tg1632_hw2_q1q2.pdf

Question 1

a) $5n^3 + 2n^2 + 3n = O(n^3)$

$f(n) = 5n^3 + 2n^2 + 3n \qquad g(n) = n^3$

For O: $\quad 0 \le f(n) \le c \cdot g(n) \quad \forall n \ge k$

$c = (5+2+3) = 10$

$0 \le 5n^3 + 2n^2 + 3n \le 10n^3 \quad$ for $n \ge 1$; $\quad c = 10, k = 1$

$5n^3 + 2n^2 + 3n \le 10n^3$

$5n^3 - 2n^2 - 3n \ge 0$

$5n^3 - 2n^2 - 3n \ge 0$

$n(5n^2 - 2n - 3n) = 0$

$n = 0 \quad \bigg| \quad 5n^2 - 2n - 3n = 0$

$\qquad \qquad n = \dfrac{2 \pm \sqrt{64}}{10} = 1 \text{ or } \dfrac{-3}{5}$

Since $0 \le 5n^3 + 2n^2 + 3n \le 10n^3$ for $n \ge 1$ (where $c \ne 0, k = 1$),

$5n^3 + 2n^2 + 3n = O(n^3)$

b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

If we take $\quad C_1 = 3$

$\qquad \qquad \quad C_2 = \sqrt{7}$

$\qquad \qquad \quad n_0 = 4$

$\sqrt{7n} \le \sqrt{7n^2 + 2n - 8} < 3n$

$7n^2 \le 7n^2 + 2n - 8 < 9n^2$

$7n^2 \le 7n^2 + 2n - 8$	$7n^2 + 2n - 8 \le 7n^2 + 2n$
$2n - 8 \ge 0$	$7n^2 + 2n - 8 \le 7n^2 + 2n^2$
$n \ge 4$	$7n^2 + 2n - 8 \le 9n^2$

Therefore $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

c), $d(n) = O(f(n))$
$\hookrightarrow 0 \le d(n) \le C_1 \cdot f(n)$
$e(n) = O(g(n))$
$\hookrightarrow 0 \le e(n) \le C_2 \cdot g(n)$
Split it up into 2 parts

Part 1
$\begin{cases} 0 \le d(n) \\ 0 \le e(n) \\ \therefore \ 0 \le e(n) \cdot d(n) \end{cases}$

Part 2
$\begin{cases} d(n) \le C_1 \cdot f(n) \\ e(n) \le C_2 \cdot g(n) \\ \ d(n) \cdot e(n) \le C_1 C_2 f(n) g(n) \\ C_1 \cdot C_2 \text{ is also a constant which we will call } C \\ \ d(n) \cdot e(n) \le C \cdot f(n) \cdot g(n) \end{cases}$

Combining both parts we get $0 \le d(n) \cdot e(n) \le C \cdot f(n) \cdot g(n)$.
Hence, by the definition of Big O, $d(n) e(n)$ is $O(f(n) g(n))$

Question 2:
Program 1) example1 is $\Theta(n^2)$

Program 2) example 2 is $\Theta(n)$

Program 3) example 3 is $\Theta(\log(n^2))$

Program 4) example 4 is $\Theta(n \log n)$