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1 Hamiltoniaan

$$\begin{aligned}
\hat{H} = & \sum_{\alpha} \sum_{\mathbf{k}} \sum_s \tilde{\varepsilon}_{\alpha}(\mathbf{k}) a_{\alpha\mathbf{k}s}^{\dagger} a_{\alpha\mathbf{k}s} + \\
& + \sum_{\alpha} \sum_{\mathbf{k}, \mathbf{k}'} V_{\alpha\alpha}(\mathbf{k}, \mathbf{k}') a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha\mathbf{k}'\uparrow} + \\
& + \sum_{\alpha} \sum_{\mathbf{k}, \mathbf{k}'} U_{\alpha\alpha}(\mathbf{k}, \mathbf{k}') a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha\mathbf{k}'\uparrow} + \\
& + \sum_{\alpha} \sum_{\mathbf{k}, \mathbf{k}'} V_{\alpha\alpha'}(\mathbf{k}, \mathbf{k}') a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} a_{\alpha'-\mathbf{k}'\downarrow} a_{\alpha'\mathbf{k}'\uparrow} + \\
& + \sum_{\alpha} \sum_{\mathbf{k}, \mathbf{k}'} U_{\alpha\alpha'}(\mathbf{k}, \mathbf{k}') a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} a_{\alpha'-\mathbf{k}'\downarrow} a_{\alpha'\mathbf{k}'\uparrow}
\end{aligned} \tag{1.1}$$

Siin a^{\dagger} ja a on ülijuhtivus elektroni tekke ja kadumise operaatorid, $\alpha = 1, 2$ on tsooni indeks, \mathbf{k} on elektroni lainevektor, $s = \uparrow, \downarrow$ on spinni indeks, $\tilde{\varepsilon}_{\alpha}(\mathbf{k}) = \varepsilon_{\alpha}(\mathbf{k}) - \mu$ on elektroni energia tsoonis α , μ on keemiline potentsiaal, $V_{\alpha\alpha'}(\mathbf{k}, \mathbf{k}')$ on tsooniseesmise ($\alpha = \alpha'$) või tsoonidevahelise ($\alpha \neq \alpha'$) efektiivse tõmbeinteraktsiooni konstant ja $U_{\alpha\alpha'}(\mathbf{k}, \mathbf{k}')$ on tsooniseesmise ($\alpha = \alpha'$) või tsoonidevahelise ($\alpha \neq \alpha'$) tõukeinteraktsiooni konstant. Efektiivne elektronidevaheline tõmbeinteraktsioon on indutseeritud elektron-foonon interaktsiooni poolt.

2 Keskmise välja lähendus

Kasutame nüüd keskmise välja lähendust, mille käigus defineerime ka ülijuhtivuspilud, mis kirjeldavad ülijuhtivat faasi ja faasisiiret. Selleks teeme järgmise asenduse

$$a^{\dagger} a^{\dagger} a a \rightarrow \langle a^{\dagger} a^{\dagger} \rangle a a + a^{\dagger} a^{\dagger} \langle a a \rangle - \langle a^{\dagger} a^{\dagger} \rangle \langle a a \rangle \tag{2.1}$$

$$\begin{aligned}
\hat{H} = & \sum_{\alpha} \sum_{\mathbf{k}} \sum_s \tilde{\epsilon}_{\alpha}(\mathbf{k}) a_{\alpha \mathbf{k} s}^{\dagger} a_{\alpha \mathbf{k} s} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} V_{11}(\mathbf{k}, \mathbf{k}') a_{1 \mathbf{k} \uparrow}^{\dagger} a_{1 -\mathbf{k} \downarrow}^{\dagger} a_{1 -\mathbf{k}' \downarrow} a_{1 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} V_{22}(\mathbf{k}, \mathbf{k}') a_{2 \mathbf{k} \uparrow}^{\dagger} a_{2 -\mathbf{k} \downarrow}^{\dagger} a_{2 -\mathbf{k}' \downarrow} a_{2 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} U_{11}(\mathbf{k}, \mathbf{k}') a_{1 \mathbf{k} \uparrow}^{\dagger} a_{1 -\mathbf{k} \downarrow}^{\dagger} a_{1 -\mathbf{k}' \downarrow} a_{1 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} U_{22}(\mathbf{k}, \mathbf{k}') a_{2 \mathbf{k} \uparrow}^{\dagger} a_{2 -\mathbf{k} \downarrow}^{\dagger} a_{2 -\mathbf{k}' \downarrow} a_{2 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} V_{12}(\mathbf{k}, \mathbf{k}') a_{1 \mathbf{k} \uparrow}^{\dagger} a_{1 -\mathbf{k} \downarrow}^{\dagger} a_{2 -\mathbf{k}' \downarrow} a_{2 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} U_{12}(\mathbf{k}, \mathbf{k}') a_{1 \mathbf{k} \uparrow}^{\dagger} a_{1 -\mathbf{k} \downarrow}^{\dagger} a_{2 -\mathbf{k}' \downarrow} a_{2 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} V_{21}(\mathbf{k}, \mathbf{k}') a_{2 \mathbf{k} \uparrow}^{\dagger} a_{2 -\mathbf{k} \downarrow}^{\dagger} a_{1 -\mathbf{k}' \downarrow} a_{1 \mathbf{k}' \uparrow} + \\
& + \sum_{\mathbf{k}, \mathbf{k}'} U_{21}(\mathbf{k}, \mathbf{k}') a_{2 \mathbf{k} \uparrow}^{\dagger} a_{2 -\mathbf{k} \downarrow}^{\dagger} a_{1 -\mathbf{k}' \downarrow} a_{1 \mathbf{k}' \uparrow}
\end{aligned} \tag{2.2}$$

Seega

$$\begin{aligned}
\hat{H}_{mf} &= \sum_{\alpha} \sum_{\mathbf{k}} \sum_s \tilde{\epsilon}_{\alpha}(\mathbf{k}) a_{\alpha\mathbf{k}s}^{\dagger} a_{\alpha\mathbf{k}s} + \\
&+ \sum_{\mathbf{k}, \mathbf{k}'} \{ \Delta_{1\mathbf{k}} a_{1\mathbf{k}\uparrow}^{\dagger} a_{1-\mathbf{k}\downarrow}^{\dagger} + \Delta_{2\mathbf{k}} a_{2\mathbf{k}\uparrow}^{\dagger} a_{2-\mathbf{k}\downarrow}^{\dagger} + \\
&+ \Delta_{1\mathbf{k}}^* a_{1-\mathbf{k}'\downarrow} a_{1\mathbf{k}'\uparrow} + \Delta_{2\mathbf{k}}^* a_{2-\mathbf{k}'\downarrow} a_{2\mathbf{k}'\uparrow} - \\
&- \Delta_{1\mathbf{k}} \langle a_{1\mathbf{k}\uparrow}^{\dagger} a_{1-\mathbf{k}\downarrow}^{\dagger} \rangle - \Delta_{2\mathbf{k}} \langle a_{2\mathbf{k}\uparrow}^{\dagger} a_{2-\mathbf{k}\downarrow}^{\dagger} \rangle \} = \\
&= \sum_{\alpha} \sum_{\mathbf{k}} \sum_s \tilde{\epsilon}_{\alpha}(\mathbf{k}) a_{\alpha\mathbf{k}s}^{\dagger} a_{\alpha\mathbf{k}s} + \\
&+ \sum_{\alpha} \{ \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} + \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha\mathbf{k}'\uparrow} - \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} \langle a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} \rangle \}
\end{aligned} \tag{2.6}$$

3 Bogoljubovi-Valantini teisendus

$$\begin{cases} a_{\alpha\mathbf{k}\uparrow} = u_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow} + v_{\alpha\mathbf{k}}^* \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \\ a_{\alpha-\mathbf{k}\downarrow} = u_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow} - v_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \end{cases} \tag{3.1}$$

$$|u_{\mathbf{k}}|^2 + |v_{\mathbf{k}}|^2 = 1; \quad u_{-\mathbf{k}} = u_{\mathbf{k}}; \quad v_{-\mathbf{k}} = -v_{\mathbf{k}} \tag{3.2}$$

Fermionide statistikast tulenevad seosed, kus $[A, B]_{+}$ on operaatorite A ja B antikommutatsioon.

$$[\alpha_{\mathbf{k}s}, \alpha_{\mathbf{k}'s'}^{\dagger}]_{+} = \delta_{\mathbf{k}\mathbf{k}'} \delta_{ss'}, \quad [\alpha_{\mathbf{k}s}, \alpha_{\mathbf{k}'s'}]_{+} = 0, \quad [\alpha_{\mathbf{k}s}^{\dagger}, \alpha_{\mathbf{k}'s'}^{\dagger}]_{+} = 0 \tag{3.3}$$

Teisendame liikmed

I liige

$$\begin{aligned}
& \sum_{\alpha} \sum_{\mathbf{k}} \sum_s \tilde{\varepsilon}_{\alpha}(\mathbf{k}) a_{\alpha\mathbf{k}s}^{\dagger} a_{\alpha\mathbf{k}s} = \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\varepsilon}_{\alpha}(\mathbf{k}) \left[a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha\mathbf{k}\uparrow} + a_{\alpha-\mathbf{k}\downarrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow} \right] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\varepsilon}_{\alpha}(\mathbf{k}) \left[\left(u_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} + v_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow} \right) \left(u_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow} + v_{\alpha\mathbf{k}}^* \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \right) + \right. \\
& + \left. \left(u_{\alpha\mathbf{k}}^* \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - v_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow} \right) \left(u_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow} - v_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \right) \right] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\varepsilon}_{\alpha}(\mathbf{k}) \left[\underbrace{u_{\alpha\mathbf{k}}^* u_{\alpha\mathbf{k}}}_{|u_{\alpha\mathbf{k}}|^2} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} + \right. \\
& + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} + \underbrace{v_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}^*}_{|v_{\alpha\mathbf{k}}|^2} \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} + \\
& + \underbrace{u_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^*}_{|u_{\alpha\mathbf{k}}|^2} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} - u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}}^* \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} - \\
& - v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow} \alpha_{\alpha-\mathbf{k}\downarrow} + \underbrace{v_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}^*}_{|v_{\alpha\mathbf{k}}|^2} \alpha_{\alpha\mathbf{k}\uparrow} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \left. \right] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\varepsilon}_{\alpha}(\mathbf{k}) \left[|u_{\alpha\mathbf{k}}|^2 \left(\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} \right) + \right. \\
& + |v_{\alpha\mathbf{k}}|^2 \left(\underbrace{\alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger}}_{1 - \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}} + \underbrace{\alpha_{\alpha\mathbf{k}\uparrow} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger}}_{1 - \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow}} \right) + \\
& + u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}}^* \left(\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - \underbrace{\alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger}}_{-\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger}} \right) + \\
& + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} \left(\alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} - \underbrace{\alpha_{\alpha\mathbf{k}\uparrow} \alpha_{\alpha-\mathbf{k}\downarrow}}_{-\alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow}} \right) \left. \right] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\varepsilon}_{\alpha}(\mathbf{k}) \left[|u_{\alpha\mathbf{k}}|^2 \left(\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} \right) + \right. \\
& + |v_{\alpha\mathbf{k}}|^2 (1 - \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} + 1 - \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow}) + \\
& + u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}}^* (\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} + \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger}) + \\
& + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} (\alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} + \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow}) \left. \right] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\varepsilon}_{\alpha}(\mathbf{k}) [2 |v_{\alpha\mathbf{k}}|^2 + (|u_{\alpha\mathbf{k}}|^2 - |v_{\alpha\mathbf{k}}|^2) \left(\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} \right) + \\
& + 2 u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} + 2 v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow}]
\end{aligned} \tag{3.4}$$

II liige

$$\begin{aligned}
& \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} = \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} [(u_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} + v_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow}) (u_{\alpha\mathbf{k}}^* \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - v_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow})] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} \underbrace{[u_{\alpha\mathbf{k}}^* u_{\alpha\mathbf{k}}^* \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} +}_{(u_{\alpha\mathbf{k}}^*)^2} \\
& + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* \underbrace{\alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger}}_{1 - \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}} - \underbrace{v_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}}_{(v_{\alpha\mathbf{k}})^2} \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow}] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} [(u_{\alpha\mathbf{k}}^*)^2 \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + \\
& + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* - v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} - (v_{\alpha\mathbf{k}})^2 \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow}] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} [(u_{\alpha\mathbf{k}}^*)^2 \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - (v_{\alpha\mathbf{k}})^2 \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} - \\
& - v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* (\alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} + \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow}) + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^*]
\end{aligned} \tag{3.5}$$

III liige

$$\begin{aligned}
& \sum_{\alpha} \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha\mathbf{k}'\uparrow} = \\
& = \sum_{\alpha} \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* (u_{\alpha\mathbf{k}'} \alpha_{\alpha-\mathbf{k}'\downarrow} - v_{\alpha\mathbf{k}'}^* \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger}) (u_{\alpha\mathbf{k}'} \alpha_{\alpha\mathbf{k}'\uparrow} + v_{\alpha\mathbf{k}'}^* \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger}) = \\
& = \sum_{\alpha} \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* [u_{\alpha\mathbf{k}'} u_{\alpha\mathbf{k}'} \alpha_{\alpha-\mathbf{k}'\downarrow} \alpha_{\alpha\mathbf{k}'\uparrow} + u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^* \underbrace{\alpha_{\alpha-\mathbf{k}'\downarrow} \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger}}_{1 - \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow}} - \\
& - v_{\alpha\mathbf{k}'}^* u_{\alpha\mathbf{k}'} \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}'\uparrow} - v_{\alpha\mathbf{k}'}^* v_{\alpha\mathbf{k}'}^* \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger}] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* [(u_{\alpha\mathbf{k}'}^*)^2 \alpha_{\alpha-\mathbf{k}'\downarrow} \alpha_{\alpha\mathbf{k}'\uparrow} - (v_{\alpha\mathbf{k}'}^*)^2 \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} + \\
& + u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^* - u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^* \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow} - v_{\alpha\mathbf{k}'}^* u_{\alpha\mathbf{k}'} \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}'\uparrow}] = \\
& = \sum_{\alpha} \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* [(u_{\alpha\mathbf{k}'}^*)^2 \alpha_{\alpha-\mathbf{k}'\downarrow} \alpha_{\alpha\mathbf{k}'\uparrow} - (v_{\alpha\mathbf{k}'}^*)^2 \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} - \\
& - u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^* (\alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow} + \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}'\uparrow}) + u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^*]
\end{aligned} \tag{3.6}$$

IV liige jääb hetkel samaks, hiljem arvutame keskväärtuse.

Kokku saame

$$\begin{aligned}
\hat{H}_{mf} = & \sum_{\alpha} \sum_{\mathbf{k}} \tilde{\epsilon}_{\alpha}(\mathbf{k}) \left[2|v_{\alpha\mathbf{k}}|^2 + (|u_{\alpha\mathbf{k}}|^2 - |v_{\alpha\mathbf{k}}|^2) \left(\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} \right) \right] + \\
& + 2u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} + 2v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} + \\
& + \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} [(u_{\alpha\mathbf{k}}^*)^2 \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} - (v_{\alpha\mathbf{k}})^2 \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} - \\
& - v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* \left(\alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} + \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} \right) + v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^*] + \\
& + \sum_{\alpha} \sum_{\mathbf{k}'} \Delta_{\alpha\mathbf{k}'}^* [(u_{\alpha\mathbf{k}'}^*)^2 \alpha_{\alpha-\mathbf{k}'\downarrow} \alpha_{\alpha\mathbf{k}'\uparrow} - (v_{\alpha\mathbf{k}'}^*)^2 \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} - \\
& - u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^* \left(\alpha_{\alpha-\mathbf{k}'\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}'\downarrow} + \alpha_{\alpha\mathbf{k}'\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}'\uparrow} \right) + u_{\alpha\mathbf{k}'} v_{\alpha\mathbf{k}'}^*] \\
& - \sum_{\alpha} \sum_{\mathbf{k}} \Delta_{\alpha\mathbf{k}} \left\langle a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} \right\rangle = \\
\stackrel{\mathbf{k}'=\mathbf{k}}{=} & \sum_{\alpha} \sum_{\mathbf{k}} \left\{ 2\tilde{\epsilon}_{\alpha}(\mathbf{k}) |v_{\alpha\mathbf{k}}|^2 + \Delta_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* + \Delta_{\alpha\mathbf{k}}^* u_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}^* - \Delta_{\alpha\mathbf{k}} \left\langle a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} \right\rangle + \right. \\
& + \left[\tilde{\epsilon}_{\alpha}(\mathbf{k}) (|u_{\alpha\mathbf{k}}|^2 - |v_{\alpha\mathbf{k}}|^2) - \Delta_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* - \Delta_{\alpha\mathbf{k}}^* u_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}^* \right] \times \\
& \times \left(\alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} + \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow} \right) + \\
& + \left(2\tilde{\epsilon}_{\alpha}(\mathbf{k}) u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}} + \Delta_{\alpha\mathbf{k}} (u_{\alpha\mathbf{k}}^*)^2 - \Delta_{\alpha\mathbf{k}}^* (v_{\alpha\mathbf{k}}^*)^2 \right) \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha-\mathbf{k}\downarrow}^{\dagger} + \\
& + \left. \left(2\tilde{\epsilon}_{\alpha}(\mathbf{k}) v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} - \Delta_{\alpha\mathbf{k}} (v_{\alpha\mathbf{k}})^2 + \Delta_{\alpha\mathbf{k}}^* (u_{\alpha\mathbf{k}})^2 \right) \alpha_{\alpha-\mathbf{k}\downarrow} \alpha_{\alpha\mathbf{k}\uparrow} \right\}
\end{aligned} \tag{3.7}$$

Liikmed operaatoritega $\alpha\alpha$ ja $\alpha^{\dagger}\alpha^{\dagger}$ ei ole diagonaalil, nõuame, et kehtiks tingimused

$$\begin{aligned}
2\tilde{\epsilon}_{\alpha}(\mathbf{k}) v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}} - \Delta_{\alpha\mathbf{k}} (v_{\alpha\mathbf{k}})^2 + \Delta_{\alpha\mathbf{k}}^* (u_{\alpha\mathbf{k}})^2 &= 0 \\
2\tilde{\epsilon}_{\alpha}(\mathbf{k}) u_{\alpha\mathbf{k}}^* v_{\alpha\mathbf{k}} + \Delta_{\alpha\mathbf{k}} (u_{\alpha\mathbf{k}}^*)^2 - \Delta_{\alpha\mathbf{k}}^* (v_{\alpha\mathbf{k}}^*)^2 &= 0
\end{aligned} \tag{3.8}$$

Tähistame

$$\begin{aligned}
E_0 &= \sum_{\alpha} \sum_{\mathbf{k}} \left\{ 2\tilde{\epsilon}_{\alpha}(\mathbf{k}) |v_{\alpha\mathbf{k}}|^2 + \Delta_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* + \Delta_{\alpha\mathbf{k}}^* u_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}^* - \Delta_{\alpha\mathbf{k}} \left\langle a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} \right\rangle \right\} \\
E_{\alpha}(\mathbf{k}) &= \tilde{\epsilon}_{\alpha}(\mathbf{k}) (|u_{\alpha\mathbf{k}}|^2 - |v_{\alpha\mathbf{k}}|^2) - \Delta_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}} u_{\alpha\mathbf{k}}^* - \Delta_{\alpha\mathbf{k}}^* u_{\alpha\mathbf{k}} v_{\alpha\mathbf{k}}^*
\end{aligned} \tag{3.9}$$

Seega

$$H_{mf} = E_0 + \sum_{\alpha} \sum_{\mathbf{k}} \sum_s E_{\alpha}(\mathbf{k}) \alpha_{\alpha\mathbf{k}s}^{\dagger} \alpha_{\alpha\mathbf{k}s} \tag{3.10}$$

3.1 Bogoljubov-Valatini teisenduse kordajad

Kkasutame seoseid (3.8), lihtsuse mõttes jätame indeksid ära

$$\begin{aligned}
2\varepsilon vu - \Delta v^2 + \Delta^* u^2 &= 0 \quad | \cdot \frac{\Delta}{u^2} \\
\frac{2\varepsilon vu\Delta}{u^2} - \frac{\Delta v^2 \Delta}{u^2} + \frac{\Delta^* u^2 \Delta}{u^2} &= 0 \quad | \cdot (-1) \\
\left(\frac{\Delta v}{u}\right)^2 - 2\varepsilon \frac{\Delta v}{u} - |\Delta|^2 &= 0 \\
\Rightarrow \Delta \frac{v}{u} = \varepsilon \pm \sqrt{\varepsilon^2 + |\Delta|^2} \equiv E \pm \varepsilon \Rightarrow \frac{v}{u} &= \frac{\varepsilon - \sqrt{\varepsilon^2 + |\Delta|^2}}{\Delta}
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
2\varepsilon u^* v^* + \Delta (u^*)^2 - \Delta^* (v^*)^2 &= 0 \quad | \cdot \frac{\Delta^*}{(u^*)^2} \\
2\varepsilon u^* v^* \frac{\Delta^*}{(u^*)^2} + \Delta (u^*)^2 \frac{\Delta^*}{(u^*)^2} - \Delta^* (v^*)^2 \frac{\Delta^*}{(u^*)^2} &= 0 \quad | \cdot (-1) \\
\left(\frac{\Delta^* v^*}{u^*}\right)^2 - 2\varepsilon \frac{\Delta^* v^*}{u^*} - |\Delta|^2 &= 0 \\
\Rightarrow \frac{\Delta^* v^*}{u^*} = \varepsilon \pm \sqrt{\varepsilon^2 + |\Delta|^2} \equiv \varepsilon \pm E \Rightarrow \frac{v^*}{u^*} &= \frac{\varepsilon - \sqrt{\varepsilon^2 + |\Delta|^2}}{\Delta^*}
\end{aligned} \tag{3.12}$$

Leiame $|u|^2$ ja $|v|^2$, kasutame seostest (3.2) esimest

$$|u|^2 + |v|^2 = 1 \Rightarrow |u|^2 = 1 - |v|^2 \tag{3.13}$$

$$\frac{1}{1 + \left|\frac{v}{u}\right|^2} = \frac{1}{\frac{|u|^2 + |v|^2}{|u|^2}} = \frac{|u|^2}{\underbrace{|u|^2 + |v|^2}_{=1}} = |u|^2 \tag{3.14}$$

Asendame seosed (3.11) (3.12) võrrandisse (3.14)

$$\frac{v}{u} \frac{v^*}{u^*} = \frac{\varepsilon - E}{\Delta} \frac{\varepsilon - E}{\Delta^*} = \frac{(\varepsilon - E)^2}{|\Delta|^2} = \left|\frac{v}{u}\right|^2 \tag{3.15}$$

$$\begin{aligned}
|u|^2 &= \left(1 + \left|\frac{v}{u}\right|^2\right)^{-1} = \left(1 + \left|\frac{\sqrt{\varepsilon^2 + |\Delta|^2} - \varepsilon}{\Delta}\right|^2\right)^{-1} = \\
&= \left(1 + \frac{(\varepsilon - E)^2}{|\Delta|^2}\right)^{-1} = \left(\frac{|\Delta|^2 + \varepsilon^2 - 2\varepsilon E + E^2}{E^2 - \varepsilon^2}\right)^{-1} = \left(\frac{2E^2 - 2\varepsilon E}{E^2 - \varepsilon^2}\right)^{-1} = \\
&= \left(\frac{2E^2(1 - \varepsilon/E)}{E^2(1 - (\varepsilon^2/E^2))}\right)^{-1} = \left(\frac{2}{1 + \varepsilon/E}\right)^{-1} = \frac{1}{2} \left(1 + \frac{\varepsilon}{E}\right)
\end{aligned} \tag{3.16}$$

$$|u|^2 + |v|^2 = 1 \Rightarrow |v|^2 = 1 - |u|^2 = 1 - \frac{1}{2} \left(1 + \frac{\varepsilon}{E}\right) = \frac{1}{2} \left(1 - \frac{\varepsilon}{E}\right) \quad (3.17)$$

Leiame vu^* ja uv^* . Seosest (3.11) ja (3.12) saame

$$\begin{aligned} \frac{v}{u} &= \frac{v u^*}{u u^*} = \frac{vu^*}{|u|^2} = \frac{\varepsilon - E}{\Delta} \Rightarrow vu^* = |u|^2 \frac{\varepsilon - E}{\Delta} \stackrel{(3.16)}{=} \frac{1}{2} \left(1 + \frac{\varepsilon}{E}\right) \frac{\varepsilon - E}{\Delta} = \\ &= \frac{1}{2} \frac{E + \varepsilon}{E} \frac{\varepsilon - E}{\Delta} = \frac{1}{2} \frac{\varepsilon^2 - E^2}{E\Delta} = \frac{1}{2} \frac{\varepsilon^2 - \varepsilon^2 - |\Delta|^2}{E\Delta} = -\frac{\Delta^*}{2E} \end{aligned} \quad (3.18)$$

$$\begin{aligned} \frac{v^*}{u^*} &= \frac{v^* u}{u^* u} = \frac{v^* u}{|u|^2} = \frac{\varepsilon - E}{\Delta^*} \\ \Rightarrow uv^* &= \frac{\varepsilon - E}{\Delta^*} |u|^2 = \frac{\varepsilon - E}{\Delta^*} \frac{1}{2} \left(1 + \frac{\varepsilon}{E}\right) = \frac{1}{2E\Delta^*} (\varepsilon^2 - E^2) = \frac{-|\Delta|^2}{2E\Delta^*} = -\frac{\Delta}{2E} \end{aligned} \quad (3.19)$$

Kokku saime

$$|u|^2 = \frac{1}{2} \left(1 + \frac{\varepsilon}{E}\right), \quad |v|^2 = \frac{1}{2} \left(1 - \frac{\varepsilon}{E}\right), \quad vu^* = -\frac{\Delta^*}{2E}, \quad uv^* = -\frac{\Delta}{2E} \quad (3.20)$$

3.2 Ergastusenergia ja põhiseisundi energia

Saadud seostega (3.20) saame arvutada E_0 -s oleva keskvaartuse, lihtsuse mõttes jätame alles ainult spinni indeksi, pidades meeles, et \downarrow spinnile vastab $-\mathbf{k}$ ja \uparrow spinnile vastab \mathbf{k}

$$\begin{aligned} \langle a_{\uparrow} a_{\downarrow} \rangle &= \langle (u\alpha_{\uparrow} + v^* \alpha_{\downarrow}^{\dagger})(u\alpha_{\downarrow} - v^* \alpha_{\uparrow}^{\dagger}) \rangle = \\ &= \langle u\alpha_{\uparrow} u\alpha_{\downarrow} - u\alpha_{\uparrow} v^* \alpha_{\downarrow}^{\dagger} + v^* \alpha_{\downarrow}^{\dagger} u\alpha_{\downarrow} - v^* \alpha_{\downarrow}^{\dagger} v^* \alpha_{\uparrow}^{\dagger} \rangle = \\ &= u^2 \underbrace{\langle \alpha_{\uparrow} \alpha_{\downarrow} \rangle}_0 - uv^* \underbrace{\langle \alpha_{\uparrow} \alpha_{\downarrow}^{\dagger} \rangle}_{1 - \langle \alpha_{\uparrow}^{\dagger} \alpha_{\uparrow} \rangle} + v^* u \underbrace{\langle \alpha_{\downarrow}^{\dagger} \alpha_{\downarrow} \rangle}_0 - (v^*)^2 \underbrace{\langle \alpha_{\downarrow}^{\dagger} \alpha_{\uparrow}^{\dagger} \rangle}_0 = \\ &= uv^* \left(-1 + \langle \alpha_{\uparrow}^{\dagger} \alpha_{\uparrow} \rangle + \langle \alpha_{\downarrow}^{\dagger} \alpha_{\downarrow} \rangle \right) = uv^* (2f(E) - 1) = \\ &= -\frac{\Delta}{2E} \left(\frac{2}{\exp(\beta E) + 1} - 1 \right) = -\frac{\Delta}{2E} \left(\frac{1 - \exp(\beta E)}{\exp(\beta E) + 1} \right) = \\ &= \frac{\Delta}{2E} \frac{\exp(\beta E) - 1}{\exp(\beta E) + 1} = \frac{\Delta}{2E} \tanh\left(\frac{\beta E}{2}\right) \end{aligned} \quad (3.21)$$

ehk

$$\langle a_{\alpha\mathbf{k}\uparrow} a_{\alpha-\mathbf{k}\downarrow} \rangle = \frac{1}{2} \Delta_{\alpha\mathbf{k}} \xi_{\alpha\mathbf{k}} \Rightarrow \langle a_{\alpha\mathbf{k}\uparrow}^{\dagger} a_{\alpha-\mathbf{k}\downarrow}^{\dagger} \rangle = \frac{1}{2} \Delta_{\alpha\mathbf{k}}^* \xi_{\alpha\mathbf{k}}, \quad (3.22)$$

kus

$$\xi_{\alpha\mathbf{k}} = E_{\alpha}^{-1}(\mathbf{k}) \tanh \frac{E_{\alpha}(\mathbf{k})}{2k_B T} \quad (3.23)$$

Analoogselt

$$\begin{aligned}
\langle a_{\downarrow} a_{\uparrow} \rangle &= \langle (u\alpha_{\downarrow} - v^* \alpha_{\uparrow}^{\dagger})(u\alpha_{\uparrow} + v^* \alpha_{\downarrow}^{\dagger}) \rangle = \\
&= \langle u\alpha_{\downarrow} u\alpha_{\uparrow} + u\alpha_{\downarrow} v^* \alpha_{\downarrow}^{\dagger} - v^* \alpha_{\uparrow}^{\dagger} u\alpha_{\uparrow} - v^* \alpha_{\uparrow}^{\dagger} v^* \alpha_{\downarrow}^{\dagger} \rangle = \\
&= u^2 \underbrace{\langle \alpha_{\downarrow} \alpha_{\downarrow} \rangle}_0 + uv^* \underbrace{\langle \alpha_{\alpha\mathbf{k}\uparrow} \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \rangle}_{1 - \langle \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} \rangle} - v^* u \langle \alpha_{\uparrow}^{\dagger} \alpha_{\uparrow} \rangle - (v^*)^2 \underbrace{\langle \alpha_{\uparrow}^{\dagger} \alpha_{\downarrow}^{\dagger} \rangle}_0 = \\
&= uv^* (1 - 2 \langle \alpha_{\alpha\mathbf{k}\uparrow}^{\dagger} \alpha_{\alpha\mathbf{k}\uparrow} \rangle) = uv^* (1 - 2f(E)) = \\
&= -\frac{\Delta}{2E} \left(1 - \frac{2}{\exp(\beta E) - 1} \right) = -\frac{\Delta}{2E} \left(\frac{\exp(\beta E) - 1 + 2}{\exp(\beta E) - 1} \right) = \\
&= -\frac{\Delta}{2E} \left(\frac{\exp(\beta E) + 1}{\exp(\beta E) - 1} \right) = -\frac{\Delta}{2E} \tanh \left(\frac{\beta E}{2} \right)
\end{aligned} \tag{3.24}$$

ehk

$$\langle a_{\alpha-\mathbf{k}\downarrow} a_{\alpha\mathbf{k}\uparrow} \rangle = -\frac{1}{2} \Delta_{\alpha\mathbf{k}} \xi_{\alpha\mathbf{k}} \tag{3.25}$$

Arvutame E_0 ja $E_{\alpha}(\mathbf{k})$

$$\begin{aligned}
E_0 &= 2\varepsilon|v|^2 + \Delta v u^* + \Delta^* u v^* + \frac{1}{2} |\Delta|^2 \xi = \\
&= 2\varepsilon \frac{1}{2} \left(1 - \frac{\varepsilon}{E} \right) + \Delta \left(-\frac{1}{2} \frac{\Delta^*}{E} \right) + \Delta^* \left(-\frac{1}{2} \frac{\Delta}{E} \right) + \frac{1}{2} |\Delta|^2 \xi = \\
&= \varepsilon - \frac{\varepsilon^2}{E} - \frac{1}{2} \frac{|\Delta|^2}{E} - \frac{1}{2} \frac{|\Delta|^2}{E} + \frac{1}{2} |\Delta|^2 \xi \\
&= \varepsilon - \frac{\varepsilon^2 + |\Delta|^2}{E} + \frac{1}{2} |\Delta|^2 \xi = \varepsilon - E + \frac{1}{2} |\Delta|^2 \xi
\end{aligned} \tag{3.26}$$

$$\begin{aligned}
E &= \varepsilon(|u|^2 - |v|^2) - \Delta v u^* - \Delta^* u v^* = \\
&= \varepsilon(2|u|^2 - 1) - \Delta v u^* - \Delta^* u v^* = \\
&= \varepsilon 2 \frac{1}{2} \left(1 + \frac{\varepsilon}{E} \right) - \varepsilon - \Delta \left(-\frac{1}{2} \frac{\Delta^*}{E} \right) - \Delta^* \frac{1}{2} \frac{\Delta}{E} = \\
&= \varepsilon + \frac{\varepsilon^2}{E} - \varepsilon + \frac{|\Delta|^2}{2E} - \frac{|\Delta|^2}{2E} = \frac{\varepsilon^2}{E} + \frac{|\Delta|^2}{E} = E \Rightarrow \varepsilon^2 + |\Delta|^2 = E^2
\end{aligned} \tag{3.27}$$

4 Interaktsioonikonstantide lähendused

Lähendame interaktsioonikonstandid energeetilistes piirkondades Heaviside'i funktsioonile

$$\begin{aligned}
 V_{\alpha\alpha}(\mathbf{k}, \mathbf{k}') &= V_{\alpha\alpha} \Theta(\hbar\omega_D - |\tilde{\epsilon}_\alpha(\mathbf{k})|) \Theta(\hbar\omega_D - |\tilde{\epsilon}_\alpha(\mathbf{k}')|) \\
 U_{\alpha\alpha}(\mathbf{k}, \mathbf{k}') &= U_{\alpha\alpha} \Theta(\hbar\omega_C - |\tilde{\epsilon}_\alpha(\mathbf{k})|) \Theta(\hbar\omega_C - |\tilde{\epsilon}_\alpha(\mathbf{k}')|) \\
 V_{12}(\mathbf{k}, \mathbf{k}') &= V_{12} \Theta(\hbar\omega_D - |\tilde{\epsilon}_1(\mathbf{k})|) \Theta(\hbar\omega_D - |\tilde{\epsilon}_2(\mathbf{k}')|) \\
 V_{21}(\mathbf{k}, \mathbf{k}') &= V_{21} \Theta(\hbar\omega_D - |\tilde{\epsilon}_2(\mathbf{k})|) \Theta(\hbar\omega_D - |\tilde{\epsilon}_1(\mathbf{k}')|) \\
 U_{12}(\mathbf{k}, \mathbf{k}') &= U_{12} \Theta(\hbar\omega_C - |\tilde{\epsilon}_1(\mathbf{k})|) \Theta(\hbar\omega_C - |\tilde{\epsilon}_2(\mathbf{k}')|) \\
 U_{21}(\mathbf{k}, \mathbf{k}') &= U_{21} \Theta(\hbar\omega_C - |\tilde{\epsilon}_2(\mathbf{k})|) \Theta(\hbar\omega_C - |\tilde{\epsilon}_1(\mathbf{k}')|),
 \end{aligned} \tag{4.1}$$

$\hbar\omega_C \geq \hbar\omega_D$, $V_{\alpha\alpha} < 0$, $V_{12} = V_{21} < 0$, $U_{\alpha\alpha} > 0$ ja $U_{12} = U_{21} > 0$.

5 Võrrandisüsteemi teisendamine

$$\begin{aligned}
 \Delta_{\alpha\mathbf{k}} &= \sum_{\mathbf{k}'} [V_{\alpha\alpha}(\mathbf{k}, \mathbf{k}') \langle a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha\mathbf{k}'\uparrow} \rangle + U_{\alpha\alpha}(\mathbf{k}, \mathbf{k}') \langle a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha\mathbf{k}'\uparrow} \rangle + \\
 &\quad + V_{\alpha\alpha'}(\mathbf{k}, \mathbf{k}') \langle a_{\alpha'-\mathbf{k}'\downarrow} a_{\alpha'\mathbf{k}'\uparrow} \rangle + U_{\alpha\alpha'}(\mathbf{k}, \mathbf{k}') \langle a_{\alpha'-\mathbf{k}'\downarrow} a_{\alpha'\mathbf{k}'\uparrow} \rangle]
 \end{aligned} \tag{5.1}$$

Lähme summeerimiselt üle integreerimisele

$$\sum_{\mathbf{x}} f(y(\mathbf{x})) = \int f(y) \rho(y) dy, \tag{5.2}$$

kus suurus ρ on olekute tihedus.

Tähistame

$$\begin{aligned}
 \eta_\alpha(T, \Delta_\alpha) &= \int_0^{\hbar\omega_D} E^{-1}(\Delta_\alpha) \tanh\left(\frac{E(\Delta_\alpha)}{2k_B T}\right) d\tilde{\epsilon}_\alpha \equiv \eta_\alpha \\
 \tilde{\eta}_\alpha(T, \tilde{\Delta}_\alpha) &= \int_{\hbar\omega_D}^{\hbar\omega_C} E^{-1}(\tilde{\Delta}_\alpha) \tanh\left(\frac{E(\tilde{\Delta}_\alpha)}{2k_B T}\right) d\tilde{\epsilon}_\alpha \equiv \tilde{\eta}_\alpha \\
 E(\Delta_\alpha) &\equiv \sqrt{\tilde{\epsilon}_\alpha^2 + |\Delta_\alpha|^2} \\
 E(\tilde{\Delta}_\alpha) &\equiv \sqrt{\tilde{\epsilon}_\alpha^2 + |\tilde{\Delta}_\alpha|^2}
 \end{aligned} \tag{5.3}$$

Ehk

$$\Delta_{\alpha\mathbf{k}} = \sum_{\alpha} \sum_{\mathbf{k}'} W_{\alpha\alpha'} \langle a_{\alpha-\mathbf{k}'\downarrow} a_{\alpha'\mathbf{k}'\uparrow} \rangle = \sum_{\alpha} W_{\alpha\alpha'} \rho_{\alpha'} \eta_{\alpha'} \Delta_{\alpha'} \tag{5.4}$$

Kordaja 1/2 kaob ära, kuna muudame integreerimisrajasid $\int_{-\hbar\omega_D}^{\hbar\omega_D} \rightarrow 2 \int_0^{\hbar\omega_D}$ ja eeldame, et ole-

kute tihedus ρ on integreerimisvahemikus konstantne. $\alpha = 1, 2, \alpha \neq \alpha'$

$$\begin{cases} \Delta_\alpha &= -V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - \\ &- V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} \\ \tilde{\Delta}_\alpha &= -U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} \end{cases} \quad (5.5)$$

5.1 $\tilde{\Delta}_\alpha$ eemaldamine võrrandisüsteemist

Eemaldame võrrandisüsteemist $\tilde{\Delta}_\alpha$

$$\begin{aligned} \Delta_\alpha - \tilde{\Delta}_\alpha &= -V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - \\ &- V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} - \\ &- (U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\tilde{\Delta}_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\tilde{\Delta}_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'}) \Rightarrow \\ \Rightarrow \tilde{\Delta}_\alpha &= \Delta_\alpha + V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha + V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} \end{aligned} \quad (5.6)$$

Asendame $\tilde{\Delta}_\alpha$ esimesse võrrandisse

$$\begin{aligned} \Delta_\alpha &= -V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - \\ &- U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha(\Delta_\alpha + V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha + V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'}) - \\ &- V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - \\ &- U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}(\Delta_{\alpha'} + V_{\alpha'\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} + V_{\alpha'\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha) = \\ &= -V_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha\Delta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\Delta_\alpha - U_{\alpha\alpha}V_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\rho_\alpha\eta_\alpha\Delta_\alpha - \\ &- U_{\alpha\alpha}V_{\alpha\alpha'}\rho_\alpha\tilde{\eta}_\alpha\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - \\ &- U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}V_{\alpha'\alpha'}\tilde{\eta}_{\alpha'}\rho_{\alpha'}\eta_{\alpha'}\Delta_{\alpha'} - U_{\alpha\alpha'}V_{\alpha'\alpha}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_\alpha\eta_\alpha\Delta_\alpha \\ \Delta_\alpha &= \Delta_\alpha(-V_{\alpha\alpha}\rho_\alpha\eta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha - \\ &- U_{\alpha\alpha}V_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\rho_\alpha\eta_\alpha - U_{\alpha\alpha'}V_{\alpha'\alpha}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_\alpha\eta_\alpha) + \\ &+ \Delta_{\alpha'}(-U_{\alpha\alpha}V_{\alpha\alpha'}\rho_\alpha\tilde{\eta}_\alpha\rho_{\alpha'}\eta_{\alpha'} - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'} - \\ &- U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'} - U_{\alpha\alpha'}V_{\alpha'\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_{\alpha'}\eta_{\alpha'}) \end{aligned} \quad (5.7)$$

$$\begin{aligned} \Delta_\alpha &= \Delta_\alpha(-V_{\alpha\alpha}\rho_\alpha\eta_\alpha - U_{\alpha\alpha}\rho_\alpha\eta_\alpha - U_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha - \\ &- U_{\alpha\alpha}V_{\alpha\alpha}\rho_\alpha\tilde{\eta}_\alpha\rho_\alpha\eta_\alpha - U_{\alpha\alpha'}V_{\alpha'\alpha}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_\alpha\eta_\alpha) + \\ &+ \Delta_{\alpha'}(-U_{\alpha\alpha}V_{\alpha\alpha'}\rho_\alpha\tilde{\eta}_\alpha\rho_{\alpha'}\eta_{\alpha'} - V_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'} - \\ &- U_{\alpha\alpha'}\rho_{\alpha'}\eta_{\alpha'} - U_{\alpha\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'} - U_{\alpha\alpha'}V_{\alpha'\alpha'}\rho_{\alpha'}\tilde{\eta}_{\alpha'}\rho_{\alpha'}\eta_{\alpha'}) \end{aligned} \quad (5.8)$$

Kirjutame indeksid lahti

$$\begin{aligned} \Delta_1 &= \Delta_1(-V_{11}\rho_1\eta_1 - U_{11}\rho_1\eta_1 - U_{11}\rho_1\tilde{\eta}_1 - U_{11}V_{11}\rho_1\tilde{\eta}_1\rho_1\eta_1 - U_{12}V_{21}\rho_2\tilde{\eta}_2\rho_1\eta_1) + \\ &+ \Delta_2(-U_{11}V_{12}\rho_1\tilde{\eta}_1\rho_2\eta_2 - V_{12}\rho_2\eta_2 - U_{12}\rho_2\eta_2 - U_{12}\rho_2\tilde{\eta}_2 - U_{12}V_{22}\rho_2\tilde{\eta}_2\rho_2\eta_2) \\ \Delta_2 &= \Delta_2(-V_{22}\rho_2\eta_2 - U_{22}\rho_2\eta_2 - U_{22}\rho_2\tilde{\eta}_2 - U_{22}V_{22}\rho_2\tilde{\eta}_2\rho_2\eta_2 - U_{21}V_{12}\rho_1\tilde{\eta}_1\rho_2\eta_2) + \\ &+ \Delta_1(-U_{22}V_{21}\rho_2\tilde{\eta}_2\rho_1\eta_1 - V_{21}\rho_1\eta_1 - U_{21}\rho_1\eta_1 - U_{21}\rho_1\tilde{\eta}_1 - U_{21}V_{11}\rho_1\tilde{\eta}_1\rho_1\eta_1) \end{aligned} \quad (5.9)$$

6 Võrrandisüsteemi ümbertähistamine

Tähistame

$$\begin{aligned}
 \Xi_{11} &= V_{11}\rho_1 + U_{11}\rho_1 + U_{11}V_{11}\rho_1\tilde{\eta}_1\rho_1 + U_{12}V_{21}\rho_2\tilde{\eta}_2\rho_1 \\
 \Xi_{12} &= V_{12}\rho_2 + U_{12}\rho_2 + U_{11}V_{12}\rho_1\tilde{\eta}_1\rho_2 + U_{12}V_{22}\rho_2\tilde{\eta}_2\rho_2 \\
 \Xi_{21} &= V_{21}\rho_1 + U_{21}\rho_1 + U_{22}V_{21}\rho_2\tilde{\eta}_2\rho_1 + U_{21}V_{11}\rho_1\tilde{\eta}_1\rho_1 \\
 \Xi_{22} &= V_{22}\rho_2 + U_{22}\rho_2 + U_{22}V_{22}\rho_2\tilde{\eta}_2\rho_2 + U_{21}V_{12}\rho_1\tilde{\eta}_1\rho_2
 \end{aligned} \tag{6.1}$$

$$\begin{cases} \Delta_1 = (-\Xi_{11}\eta_1 - U_{11}\rho_1\tilde{\eta}_1)\Delta_1 + (-\Xi_{12}\eta_2 - U_{12}\rho_2\tilde{\eta}_2)\Delta_2 \\ \Delta_2 = (-\Xi_{21}\eta_1 - U_{21}\rho_1\tilde{\eta}_1)\Delta_1 + (-\Xi_{22}\eta_2 - U_{22}\rho_2\tilde{\eta}_2)\Delta_2 \end{cases} \tag{6.2}$$

$$\begin{cases} \Delta_1 = -\Xi_{11}\eta_1\Delta_1 - U_{11}\rho_1\tilde{\eta}_1\Delta_1 - \Xi_{12}\eta_2\Delta_2 - U_{12}\rho_2\tilde{\eta}_2\Delta_2 \\ \Delta_2 = -\Xi_{21}\eta_1\Delta_1 - U_{21}\rho_1\tilde{\eta}_1\Delta_1 - \Xi_{22}\eta_2\Delta_2 - U_{22}\rho_2\tilde{\eta}_2\Delta_2 \end{cases} \tag{6.3}$$

Arvutame $\tilde{\eta}_\alpha$. Arvestame, et $k_B T_c, |\Delta_\alpha| \ll \hbar\omega_D$

$$\begin{aligned}
 \tilde{\eta}_\alpha(T, \tilde{\Delta}_\alpha) &= \int_{\hbar\omega_D}^{\hbar\omega_C} E^{-1}(\tilde{\Delta}_\alpha) \tanh\left(\frac{E(\tilde{\Delta}_\alpha)}{2k_B T}\right) d\tilde{\epsilon}_\alpha \equiv \tilde{\eta}_\alpha \\
 \tilde{\eta}_\alpha(T, 0) &= \int_{\hbar\omega_D}^{\hbar\omega_C} \tilde{\epsilon}^{-1} \tanh\left(\frac{\tilde{\epsilon}}{2k_B T}\right) d\tilde{\epsilon}_\alpha \approx \int_{\hbar\omega_D}^{\hbar\omega_C} \tilde{\epsilon}^{-1} d\tilde{\epsilon}_\alpha = \ln|\tilde{\epsilon}| \Big|_{\hbar\omega_C}^{\hbar\omega_D} = \ln \frac{\hbar\omega_C}{\hbar\omega_D}
 \end{aligned} \tag{6.4}$$

$$\tilde{\eta}_\alpha \approx \ln\left(\frac{\hbar\omega_C}{\hbar\omega_D}\right) \equiv \tilde{\eta} \tag{6.5}$$

Tähistame

$$\begin{aligned}
 P &= \sqrt{\frac{\rho_1}{\rho_2}} \\
 v_{\alpha\alpha} &= V_{\alpha\alpha}\rho_\alpha \\
 \mu_{\alpha\alpha} &= U_{\alpha\alpha}\rho_\alpha \\
 v_{\alpha\alpha'} &= V_{\alpha\alpha'}\sqrt{\rho_\alpha\rho_{\alpha'}} \\
 \mu_{\alpha\alpha'} &= U_{\alpha\alpha'}\sqrt{\rho_\alpha\rho_{\alpha'}}
 \end{aligned} \tag{6.6}$$

$$\begin{aligned}
\Xi_{11} &= v_{11} + \mu_{11} + \mu_{11} v_{11} \tilde{\eta} + \mu_{12} v_{21} \tilde{\eta} \\
\Xi_{12} &= v_{12} P^{-1} + \mu_{12} P^{-1} + \mu_{11} v_{12} P^{-1} \tilde{\eta} + \mu_{12} P^{-1} v_{22} \tilde{\eta} \\
\Xi_{21} &= v_{21} P + \mu_{21} P + \mu_{22} v_{21} P \tilde{\eta} + \mu_{21} P v_{11} \tilde{\eta} \\
\Xi_{22} &= v_{22} + \mu_{22} + \mu_{22} v_{22} \tilde{\eta} + \mu_{21} v_{12} \tilde{\eta}
\end{aligned} \tag{6.7}$$

$$\begin{aligned}
\Xi_{11} &= v_{11} + \mu_{11} + \mu_{11} v_{11} \tilde{\eta} + \mu_{12} v_{21} \tilde{\eta} \\
\Xi_{12} &= (v_{12} + \mu_{12} + \mu_{11} v_{12} \tilde{\eta} + \mu_{12} v_{22} \tilde{\eta}) P^{-1} \\
\Xi_{21} &= (v_{21} + \mu_{21} + \mu_{22} v_{21} \tilde{\eta} + \mu_{21} v_{11} \tilde{\eta}) P \\
\Xi_{22} &= v_{22} + \mu_{22} + \mu_{22} v_{22} \tilde{\eta} + \mu_{21} v_{12} \tilde{\eta}
\end{aligned} \tag{6.8}$$

Toome P Ξ_{12} ja Ξ_{21} seest välja

$$\begin{cases} \Delta_1 = -\Xi_{11} \eta_1 \Delta_1 - \mu_{11} \tilde{\eta} \Delta_1 - \Xi_{12} P^{-1} \eta_2 \Delta_2 - \mu_{12} P^{-1} \tilde{\eta} \Delta_2 \\ \Delta_2 = -\Xi_{21} P \eta_1 \Delta_1 - \mu_{21} P \tilde{\eta} \Delta_1 - \Xi_{22} \eta_2 \Delta_2 - \mu_{22} \tilde{\eta} \Delta_2 \end{cases} \tag{6.9}$$

Tähistame

$$Y_{\alpha\alpha} = 1 + \mu_{\alpha\alpha} \tilde{\eta} \tag{6.10}$$

$$\begin{cases} Y_{11} \Delta_1 = -\Xi_{11} \eta_1 \Delta_1 - \Xi_{12} P^{-1} \eta_2 \Delta_2 - \mu_{12} P^{-1} \tilde{\eta} \Delta_2 \\ Y_{22} \Delta_2 = -\Xi_{21} P \eta_1 \Delta_1 - \mu_{21} P \tilde{\eta} \Delta_1 - \Xi_{22} \eta_2 \Delta_2 \end{cases} \tag{6.11}$$

$$\begin{cases} Y_{11} \Delta_1 + \Xi_{11} \eta_1 \Delta_1 + \Xi_{12} P^{-1} \eta_2 \Delta_2 + \mu_{12} P^{-1} \tilde{\eta} \Delta_2 = 0 \\ Y_{22} \Delta_2 + \Xi_{22} \eta_2 \Delta_2 + \Xi_{21} P \eta_1 \Delta_1 + \mu_{21} P \tilde{\eta} \Delta_1 = 0 \end{cases} \tag{6.12}$$

Korrutame esimest võrrandit $\mu_{21} \tilde{\eta} P$ -ga ja teist Y_{11} -ga

$$\begin{cases} \mu_{21} \tilde{\eta} P Y_{11} \Delta_1 + \mu_{21} \tilde{\eta} P \Xi_{11} \eta_1 \Delta_1 + \mu_{21} \tilde{\eta} P \Xi_{12} P^{-1} \eta_2 \Delta_2 + \mu_{21} \tilde{\eta} \mu_{12} \tilde{\eta} \Delta_2 = 0 \\ Y_{11} Y_{22} \Delta_2 + Y_{11} \Xi_{22} \eta_2 \Delta_2 + Y_{11} \Xi_{21} P \eta_1 \Delta_1 + Y_{11} \mu_{21} P \tilde{\eta} \Delta_1 = 0 \end{cases} \tag{6.13}$$

Lahutame esimesest võrrandist teise

$$\begin{aligned}
&\mu_{21} \tilde{\eta} P Y_{11} \Delta_1 + \mu_{21} \tilde{\eta} P \Xi_{11} \eta_1 \Delta_1 + \mu_{21} P^{-1} \tilde{\eta} P \Xi_{12} \eta_2 \Delta_2 + \mu_{21} \tilde{\eta} \mu_{12} \tilde{\eta} \Delta_2 - \\
&-(Y_{11} Y_{22} \Delta_2 + Y_{11} \Xi_{22} \eta_2 \Delta_2 + Y_{11} \Xi_{21} P \eta_1 \Delta_1 + Y_{11} \mu_{21} P \tilde{\eta} \Delta_1) = 0
\end{aligned} \tag{6.14}$$

Grupeerime liikmed

$$\Delta_1 \eta_1 P (\mu_{21} \tilde{\eta} \Xi_{11} - Y_{11} \Xi_{21}) + \Delta_2 \eta_2 (\mu_{21} \tilde{\eta} \Xi_{12} - Y_{11} \Xi_{22}) + \Delta_2 (\mu_{21} \mu_{12} \tilde{\eta}^2 - Y_{11} Y_{22}) = 0 \tag{6.15}$$

$$\begin{cases} \Upsilon_{11}\Delta_1 + \Xi_{11}\eta_1\Delta_1 + \Xi_{12}P^{-1}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Delta_2 = 0 \\ \Upsilon_{22}\Delta_2 + \Xi_{22}\eta_2\Delta_2 + \Xi_{21}P\eta_1\Delta_1 + \mu_{21}P\tilde{\eta}\Delta_1 = 0 \end{cases} \quad (6.16)$$

Korrutame esimest võrrandit Υ_{22} -ga ja teist $\mu_{12}P^{-1}\tilde{\eta}$ -ga

$$\begin{cases} \Upsilon_{22}\Upsilon_{11}\Delta_1 + \Upsilon_{22}\Xi_{11}\eta_1\Delta_1 + \Upsilon_{22}\Xi_{12}P^{-1}\eta_2\Delta_2 + \Upsilon_{22}\mu_{12}P^{-1}\tilde{\eta}\Delta_2 = 0 \\ \mu_{12}P^{-1}\tilde{\eta}\Upsilon_{22}\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{22}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{21}P\eta_1\Delta_1 + \mu_{12}P^{-1}\tilde{\eta}\mu_{21}P\tilde{\eta}\Delta_1 = 0 \end{cases} \quad (6.17)$$

Lahutame teisest võrrandist esimese

$$\begin{aligned} & \mu_{12}P^{-1}\tilde{\eta}\Upsilon_{22}\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{22}\eta_2\Delta_2 + \mu_{12}P^{-1}\tilde{\eta}\Xi_{21}P\eta_1\Delta_1 + \mu_{12}\tilde{\eta}\mu_{21}\tilde{\eta}\Delta_1 - \\ & - (\Upsilon_{22}\Upsilon_{11}\Delta_1 + \Upsilon_{22}\Xi_{11}\eta_1\Delta_1 + \Upsilon_{22}\Xi_{12}P^{-1}\eta_2\Delta_2 + \Upsilon_{22}\mu_{12}P^{-1}\tilde{\eta}\Delta_2) = 0 \end{aligned} \quad (6.18)$$

Grupeerime liikmed

$$\Delta_1\eta_1(\mu_{12}\tilde{\eta}\Xi_{21} - \Upsilon_{22}\Xi_{11}) + \Delta_1(\mu_{12}\mu_{21}\tilde{\eta}^2 - \Upsilon_{22}\Upsilon_{11}) + \Delta_2\eta_2P^{-1}(\mu_{12}\tilde{\eta}\Xi_{22} - \Upsilon_{22}\Xi_{12}) = 0 \quad (6.19)$$

Kokku saame

$$\begin{cases} \Delta_1\eta_1P(\mu_{21}\tilde{\eta}\Xi_{11} - \Upsilon_{11}\Xi_{21}) + \Delta_2\eta_2(\mu_{21}\tilde{\eta}\Xi_{12} - \Upsilon_{11}\Xi_{22}) = \Delta_2(\Upsilon_{11}\Upsilon_{22} - \mu_{21}\mu_{12}\tilde{\eta}^2) \\ \Delta_1\eta_1(\mu_{12}\tilde{\eta}\Xi_{21} - \Upsilon_{22}\Xi_{11}) + \Delta_2\eta_2P^{-1}(\mu_{12}\tilde{\eta}\Xi_{22} - \Upsilon_{22}\Xi_{12}) = \Delta_1(\Upsilon_{22}\Upsilon_{11} - \mu_{12}\mu_{21}\tilde{\eta}^2) \end{cases} \quad (6.20)$$

Tähistame

$$\begin{aligned} \Theta_{11} &= \mu_{12}\tilde{\eta}\Xi_{21} - \Upsilon_{22}\Xi_{11} \\ \Theta_{12} &= \mu_{12}\tilde{\eta}\Xi_{22} - \Upsilon_{22}\Xi_{12} \\ \Phi &= \Upsilon_{11}\Upsilon_{22} - \mu_{21}\mu_{12}\tilde{\eta}^2 \\ \Theta_{21} &= \mu_{21}\tilde{\eta}\Xi_{11} - \Upsilon_{11}\Xi_{21} \\ \Theta_{22} &= \mu_{21}\tilde{\eta}\Xi_{12} - \Upsilon_{11}\Xi_{22} \end{aligned} \quad (6.21)$$

$$\begin{cases} \Delta_1\eta_1P\Theta_{21} + \Delta_2\eta_2\Theta_{22} = \Delta_2\Phi \\ \Delta_1\eta_1\Theta_{11} + \Delta_2\eta_2P^{-1}\Theta_{12} = \Delta_1\Phi \end{cases} \quad (6.22)$$

$$\begin{cases} \Delta_1 = \Delta_1\eta_1\frac{\Theta_{11}}{\Phi} + \Delta_2\eta_2\frac{\Theta_{12}}{P\Phi} \\ \Delta_2 = \Delta_1\eta_1\frac{P\Theta_{21}}{\Phi} + \Delta_2\eta_2\frac{\Theta_{22}}{\Phi} \end{cases} \quad (6.23)$$

Võtame arvesse, et $v_{12} = v_{21}$ ja $\mu_{12} = \mu_{21}$ ning asendame tähistused (6.8),(6.10) tagasi tähistusse (6.21) ning viime olekute tihedused tähistuse sisse

$$\begin{aligned}
\Phi &= 1 + \tilde{\eta}\mu_{11} - \tilde{\eta}^2\mu_{12}^2 + \tilde{\eta}\mu_{22} + \tilde{\eta}^2\mu_{11}\mu_{22} \\
\Theta_{11} &= -\mu_{11} + \tilde{\eta}\mu_{12}^2 - \tilde{\eta}\mu_{11}\mu_{22} - v_{11} - \tilde{\eta}\mu_{11}v_{11} + \tilde{\eta}^2\mu_{12}^2v_{11} - \tilde{\eta}\mu_{22}v_{11} - \tilde{\eta}^2\mu_{11}\mu_{22}v_{11} \\
\Theta_{22} &= -\mu_{22} + \tilde{\eta}\mu_{12}^2 - \tilde{\eta}\mu_{11}\mu_{22} - v_{22} - \tilde{\eta}\mu_{11}v_{22} + \tilde{\eta}^2\mu_{12}^2v_{22} - \tilde{\eta}\mu_{22}v_{22} - \tilde{\eta}^2\mu_{11}\mu_{22}v_{22} \\
\Theta_{12} = \Theta_{21} &= -\mu_{12} - v_{12} - \tilde{\eta}\mu_{11}v_{12} + \tilde{\eta}^2\mu_{12}^2v_{12} - \tilde{\eta}\mu_{22}v_{12} - \tilde{\eta}^2\mu_{11}\mu_{22}v_{12}
\end{aligned} \tag{6.24}$$

Tähistame

$$\Gamma_{\alpha\alpha} = \frac{\Theta_{\alpha\alpha}}{\Phi}, \quad \Gamma_{12} = \frac{\Theta_{12}}{P\Phi}, \quad \Gamma_{21} = \frac{P\Theta_{21}}{\Phi} \tag{6.25}$$

$$\begin{cases} \Delta_1 = \Delta_1\eta_1\Gamma_{11} + \Delta_2\eta_2\Gamma_{12} \\ \Delta_2 = \Delta_1\eta_1\Gamma_{21} + \Delta_2\eta_2\Gamma_{22} \end{cases} \tag{6.26}$$

7 Efektiivsete interaktsioonikonstantide analüüs

Efektiivsete interaktsioonikonstantide sõltuvus interaktsioonikonstantidest

$$\begin{aligned}
\Phi &= \Phi(\mu_{11}, \mu_{22}, \mu_{12}, \tilde{\eta}) \\
\Theta_{11} &= \Theta_{11}(\mu_{11}, \mu_{22}, \mu_{12}, v_{11}, \tilde{\eta}) \\
\Theta_{22} &= \Theta_{22}(\mu_{11}, \mu_{22}, \mu_{12}, v_{22}, \tilde{\eta}) \\
\Theta_{12} &= \Theta(\mu_{11}, \mu_{22}, \mu_{12}, v_{12}, \tilde{\eta}) \\
\Theta_{21} &= \Theta(\mu_{11}, \mu_{22}, \mu_{12}, v_{12}, \tilde{\eta}) \\
\Gamma_{11} &= \Gamma_{11}(\mu_{11}, \mu_{22}, \mu_{12}, v_{11}, \tilde{\eta}) \\
\Gamma_{22} &= \Gamma_{22}(\mu_{11}, \mu_{22}, \mu_{12}, v_{22}, \tilde{\eta}) \\
\Gamma_{12} &= \Gamma_{12}(\mu_{11}, \mu_{22}, \mu_{12}, v_{12}, \tilde{\eta}, P) \\
\Gamma_{21} &= \Gamma_{21}(\mu_{11}, \mu_{22}, \mu_{12}, v_{12}, \tilde{\eta}, P)
\end{aligned} \tag{7.1}$$

GRAAFIKUD TEISES FAILIS

Arvestame interaktsioonikonstantide märgiga $\mu > 0$ ja $v < 0$ ja kirjutame efektiivsete interakt-

sioonikonstantide positiivsed ja negatiivsed liikmed eraldi välja

$$\begin{aligned}
\Phi^+ &= 1 + \tilde{\eta}\mu_{11} + \tilde{\eta}\mu_{22} + \tilde{\eta}^2\mu_{11}\mu_{22} \\
\Phi^- &= -\tilde{\eta}^2\mu_{12}^2 \\
\Theta_{11}^+ &= \tilde{\eta}\mu_{12}^2 - v_{11} - \tilde{\eta}\mu_{11}v_{11} - \tilde{\eta}\mu_{22}v_{11} - \tilde{\eta}^2\mu_{11}\mu_{22}v_{11} \\
\Theta_{11}^- &= -\mu_{11} - \tilde{\eta}\mu_{11}\mu_{22} + \tilde{\eta}^2\mu_{12}^2v_{11} \\
\Theta_{22}^+ &= \tilde{\eta}\mu_{12}^2 - v_{22} - \tilde{\eta}\mu_{11}v_{22} - \tilde{\eta}\mu_{22}v_{22} - \tilde{\eta}^2\mu_{11}\mu_{22}v_{22} \\
\Theta_{22}^- &= -\mu_{22} - \tilde{\eta}\mu_{11}\mu_{22} + \tilde{\eta}^2\mu_{12}^2v_{22} \\
\Theta^+ &= -v_{12} - \tilde{\eta}\mu_{11}v_{12} - \tilde{\eta}\mu_{22}v_{12} - \tilde{\eta}^2\mu_{11}\mu_{22}v_{12} \\
\Theta^- &= -\mu_{12} + \tilde{\eta}^2\mu_{12}^2v_{12}
\end{aligned} \tag{7.2}$$

Avaldame, mis punktis toimub efektiivsetel interaktsioonikonstantidel märgimuutus

$$\begin{aligned}
\Phi^+ &= \Phi^- \\
1 + \tilde{\eta}\mu_{11} + \tilde{\eta}\mu_{22} + \tilde{\eta}^2\mu_{11}\mu_{22} &= -\tilde{\eta}^2\mu_{12}^2
\end{aligned} \tag{7.3}$$

$$\Theta_{11}^+ = \Theta_{11}^- \tag{7.4}$$

$$\Theta_{22}^+ = \Theta_{22}^- \tag{7.5}$$

$$\Theta^+ = \Theta^- \tag{7.6}$$

8 Faasisiirde temperatuur

Faasisiirde temperatuuri lähedal saame ligikaudu integreerida integraali (5.3) arvestades, et $\hbar\omega_D \gg 2k_B T_c$ ja $\Delta_\alpha(T = T_c) = 0$

$$\eta(T_c, 0) = \int_0^{\hbar\omega_D} \tilde{\epsilon}_\alpha^{-1} \tanh\left(\frac{\tilde{\epsilon}_\alpha}{2k_B T_c}\right) d\tilde{\epsilon}_\alpha \tag{8.1}$$

Teeme muutujavahetuse

$$x = \frac{\tilde{\epsilon}_\alpha}{2k_B T_c} \tag{8.2}$$

$$\begin{aligned}
\eta(T_c, 0) &= \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} \frac{1}{x} \tanh(x) dx = \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} \tanh(x) d[\ln(x)] = \\
&= \ln(x) \tanh(x) \Big|_0^{\frac{\hbar\omega_D}{2k_B T_c}} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} \ln(x) d[\tanh(x)] = \\
&= \ln\left(\frac{\hbar\omega_D}{2k_B T_c}\right) \underbrace{\tanh\left(\frac{\hbar\omega_D}{2k_B T_c}\right)}_{\approx 1} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} \frac{\ln(x)}{\cosh^2(x)} dx
\end{aligned} \tag{8.3}$$

Kui $\hbar\omega_D \gg 2k_B T_c \Rightarrow \frac{\hbar\omega_D}{2k_B T_c} \rightarrow \infty$

$$- \int_0^\infty \frac{\ln(x)}{\cosh^2(x)} dx = \gamma - \ln\left(\frac{\pi}{4}\right) = \ln(e^\gamma) - \ln\left(\frac{\pi}{4}\right) = \ln\left(\frac{4e^\gamma}{\pi}\right) \tag{8.4}$$

$$\eta(T_c, 0) = \ln\left(\frac{\hbar\omega_D}{2k_B T_c}\right) + \ln\left(\frac{4e^\gamma}{\pi}\right) = \ln\left(\frac{4\hbar\omega_D e^\gamma}{k_B T_c}\right) \equiv \eta, \quad \gamma = 0.577 \dots \tag{8.5}$$

Teisendame võrrandisüsteemi (6.26)

$$\begin{cases} \Delta_1 (\eta \Gamma_{11} - 1) + \Delta_2 \eta \Gamma_{12} = 0 \\ \Delta_1 \eta \Gamma_{21} + \Delta_2 (\eta \Gamma_{22} - 1) = 0 \end{cases} \tag{8.6}$$

Võrrandisüsteemil (6.26) leiduvad mittetriviaalsed lahendid, kui

$$\begin{vmatrix} \eta \Gamma_{11} - 1 & \eta \Gamma_{12} \\ \eta \Gamma_{21} & \eta \Gamma_{22} - 1 \end{vmatrix} = 0 \tag{8.7}$$

$$\begin{aligned}
(\eta \Gamma_{11} - 1)(\eta \Gamma_{22} - 1) - \eta^2 \Gamma_{21} \Gamma_{12} &= 0 \\
\eta^2 \Gamma_{11} \Gamma_{22} - \eta \Gamma_{11} - \eta \Gamma_{22} + 1 - \eta^2 \Gamma_{12} \Gamma_{21} &= 0 \\
\eta^2 (\Gamma_{11} \Gamma_{22} - \Gamma_{12} \Gamma_{21}) - \eta (\Gamma_{11} + \Gamma_{22}) + 1 &= 0
\end{aligned} \tag{8.8}$$

Lahendame η suhtes ruutvõrrandi

$$\eta^\pm = \frac{\Gamma_{11} + \Gamma_{22} \pm \sqrt{(\Gamma_{11} - \Gamma_{22})^2 + 4\Gamma_{12}\Gamma_{21}}}{2(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} = \frac{2}{\Gamma_{11} + \Gamma_{22} \mp \sqrt{(\Gamma_{11} - \Gamma_{22})^2 + 4\Gamma_{12}\Gamma_{21}}} \tag{8.9}$$

Avaldame võrrandist (8.5) T_c

$$T_c^\pm = \frac{2\hbar\omega_D e^\gamma}{k_B \pi} \exp(-\eta^\pm) \tag{8.10}$$

9 Ülijuhtivuspilude faaside vahe

Teeme võrrandisüsteemis (8.6) asenduse $\Delta_{\alpha\pm} = |\Delta_{\alpha\pm}| e^{\phi_{\alpha\pm}}$

$$\begin{cases} |\Delta_{1\pm}| e^{\phi_{1\pm}} (\eta_{\pm}\Gamma_{11} - 1) + |\Delta_{2\pm}| e^{\phi_{2\pm}} \eta_{\pm}\Gamma_{12} = 0 \\ |\Delta_{1\pm}| e^{\phi_{1\pm}} \eta_{\pm}\Gamma_{21} + |\Delta_{2\pm}| e^{\phi_{2\pm}} (\eta_{\pm}\Gamma_{22} - 1) = 0 \end{cases} : |\Delta_{2\pm}| e^{\phi_{2\pm}} \quad (9.1)$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} (\eta_{\pm}\Gamma_{11} - 1) + \eta_{\pm}\Gamma_{12} = 0 \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} \eta_{\pm}\Gamma_{21} + \eta_{\pm}\Gamma_{22} - 1 = 0 \end{cases} \quad (9.2)$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} (\eta_{\pm}\Gamma_{11} - 1) = -\eta_{\pm}\Gamma_{12} \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} \eta_{\pm}\Gamma_{21} = -(\eta_{\pm}\Gamma_{22} - 1) \end{cases} \quad (9.3)$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} = -\frac{\eta_{\pm}\Gamma_{12}}{\eta_{\pm}\Gamma_{11} - 1} \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} e^{\phi_{1\pm} - \phi_{2\pm}} = -\frac{\eta_{\pm}\Gamma_{22} - 1}{\eta_{\pm}\Gamma_{21}} \end{cases} \quad (9.4)$$

$$\begin{cases} \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} \frac{\eta_{\pm}\Gamma_{11} - 1}{\eta_{\pm}\Gamma_{12}} = -e^{\phi_{1\pm} - \phi_{2\pm}} = \mp 1 \\ \frac{|\Delta_{1\pm}|}{|\Delta_{2\pm}|} \frac{\eta_{\pm}\Gamma_{21}}{\eta_{\pm}\Gamma_{22} - 1} = -e^{\phi_{1\pm} - \phi_{2\pm}} = \mp 1 \end{cases} \quad (9.5)$$

Kuna võrrandi vasakul pool olev liige on reaalne suurus, siis saab faaside vahe olla, kas $2n\pi$ või $(2n+1)\pi$, kus $n \in \mathbb{Z}$.

10 Nulltemperatuurne ülijuhtivus

Juhul, kui temperatuur on 0 K, saame lihtsustada integraali (5.3)

$$\lim_{T \rightarrow 0} \tanh \frac{E(\Delta_\alpha)}{2k_B T} = 1 \quad (10.1)$$

Seega

$$\begin{aligned} \eta_\alpha(0, \Delta_\alpha) &= \int_0^{\hbar\omega_D} \frac{d\tilde{\epsilon}_\alpha}{\sqrt{\tilde{\epsilon}_\alpha^2 + |\Delta_\alpha|^2}} = \ln \left(\tilde{\epsilon}_\alpha + \sqrt{\tilde{\epsilon}_\alpha^2 + |\Delta_\alpha|^2} \right) \Big|_0^{\hbar\omega_D} = \\ &= \ln \left(\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 + |\Delta_\alpha|^2} \right) - 2 \ln |\Delta_\alpha| = \\ &= \ln \left(\frac{\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 + |\Delta_\alpha|^2}}{|\Delta_\alpha|} \right) = \\ &= \ln \left(\frac{\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 \left(1 + \frac{|\Delta_\alpha|^2}{(\hbar\omega_D)^2} \right)}}{|\Delta_\alpha|} \right) \stackrel{(\hbar\omega_D)^2 \gg |\Delta_\alpha|^2}{\approx} \ln \left(\frac{2\hbar\omega_D}{|\Delta_\alpha|} \right) \end{aligned} \quad (10.2)$$

Asendame saadud tulemuse võrrandisüsteemi (6.26)

$$\begin{cases} \Delta_1 = \Delta_1 \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{11} + \Delta_2 \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{12} \\ \Delta_2 = \Delta_1 \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{21} + \Delta_2 \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{22} \end{cases} \quad (10.3)$$

$$\Delta_1 = |\Delta_1| e^{i\phi_1}, \Delta_2 = |\Delta_2| e^{i\phi_2} \Rightarrow \kappa \equiv \frac{\Delta_1}{\Delta_2} = \frac{|\Delta_1|}{|\Delta_2|} e^{i(\phi_1 - \phi_2)} \quad (10.4)$$

$$\begin{cases} |\Delta_1| e^{i\phi_1} = |\Delta_1| e^{i\phi_1} \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{11} + |\Delta_2| e^{i\phi_2} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{12} \\ |\Delta_2| e^{i\phi_2} = |\Delta_1| e^{i\phi_1} \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{21} + |\Delta_2| e^{i\phi_2} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{22} \end{cases} \quad (10.5)$$

$$\begin{cases} 1 = \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{11} + \kappa^{-1} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{12} \\ 1 = \kappa \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{21} + \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{22} \end{cases} \quad (10.6)$$

Avaldame esimesest võrrandist $\ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right)$

$$\ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) = \left(1 - \kappa^{-1} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{12} \right) \Gamma_{11}^{-1} \quad (10.7)$$

Asendame teise võrrandisse

$$\begin{aligned}
 1 &= \left(1 - \kappa^{-1} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{12} \right) \frac{\kappa\Gamma_{21}}{\Gamma_{11}} + \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{22} = \\
 &= \frac{\kappa\Gamma_{21}}{\Gamma_{11}} - \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{11}} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) + \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) \Gamma_{22} = \\
 &= \frac{\kappa\Gamma_{21}}{\Gamma_{11}} + \left[\Gamma_{22} - \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{11}} \right] \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right)
 \end{aligned} \tag{10.8}$$

$$\Rightarrow \frac{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}}{\Gamma_{11}} \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) = 1 - \frac{\kappa\Gamma_{21}}{\Gamma_{11}} = \frac{\Gamma_{11} - \kappa\Gamma_{21}}{\Gamma_{11}} \tag{10.9}$$

$$\Rightarrow \ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) = \frac{\Gamma_{11} - \kappa\Gamma_{21}}{\Gamma_{11}} \frac{\Gamma_{11}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}} = \frac{\Gamma_{11} - \kappa\Gamma_{21}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}} \tag{10.10}$$

$$\Rightarrow \frac{2\hbar\omega_D}{|\Delta_2|^2} = \exp \frac{\Gamma_{11} - \kappa\Gamma_{21}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}} \Rightarrow |\Delta_2|^2 = 2\hbar\omega_D \exp \frac{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}}{\Gamma_{11} - \kappa\Gamma_{21}} \tag{10.11}$$

Avaldame teisest võrrandist $\ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right)$

$$\ln \left(\frac{2\hbar\omega_D}{|\Delta_2|^2} \right) = \left(1 - \kappa \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{21} \right) \Gamma_{22}^{-1} \tag{10.12}$$

Asendame esimesse võrrandisse

$$\begin{aligned}
 1 &= \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{11} + \left(1 - \kappa \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{21} \right) \frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}} = \\
 &= \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \Gamma_{11} + \frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}} - \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{22}} = \\
 &= \frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}} + \left[\Gamma_{11} - \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{22}} \right] \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right)
 \end{aligned} \tag{10.13}$$

$$\Rightarrow \frac{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}}{\Gamma_{22}} \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) = 1 - \frac{\kappa^{-1}\Gamma_{12}}{\Gamma_{22}} = \frac{\Gamma_{22} - \kappa^{-1}\Gamma_{12}}{\Gamma_{22}} \tag{10.14}$$

$$\Rightarrow \ln \left(\frac{2\hbar\omega_D}{|\Delta_1|^2} \right) = \frac{\Gamma_{22} - \kappa^{-1}\Gamma_{12}}{\Gamma_{22}} \frac{\Gamma_{22}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}} = \frac{\Gamma_{22} - \kappa^{-1}\Gamma_{12}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}} \tag{10.15}$$

$$\Rightarrow \frac{2\hbar\omega_D}{|\Delta_1|^2} = \exp \frac{\Gamma_{22} - \kappa^{-1}\Gamma_{12}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}} \Rightarrow |\Delta_1|^2 = 2\hbar\omega_D \exp \frac{\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}}{\Gamma_{22} - \kappa^{-1}\Gamma_{12}} \tag{10.16}$$

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$$\begin{cases} |\Delta_1|^2 = 2\hbar\omega_D \exp \frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{22}-\kappa^{-1}\Gamma_{12}} \\ |\Delta_2|^2 = 2\hbar\omega_D \exp \frac{\Gamma_{11}\Gamma_{22}-\Gamma_{12}\Gamma_{21}}{\Gamma_{11}-\kappa\Gamma_{21}} \end{cases} \quad (10.17)$$