T,\tilde{\Delta}_\alpha) &= \int_{\hbar\omega_D}^{\hbar\omega_C} E^{-1}(\tilde{\Delta}_\alpha)\tanh\left(\frac{E(\tilde{\Delta}_\alpha)}{2k_BT}\right)d\tilde{\varepsilon}_\alpha \equiv \tilde{\eta}_\alpha\\
E(\Delta_\alpha) &\equiv \sqrt{\tilde{\varepsilon}_\alpha^2 + |\Delta_\alpha|^2}\\
E(\tilde{\Delta}_\alpha) &\equiv \sqrt{\tilde{\varepsilon}_\alpha^2 + |\tilde{\Delta}_\alpha|^2}
\end{aligned}
\tag{5.3}
$$

Ehk

$$
\Delta_{\alpha\mathbf{k}} = \sum_\alpha\sum_{\mathbf{k}'} W_{\alpha\alpha'}\left\langle a_{\alpha-\mathbf{k}'\downarrow}a_{\alpha'\mathbf{k}'\uparrow}\right\rangle = \sum_\alpha W_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} \tag{5.4}
$$

Kordaja $1/2$ kaob ära, kuna muudame integreerimisrajasid $\int_{-\hbar\omega_D}^{\hbar\omega_D} \to 2\int_0^{\hbar\omega_D}$ ja eeldame, et olekute tihedus ρ on integreerimisvahemikus konstantne. $\alpha = 1,2$, $\alpha \neq \alpha'$

$$\begin{cases} \Delta_\alpha & = -V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - \\ & -V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} \\ \tilde{\Delta}_\alpha & = -U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} \end{cases} \tag{5.5}$$

5.1 $\tilde{\Delta}_\alpha$ eemaldamine võrrandisüsteemist

Eemaldame võrrandisüsteemist $\tilde{\Delta}_\alpha$

$$\begin{aligned} \Delta_\alpha - \tilde{\Delta}_\alpha = & - V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - \\ & - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} - \\ & - (U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'}) \Rightarrow \\ \Rightarrow \tilde{\Delta}_\alpha = & \Delta_\alpha + V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha + V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} \end{aligned} \tag{5.6}$$

Asendame $\tilde{\Delta}_\alpha$ esimesse võrrandisse

$$\begin{aligned} \Delta_\alpha = & - V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - \\ & - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha(\Delta_\alpha + V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha + V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'}) - \\ & - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - \\ & - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}(\Delta_{\alpha'} + V_{\alpha'\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} + V_{\alpha'\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha) = \\ = & - V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\Delta_\alpha - U_{\alpha\alpha}V_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\rho_\alpha\eta_\alpha\Delta_\alpha - \\ & - U_{\alpha\alpha}V_{\alpha\alpha'}\rho_\alpha\tilde{\eta}_\alpha\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - \\ & - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}V_{\alpha'\alpha'}\tilde{\eta}_{\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}V_{\alpha'\alpha}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_\alpha\eta_\alpha\Delta_\alpha \end{aligned} \tag{5.7}$$

$$\begin{aligned} \Delta_\alpha = & \Delta_\alpha(-V_{\alpha\alpha}\rho_\alpha\eta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha - \\ & - U_{\alpha\alpha}V_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\rho_\alpha\eta_\alpha - U_{\alpha\alpha'}V_{\alpha'\alpha}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_\alpha\eta_\alpha) + \\ & + \Delta_{\alpha'}(-U_{\alpha\alpha}V_{\alpha\alpha'}\rho_\alpha\tilde{\eta}_\alpha\rho_{\alpha'}\eta_{\alpha'} - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'} - \\ & - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'} - U_{\alpha\alpha'}V_{\alpha'\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_{\alpha'}\eta_{\alpha'}) \end{aligned} \tag{5.8}$$

Kirjutame indeksid lahti

$$\begin{aligned} \Delta_1 = & \Delta_1(-V_{11}\rho_1\eta_1 - U_{11}\rho_1\eta_1 - U_{11}\rho_1\tilde{\eta}_1 - U_{11}V_{11}\rho_1\tilde{\eta}_1\rho_1\eta_1 - U_{12}V_{21}\rho_2\tilde{\eta}_2\rho_1\eta_1) + \\ & + \Delta_2(-U_{11}V_{12}\rho_1\tilde{\eta}_1\rho_2\eta_2 - V_{12}\rho_2\eta_2 - U_{12}\rho_2\eta_2 - U_{12}\rho_2\tilde{\eta}_2 - U_{12}V_{22}\rho_2\tilde{\eta}_2\rho_2\eta_2) \\ \Delta_2 = & \Delta_2(-V_{22}\rho_2\eta_2 - U_{22}\rho_2\eta_2 - U_{22}\rho_2\tilde{\eta}_2 - U_{22}V_{22}\rho_2\tilde{\eta}_2\rho_2\eta_2 - U_{21}V_{12}\rho_1\tilde{\eta}_1\rho_2\eta_2) + \\ & + \Delta_1(-U_{22}V_{21}\rho_2\tilde{\eta}_2\rho_1\eta_1 - V_{21}\rho_1\eta_1 - U_{21}\rho_1\eta_1 - U_{21}\rho_1\tilde{\eta}_1 - U_{21}V_{11}\rho_1\tilde{\eta}_1\rho_1\eta_1) \end{aligned} \tag{5.9}$$

6 Võrrandisüsteemi ümbertähistamine

Tähistame

$$
\begin{aligned}
\Xi_{11} =& V_{11}\rho_1 + U_{11}\rho_1 + U_{11}V_{11}\rho_1\tilde{\eta}_1\rho_1 + U_{12}V_{21}\rho_2\tilde{\eta}_2\rho_1 \\
\Xi_{12} =& V_{12}\rho_2 + U_{12}\rho_2 + U_{11}V_{12}\rho_1\tilde{\eta}_1\rho_2 + U_{12}V_{22}\rho_2\tilde{\eta}_2\rho_2 \\
\Xi_{21} =& V_{21}\rho_1 + U_{21}\rho_1 + U_{22}V_{21}\rho_2\tilde{\eta}_2\rho_1 + U_{21}V_{11}\rho_1\tilde{\eta}_1\rho_1 \\
\Xi_{22} =& V_{22}\rho_2 + U_{22}\rho_2 + U_{22}V_{22}\rho_2\tilde{\eta}_2\rho_2 + U_{21}V_{12}\rho_1\tilde{\eta}_1\rho_2
\end{aligned}
\tag{6.1}
$$

$$
\begin{cases}
\Delta_1 = (-\Xi_{11}\eta_1 - U_{11}\rho_1\tilde{\eta}_1)\Delta_1 + (-\Xi_{12}\eta_2 - U_{12}\rho_2\tilde{\eta}_2)\Delta_2 \\
\Delta_2 = (-\Xi_{21}\eta_1 - U_{21}\rho_1\tilde{\eta}_1)\Delta_1 + (-\Xi_{22}\eta_2 - U_{22}\rho_2\tilde{\eta}_2)\Delta_2
\end{cases}
\tag{6.2}
$$

$$
\begin{cases}
\Delta_1 = -\Xi_{11}\eta_1\Delta_1 - U_{11}\rho_1\tilde{\eta}_1\Delta_1 - \Xi_{12}\eta_2\Delta_2 - U_{12}\rho_2\tilde{\eta}_2\Delta_2 \\
\Delta_2 = -\Xi_{21}\eta_1\Delta_1 - U_{21}\rho_1\tilde{\eta}_1\Delta_1 - \Xi_{22}\eta_2\Delta_2 - U_{22}\rho_2\tilde{\eta}_2\Delta_2
\end{cases}
\tag{6.3}
$$

Arvutame $\tilde{\eta}_\alpha$. Arvestame, et $k_B T_c, |\Delta_\alpha| \ll \hbar\omega_D$

$$
\begin{aligned}
\tilde{\eta}_\alpha\left(T,\tilde{\Delta}_\alpha\right) &= \int_{\hbar\omega_D}^{\hbar\omega_C} E^{-1}(\tilde{\Delta}_\alpha)\tanh\left(\frac{E(\tilde{\Delta}_\alpha)}{2k_BT}\right)d\tilde{\varepsilon}_\alpha \equiv \tilde{\eta}_\alpha \\
\tilde{\eta}_\alpha(T,0) &= \int_{\hbar\omega_D}^{\hbar\omega_C} \tilde{\varepsilon}^{-1}\tanh\left(\frac{\tilde{\varepsilon}}{2k_BT}\right)d\tilde{\varepsilon}_\alpha \approx \int_{\hbar\omega_D}^{\hbar\omega_C}\tilde{\varepsilon}^{-1}d\tilde{\varepsilon}_\alpha = \ln|\tilde{\varepsilon}|\Big|_{\hbar\omega_C}^{\hbar\omega_D} = \ln\frac{\hbar\omega_C}{\hbar\omega_D}
\end{aligned}
\tag{6.4}
$$

$$
\tilde{\eta}_\alpha \approx \ln\left(\frac{\hbar\omega_C}{\hbar\omega_D}\right) \equiv \tilde{\eta}
\tag{6.5}
$$

Tähistame

$$
\begin{aligned}
P =& \sqrt{\frac{\rho_1}{\rho_2}} \\
\nu_{\alpha\alpha} =& V_{\alpha\alpha}\rho_\alpha \\
\mu_{\alpha\alpha} =& U_{\alpha\alpha}\rho_\alpha \\
\nu_{\alpha\alpha'} =& V_{\alpha\alpha'}\sqrt{\rho_\alpha\rho_{\alpha'}} \\
\mu_{\alpha\alpha'} =& U_{\alpha\alpha'}\sqrt{\rho_\alpha\rho_{\alpha'}}
\end{aligned}
\tag{6.6}
$$

$$\begin{aligned}
\Xi_{11} =& \nu_{11} + \mu_{11} + \mu_{11}\nu_{11}\tilde{\eta} + \mu_{12}\nu_{21}\tilde{\eta} \\
\Xi_{12} =& \nu_{12}P^{-1} + \mu_{12}P^{-1} + \mu_{11}\nu_{12}P^{-1}\tilde{\eta} + \mu_{12}P^{-1}\nu_{22}\tilde{\eta} \\
\Xi_{21} =& \nu_{21}P + \mu_{21}P + \mu_{22}\nu_{21}P\tilde{\eta} + \mu_{21}P\nu_{11}\tilde{\eta} \\
\Xi_{22} =& + \nu_{22} + \mu_{22} + \mu_{22}\nu_{22}\tilde{\eta} + \mu_{21}\nu_{12}\tilde{\eta}
\end{aligned} \tag{6.7}$$

$$\begin{aligned}
\Xi_{11} =& \nu_{11} + \mu_{11} + \mu_{11}\nu_{11}\tilde{\eta} + \mu_{12}\nu_{21}\tilde{\eta} \\
\Xi_{12} =& (\nu_{12} + \mu_{12} + \mu_{11}\nu_{12}\tilde{\eta} + \mu_{12}\nu_{22}\tilde{\eta})P^{-1} \\
\Xi_{21} =& (\nu_{21} + \mu_{21} + \mu_{22}\nu_{21}\tilde{\eta} + \mu_{21}\nu_{11}\tilde{\eta})P \\
\Xi_{22} =& \nu_{22} + \mu_{22} + \mu_{22}\nu_{22}\tilde{\eta} + \mu_{21}\nu_{12}\tilde{\eta}
\end{aligned} \tag{6.8}$$

Toome P Ξ_{12} ja Ξ_{21} seest välja

$$\begin{cases}
\Delta_1 = -\Xi_{11}\eta_1\Delta_1 - \mu_{11}\tilde{\eta}\Delta_1 - \Xi_{12}P^{-1}\eta_2\Delta_2 - \mu_{12}P^{-1}\tilde{\eta}\Delta_2 \\
\Delta_2 = -\Xi_{21}P\eta_1\Delta_1 - \mu_{21}P\tilde{\eta}\Delta_1 - \Xi_{22}\eta_2\Delta_2 - \mu_{22}\tilde{\eta}\Delta_2
\end{cases} \tag{6.9}$$

Tähistame

$$\Upsilon_{\alpha\alpha} = 1 + \mu_{\alpha\alpha}\tilde{\eta} \tag{6.10}$$

$$\begin{cases}
\Upsilon_{11}\Delta_1 = -\Xi_{11}\eta_1\Delta_1 - \Xi_{12}P^{-1}\eta_2\Delta_2 - \mu_{12}P^{-1}\tilde{\eta}\Delta_2 \\
\Upsilon_{22}\Delta_2 = -\Xi_{21}P\eta_1\Delta_1 - \mu_{21}P\tilde{\eta}\Delta_1 - \Xi_{22}\eta_2\Delta_2
\end{cases} \tag{6.11}$$

$$\begin{cases}
\Upsilon_{11}\Delta_1 + \Xi_{11}\eta_1\Delta_1 + \Xi_{12}P^{-1}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Delta_2 = 0 \\
\Upsilon_{22}\Delta_2 + \Xi_{22}\eta_2\Delta_2 + \Xi_{21}P\eta_1\Delta_1 + \mu_{21}P\tilde{\eta}\Delta_1 = 0
\end{cases} \tag{6.12}$$

Korrutame esimest võrrandit $\mu_{21}\tilde{\eta}P$-ga ja teist Υ_{11}-ga

$$\begin{cases}
\mu_{21}\tilde{\eta}P\Upsilon_{11}\Delta_1 + \mu_{21}\tilde{\eta}P\Xi_{11}\eta_1\Delta_1 + \mu_{21}\tilde{\eta}P\Xi_{12}P^{-1}\eta_2\Delta_2 + \mu_{21}\tilde{\eta}\mu_{12}\tilde{\eta}\Delta_2 = 0 \\
\Upsilon_{11}\Upsilon_{22}\Delta_2 + \Upsilon_{11}\Xi_{22}\eta_2\Delta_2 + \Upsilon_{11}\Xi_{21}P\eta_1\Delta_1 + \Upsilon_{11}\mu_{21}P\tilde{\eta}\Delta_1 = 0
\end{cases} \tag{6.13}$$

Lahutame esimesest võrrandist teise

$$\begin{aligned}
&\mu_{21}\tilde{\eta}P\Upsilon_{11}\Delta_1 + \mu_{21}\tilde{\eta}P\Xi_{11}\eta_1\Delta_1 + \mu_{21}P^{-1}\tilde{\eta}P\Xi_{12}\eta_2\Delta_2 + \mu_{21}\tilde{\eta}\mu_{12}\tilde{\eta}\Delta_2 - \\
&-(\Upsilon_{11}\Upsilon_{22}\Delta_2 + \Upsilon_{11}\Xi_{22}\eta_2\Delta_2 + \Upsilon_{11}\Xi_{21}P\eta_1\Delta_1 + \Upsilon_{11}\mu_{21}P\tilde{\eta}\Delta_1) = 0
\end{aligned} \tag{6.14}$$

Grupeerime liikmed

$$\Delta_1\eta_1P(\mu_{21}\tilde{\eta}\Xi_{11} - \Upsilon_{11}\Xi_{21}) + \Delta_2\eta_2(\mu_{21}\tilde{\eta}\Xi_{12} - \Upsilon_{11}\Xi_{22}) + \Delta_2(\mu_{21}\mu_{12}\tilde{\eta}^2 - \Upsilon_{11}\Upsilon_{22}) = 0 \tag{6.15}$$

$$
\begin{cases}
\Upsilon_{11}\Delta_1 + \Xi_{11}\eta_1\Delta_1 + \Xi_{12}P^{-1}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Delta_2 = 0 \\
\Upsilon_{22}\Delta_2 + \Xi_{22}\eta_2\Delta_2 + \Xi_{21}P\eta_1\Delta_1 + \mu_{21}P\tilde{\eta}\Delta_1 = 0
\end{cases}
\tag{6.16}
$$

Korrutame esimest võrrandit Υ_{22}-ga ja teist $\mu_{12}P^{-1}\tilde{\eta}$-ga

$$
\begin{cases}
\Upsilon_{22}\Upsilon_{11}\Delta_1 + \Upsilon_{22}\Xi_{11}\eta_1\Delta_1 + \Upsilon_{22}\Xi_{12}P^{-1}\eta_2\Delta_2 + \Upsilon_{22}\mu_{12}P^{-1}\tilde{\eta}\Delta_2 = 0 \\
\mu_{12}P^{-1}\tilde{\eta}\Upsilon_{22}\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{22}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{21}P\eta_1\Delta_1 + \mu_{12}P^{-1}\tilde{\eta}\mu_{21}P\tilde{\eta}\Delta_1 = 0
\end{cases}
\tag{6.17}
$$

Lahutame teisest võrrandist esimese

$$
\begin{aligned}
&\mu_{12}P^{-1}\tilde{\eta}\Upsilon_{22}\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{22}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{21}P\eta_1\Delta_1 + \mu_{12}\tilde{\eta}\mu_{21}\tilde{\eta}\Delta_1 - \\
&-(\Upsilon_{22}\Upsilon_{11}\Delta_1 + \Upsilon_{22}\Xi_{11}\eta_1\Delta_1 + \Upsilon_{22}\Xi_{12}P^{-1}\eta_2\Delta_2 + \Upsilon_{22}\mu_{12}P^{-1}\tilde{\eta}\Delta_2) = 0
\end{aligned}
\tag{6.18}
$$

Grupeerime liikmed

$$
\Delta_1\eta_1(\mu_{12}\tilde{\eta}\Xi_{21} - \Upsilon_{22}\Xi_{11}) + \Delta_1(\mu_{12}\mu_{21}\tilde{\eta}^2 - \Upsilon_{22}\Upsilon_{11}) + \Delta_2\eta_2P^{-1}(\mu_{12}\tilde{\eta}\Xi_{22} - \Upsilon_{22}\Xi_{12}) = 0
\tag{6.19}
$$

Kokku saame

$$
\begin{cases}
\Delta_1\eta_1P(\mu_{21}\tilde{\eta}\Xi_{11} - \Upsilon_{11}\Xi_{21}) + \Delta_2\eta_2(\mu_{21}\tilde{\eta}\Xi_{12} - \Upsilon_{11}\Xi_{22}) = \Delta_2(\Upsilon_{11}\Upsilon_{22} - \mu_{21}\mu_{12}\tilde{\eta}^2) \\
\Delta_1\eta_1(\mu_{12}\tilde{\eta}\Xi_{21} - \Upsilon_{22}\Xi_{11}) + \Delta_2\eta_2P^{-1}(\mu_{12}\tilde{\eta}\Xi_{22} - \Upsilon_{22}\Xi_{12}) = \Delta_1(\Upsilon_{22}\Upsilon_{11} - \mu_{12}\mu_{21}\tilde{\eta}^2)
\end{cases}
\tag{6.20}
$$

Tähistame

$$
\begin{aligned}
\Theta_{11} &= \mu_{12}\tilde{\eta}\Xi_{21} - \Upsilon_{22}\Xi_{11} \\
\Theta_{12} &= \mu_{12}\tilde{\eta}\Xi_{22} - \Upsilon_{22}\Xi_{12} \\
\Phi &= \Upsilon_{11}\Upsilon_{22} - \mu_{21}\mu_{12}\tilde{\eta}^2 \\
\Theta_{21} &= \mu_{21}\tilde{\eta}\Xi_{11} - \Upsilon_{11}\Xi_{21} \\
\Theta_{22} &= \mu_{21}\tilde{\eta}\Xi_{12} - \Upsilon_{11}\Xi_{22}
\end{aligned}
\tag{6.21}
$$

$$
\begin{cases}
\Delta_1\eta_1P\Theta_{21} + \Delta_2\eta_2\Theta_{22} = \Delta_2\Phi \\
\Delta_1\eta_1\Theta_{11} + \Delta_2\eta_2P^{-1}\Theta_{12} = \Delta_1\Phi
\end{cases}
\tag{6.22}
$$

$$
\begin{cases}
\Delta_1 = \Delta_1\eta_1\dfrac{\Theta_{11}}{\Phi} + \Delta_2\eta_2\dfrac{\Theta_{12}}{P\Phi} \\
\Delta_2 = \Delta_1\eta_1\dfrac{P\Theta_{21}}{\Phi} + \Delta_2\eta_2\dfrac{\Theta_{22}}{\Phi}
\end{cases}
\tag{6.23}
$$

Võtame arvesse, et $\nu_{12} = \nu_{21}$ ja $\mu_{12} = \mu_{21}$ ning asendame tähistused (6.8),(6.10) tagasi tähistusse (6.21) ning viime olekute tihedused tähistuse sisse

$$\begin{aligned}
\Phi =& 1 + \tilde{\eta}\mu_{11} - \tilde{\eta}^2\mu_{12}^2 + \tilde{\eta}\mu_{22} + \tilde{\eta}^2\mu_{11}\mu_{22} \\
\Theta_{11} =& -\mu_{11} + \tilde{\eta}\mu_{12}^2 - \tilde{\eta}\mu_{11}\mu_{22} - \nu_{11} - \tilde{\eta}\mu_{11}\nu_{11} + \tilde{\eta}^2\mu_{12}^2\nu_{11} - \tilde{\eta}\mu_{22}\nu_{11} - \tilde{\eta}^2\mu_{11}\mu_{22}\nu_{11} \\
\Theta_{22} =& -\mu_{22} + \tilde{\eta}\mu_{12}^2 - \tilde{\eta}\mu_{11}\mu_{22} - \nu_{22} - \tilde{\eta}\mu_{11}\nu_{22} + \tilde{\eta}^2\mu_{12}^2\nu_{22} - \tilde{\eta}\mu_{22}\nu_{22} - \tilde{\eta}^2\mu_{11}\mu_{22}\nu_{22} \\
\Theta_{12} =& P^{-1}\left(-\mu_{12} - \nu_{12} - \tilde{\eta}\mu_{11}\nu_{12} + \tilde{\eta}^2\mu_{12}^2\nu_{12} - \tilde{\eta}\mu_{22}\nu_{12} - \tilde{\eta}^2\mu_{11}\mu_{22}\nu_{12}\right) \\
\Theta_{21} =& P\left(-\mu_{12} - \nu_{12} - \tilde{\eta}\mu_{11}\nu_{12} + \tilde{\eta}^2\mu_{12}^2\nu_{12} - \tilde{\eta}\mu_{22}\nu_{12} - \tilde{\eta}^2\mu_{11}\mu_{22}\nu_{12}\right)
\end{aligned} \tag{6.24}$$

Tähistame

$$\Gamma_{\alpha\alpha} = \frac{\Theta_{\alpha\alpha}}{\Phi},\ \Gamma_{12} = \frac{\Theta_{12}}{P\Phi},\ \Gamma_{21} = \frac{P\Theta_{21}}{\Phi} \tag{6.25}$$

$$\begin{cases} \Delta_1 = \Delta_1\eta_1\Gamma_{11} + \Delta_2\eta_2\Gamma_{12} \\ \Delta_2 = \Delta_1\eta_1\Gamma_{21} + \Delta_2\eta_2\Gamma_{22} \end{cases} \tag{6.26}$$

7 Efektiivsete interaktsioonikonstantide analüüs

Efektiivsete interaktsioonikonstantide sõltuvus interaktsioonikonstantidest

$$\begin{aligned}
\Phi =& \Phi(\mu_{11}, \mu_{22}, \mu_{12}) \\
\Theta_{11} =& \Theta_{11}(\mu_{11}, \mu_{22}, \mu_{12}, \nu_{11}) \\
\Theta_{22} =& \Theta_{22}(\mu_{11}, \mu_{22}, \mu_{12}, \nu_{22}) \\
\Theta =& \Theta(\mu_{11}, \mu_{22}, \mu_{12}, \nu_{12}) \\
\Gamma_{11} =& \Gamma_{11}(\mu_{11}, \mu_{22}, \mu_{12}, \nu_{11}) \\
\Gamma_{22} =& \Gamma_{22}(\mu_{11}, \mu_{22}, \mu_{12}, \nu_{22}) \\
\Gamma_{12} =& \Gamma_{12}(\mu_{11}, \mu_{22}, \mu_{12}, \nu_{12})
\end{aligned} \tag{7.1}$$

GRAAFIKUD TEISES FAILIS

Arvestame interaktsioonikonstantide märgiga $\mu > 0$ ja $\nu < 0$ ja kirjutame efektiivsete interaktsioonikonstantide positiivsed ja negatiivsed liikmed eraldi välja

$$
\begin{aligned}
\Phi^{+} =& 1+\tilde{\eta}\mu_{11}+\tilde{\eta}\mu_{22}+\tilde{\eta}^{2}\mu_{11}\mu_{22} \\
\Phi^{-} =& -\tilde{\eta}^{2}\mu_{12}^{2} \\
\Theta_{11}^{+} =& \tilde{\eta}\mu_{12}^{2}-\nu_{11}-\tilde{\eta}\mu_{11}\nu_{11}-\tilde{\eta}\mu_{22}\nu_{11}-\tilde{\eta}^{2}\mu_{11}\mu_{22}\nu_{11} \\
\Theta_{11}^{-} =& -\mu_{11}-\tilde{\eta}\mu_{11}\mu_{22}+\tilde{\eta}^{2}\mu_{12}^{2}\nu_{11} \\
\Theta_{22}^{+} =& \tilde{\eta}\mu_{12}^{2}-\nu_{22}-\tilde{\eta}\mu_{11}\nu_{22}-\tilde{\eta}\mu_{22}\nu_{22}-\tilde{\eta}^{2}\mu_{11}\mu_{22}\nu_{22} \\
\Theta_{22}^{-} =& -\mu_{22}-\tilde{\eta}\mu_{11}\mu_{22}+\tilde{\eta}^{2}\mu_{12}^{2}\nu_{22} \\
\Theta^{+} =& -\nu_{12}-\tilde{\eta}\mu_{11}\nu_{12}-\tilde{\eta}\mu_{22}\nu_{12}-\tilde{\eta}^{2}\mu_{11}\mu_{22}\nu_{12} \\
\Theta^{-} =& -\mu_{12}+\tilde{\eta}^{2}\mu_{12}^{2}\nu_{12}
\end{aligned}
\tag{7.2}
$$

Avaldame, mis punktis toimub efektiivsetel interaktsioonikonstantidel märgimuutus

$$
\begin{aligned}
\Phi^{+} =& \Phi^{-} \\
1+\tilde{\eta}\mu_{11}+\tilde{\eta}\mu_{22}+\tilde{\eta}^{2}\mu_{11}\mu_{22} =& -\tilde{\eta}^{2}\mu_{12}^{2}
\end{aligned}
\tag{7.3}
$$

$$\Theta_{11}^{+} = \Theta_{11}^{-} \tag{7.4}$$

$$\Theta_{22}^{+} = \Theta_{22}^{-} \tag{7.5}$$

$$\Theta^{+} = \Theta^{-} \tag{7.6}$$

8 Faasisiirde temperatuur

Faasisiirde temperatuuri lähedal saame ligikaudu integreerida integraali (5.3) arvestades, et $\hbar\omega_D \gg 2k_BT_c$ ja $\Delta_\alpha(T=T_c)=0$

$$\eta(T_c,0)=\int_0^{\hbar\omega_D}\tilde{\varepsilon}_\alpha^{-1}\tanh\left(\frac{\tilde{\varepsilon}_\alpha}{2k_BT_c}\right)d\tilde{\varepsilon}_\alpha \tag{8.1}$$

Teeme muutujavahetuse

$$x=\frac{\tilde{\varepsilon}_\alpha}{2k_BT_c} \tag{8.2}$$

$$
\begin{aligned}
\eta(T_c,0) &= \int_0^{\frac{\hbar\omega_D}{2k_BT_c}} \frac{1}{x}\tanh(x)dx = \int_0^{\frac{\hbar\omega_D}{2k_BT_c}} \tanh(x)d[\ln(x)] = \\
&= \ln(x)\tanh(x)\Big|_0^{\frac{\hbar\omega_D}{2k_BT_c}} - \int_0^{\frac{\hbar\omega_D}{2k_BT_c}} \ln(x)d[\tanh(x)] = \\
&= \ln\left(\frac{\hbar\omega_D}{2k_BT_c}\right)\underbrace{\tanh\left(\frac{\hbar\omega_D}{2k_BT_c}\right)}_{\approx 1} - \int_0^{\frac{\hbar\omega_D}{2k_BT_c}} \frac{\ln(x)}{\cosh^2(x)}dx
\end{aligned}
\tag{8.3}
$$

Kui $\hbar\omega_D \gg 2k_BT_c \Rightarrow \frac{\hbar\omega_D}{2k_BT_c} \to \infty$

$$
-\int_0^{\infty} \frac{\ln(x)}{\cosh^2(x)}dx = \gamma - \ln\left(\frac{\pi}{4}\right) = \ln\left(e^{\gamma}\right) - \ln\left(\frac{\pi}{4}\right) = \ln\left(\frac{4e^{\gamma}}{\pi}\right) \tag{8.4}
$$

$$
\eta(T_c,0) = \ln\left(\frac{\hbar\omega_D}{2k_BT_c}\right) + \ln\left(\frac{4e^{\gamma}}{\pi}\right) = \ln\left(\frac{4\hbar\omega_D e^{\gamma}}{k_BT_c}\right) \equiv \eta,\ \gamma = 0.577\ldots \tag{8.5}
$$

Teisendame võrrandisüsteemi (6.26)

$$
\begin{cases}
\Delta_1\left(\eta\Gamma_{11} - 1\right) + \Delta_2\eta\Gamma_{12} = 0 \\
\Delta_1\eta\Gamma_{21} + \Delta_2\left(\eta\Gamma_{22} - 1\right) = 0
\end{cases}
\tag{8.6}
$$

Võrrandisüsteemil (6.26) leiduvad mittetriviaalsed lahendid, kui

$$
\begin{vmatrix}
\eta\Gamma_{11} - 1 & \eta\Gamma_{12} \\
\eta\Gamma_{21} & \eta\Gamma_{22} - 1
\end{vmatrix} = 0 \tag{8.7}
$$

$$
\begin{aligned}
\left(\eta\Gamma_{11} - 1\right)\left(\eta\Gamma_{22} - 1\right) - \eta^2\Gamma_{21}\Gamma_{12} &= 0 \\
\eta^2\Gamma_{11}\Gamma_{22} - \eta\Gamma_{11} - \eta\Gamma_{22} + 1 - \eta^2\Gamma_{12}\Gamma_{21} &= 0 \\
\eta^2\left(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}\right) - \eta\left(\Gamma_{11} + \Gamma_{22}\right) + 1 &= 0
\end{aligned}
\tag{8.8}
$$

Lahendame η suhtes ruutvõrrandi

$$
\eta^{\pm} = \frac{\Gamma_{11} + \Gamma_{22} \pm \sqrt{\left(\Gamma_{11} - \Gamma_{22}\right)^2 + 4\Gamma_{12}\Gamma_{21}}}{2\left(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}\right)} = \frac{2}{\Gamma_{11} + \Gamma_{22} \mp \sqrt{\left(\Gamma_{11} - \Gamma_{22}\right)^2 + 4\Gamma_{12}\Gamma_{21}}} \tag{8.9}
$$

Avaldame võrrandist (8.5) T_c

$$
T_c^{\pm} = \frac{2\hbar\omega_D e^{\gamma}}{k_B\pi}\exp\left(-\eta^{\pm}\right) \tag{8.10}
$$

9 Ülijuhtivuspilude faaside vahe

Teeme võrrandisüsteemis (8.6) asenduse $\Delta_{\alpha\pm} = |\Delta_{\alpha\pm}| e^{\phi_{\alpha\pm}}$

$$\begin{cases} |\Delta_{1\pm}| e^{\phi_{1\pm}} (\eta_\pm \Gamma_{11} - 1) + |\Delta_{2\pm}| e^{\phi_{2\pm}} \eta_\pm \Gamma_{12} = 0 \big| : |\Delta_{2\pm}| e^{\phi_{2\pm}} \\ |\Delta_{1\pm}| e^{\phi_{1\pm}} \eta_\pm \Gamma_{21} + |\Delta_{2\pm}| e^{\phi_{2\pm}} (\eta_\pm \Gamma_{22} - 1) = 0 \big| : |\Delta_{2\pm}| e^{\phi_{2\pm}} \end{cases} \tag{9.1}$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} (\eta_\pm \Gamma_{11} - 1) + \eta_\pm \Gamma_{12} = 0 \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} \eta_\pm \Gamma_{21} + \eta_\pm \Gamma_{22} - 1 = 0 \end{cases} \tag{9.2}$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} (\eta_\pm \Gamma_{11} - 1) = -\eta_\pm \Gamma_{12} \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} \eta_\pm \Gamma_{21} = -(\eta_\pm \Gamma_{22} - 1) \end{cases} \tag{9.3}$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} = -\frac{\eta_\pm \Gamma_{12}}{\eta_\pm \Gamma_{11} - 1} \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} = -\frac{\eta_\pm \Gamma_{22} - 1}{\eta_\pm \Gamma_{21}} \end{cases} \tag{9.4}$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} \frac{\eta_\pm \Gamma_{11} - 1}{\eta_\pm \Gamma_{12}} = -e^{\phi_{1\pm} - \phi_{2\pm}} = \mp 1 \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} \frac{\eta_\pm \Gamma_{21}}{\eta_\pm \Gamma_{22} - 1} = -e^{\phi_{1\pm} - \phi_{2\pm}} = \mp 1 \end{cases} \tag{9.5}$$

Kuna võrrandi vasakul pool olev liige on reaalne suurus, siis saab faaside vahe olla, kas $2n\pi$ või $(2n+1)\pi$, kus $n \in \mathbb{Z}$.

10 Nulletemperatuurne ülijuhtvus

Juhul, kui temperatuur on 0 K, saame lihtsustada integraali (5.3)

$$\lim_{T\to 0}\tanh\frac{E(\Delta_\alpha)}{2k_BT}=1 \tag{10.1}$$

Seega

$$\begin{aligned}\eta_\alpha(0,\Delta_\alpha)=&\int_0^{\hbar\omega_D}\frac{d\tilde{\varepsilon}_\alpha}{\sqrt{\tilde{\varepsilon}_\alpha^2+|\Delta_\alpha|^2}}=\ln\left(\tilde{\varepsilon}_\alpha+\sqrt{\tilde{\varepsilon}_\alpha^2+|\Delta_\alpha|^2}\right)\Bigg|_0^{\hbar\omega_D}=\\ =&\ln\left(\hbar\omega_D+\sqrt{(\hbar\omega_D)^2+|\Delta_\alpha|^2}\right)-2\ln|\Delta_\alpha|^2=\\ =&\ln\left(\frac{\hbar\omega_D+\sqrt{(\hbar\omega_D)^2+|\Delta_\alpha|^2}}{|\Delta_\alpha|^2}\right)=\\ =&\ln\left(\frac{\hbar\omega_D+\sqrt{(\hbar\omega_D)^2\left(1+\frac{|\Delta_\alpha|^2}{(\hbar\omega_D)^2}\right)}}{|\Delta_\alpha|^2}\right)\overset{(\hbar\omega_D)^2\gg|\Delta_\alpha|^2}{\approx}\ln\left(\frac{2\hbar\omega_D}{|\Delta_\alpha|^2}\right)\end{aligned}\tag{10.2}$$

Asendame saadud tulemuse võrrandisüsteemi (6.26)

$$\begin{cases}\Delta_1=\Delta_1\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{11}+\Delta_2\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{12}\\ \Delta_2=\Delta_1\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{21}+\Delta_2\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{22}\end{cases}\tag{10.3}$$

$$\Delta_1=|\Delta_1|e^{i\phi_1},\ \Delta_2=|\Delta_2|e^{i\phi_2}\Rightarrow\kappa\equiv\frac{\Delta_1}{\Delta_2}=\frac{|\Delta_1|}{|\Delta_2|}e^{i(\phi_1-\phi_2)}\tag{10.4}$$

$$\begin{cases}|\Delta_1|e^{i\phi_1}=|\Delta_1|e^{i\phi_1}\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{11}+|\Delta_2|e^{i\phi_2}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{12}\\ |\Delta_2|e^{i\phi_2}=|\Delta_1|e^{i\phi_1}\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{21}+|\Delta_2|e^{i\phi_2}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{22}\end{cases}\tag{10.5}$$

$$\begin{cases}1=\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{11}+\kappa^{-1}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{12}\\ 1=\kappa\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{21}+\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{22}\end{cases}\tag{10.6}$$

Avaldame esimesest võrrandist $\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)$

$$\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)=\left(1-\kappa^{-1}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{12}\right)\Gamma_{11}^{-1}\tag{10.7}$$

Asendame teise võrrandisse

$$\begin{aligned}1 &= \left(1-\kappa^{-1}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{12}\right)\frac{\kappa\Gamma_{21}}{\Gamma_{11}}+\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{22}= \\ &= \frac{\kappa\Gamma_{21}}{\Gamma_{11}}-\frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{11}}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)+\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\Gamma_{22}= \\ &= \frac{\kappa\Gamma_{21}}{\Gamma_{11}}+\left[\Gamma_{22}-\frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{11}}\right]\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)\end{aligned} \tag{10.8}$$

$$\Rightarrow \frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{11}}\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)=1-\frac{\kappa\Gamma_{21}}{\Gamma_{11}}=\frac{\Gamma_{11}-\kappa\Gamma_{21}}{\Gamma_{11}} \tag{10.9}$$

$$\Rightarrow \ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)=\frac{\Gamma_{11}-\kappa\Gamma_{21}}{\Gamma_{11}}\frac{\Gamma_{11}}{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}=\frac{\Gamma_{11}-\kappa\Gamma_{21}}{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}} \tag{10.10}$$

$$\Rightarrow \frac{2\hbar\omega_D}{|\Delta_2|^2}=\exp\frac{\Gamma_{11}-\kappa\Gamma_{21}}{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}\Rightarrow |\Delta_2|^2=2\hbar\omega_D\exp\frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{11}-\kappa\Gamma_{21}} \tag{10.11}$$

Avaldame teisest võrrandist $\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)$

$$\ln\left(\frac{2\hbar\omega_D}{|\Delta_2|^2}\right)=\left(1-\kappa\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{21}\right)\Gamma_{22}^{-1} \tag{10.12}$$

Asendame esimesse võrrandisse

$$\begin{aligned}1 &= \ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{11}+\left(1-\kappa\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{21}\right)\frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}}= \\ &= \ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\Gamma_{11}+\frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}}-\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{22}}= \\ &= \frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}}+\left[\Gamma_{11}-\frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{22}}\right]\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)\end{aligned} \tag{10.13}$$

$$\Rightarrow \frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{22}}\ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)=1-\frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}}=\frac{\Gamma_{22}-\kappa^{-1}\Gamma_{12}}{\Gamma_{22}} \tag{10.14}$$

$$\Rightarrow \ln\left(\frac{2\hbar\omega_D}{|\Delta_1|^2}\right)=\frac{\Gamma_{22}-\kappa^{-1}\Gamma_{12}}{\Gamma_{22}}\frac{\Gamma_{22}}{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}=\frac{\Gamma_{22}-\kappa^{-1}\Gamma_{12}}{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}} \tag{10.15}$$

$$\Rightarrow \frac{2\hbar\omega_D}{|\Delta_1|^2}=\exp\frac{\Gamma_{22}-\kappa^{-1}\Gamma_{12}}{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}\Rightarrow |\Delta_1|^2=2\hbar\omega_D\exp\frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{22}-\kappa^{-1}\Gamma_{12}} \tag{10.16}$$

Kokku saame

$$\begin{cases} |\Delta_1|^2 = 2\hbar\omega_D \exp \frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{22}-\kappa^{-1}\Gamma_{12}} \\ |\Delta_2|^2 = 2\hbar\omega_D \exp \frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{11}-\kappa\Gamma_{21}} \end{cases} \tag{10.17}$$