BUSI 525 Problem Set #3

Finite Sample Bias in Predictive Regressions

Taisei Noda GitHub Link

In [1]:

```
using Plots, Random, Statistics, Bootstrap, Distributions, LinearAlgebra,StatsBase

function Stambaugh(T,rho_uv;B=250,alpha=0.0,beta=0.015,sigma_u=0.053,theta=0,sigma_v=0.044,rho=0.98)
    d1 = MvNormal(zeros(2),[sigma_u^2. rho_uv*sigma_u*sigma_v;rho_uv*sigma_u*sigma_v sigma_v^2])
    est_beta = zeros(B)
    for b = 1:B
        uv = rand(d1, T)
        u = uv[1,:]
        v = uv[2,:]
        x = zeros(T)
        r = zeros(T)
        for t = 1:T
            if t == 1
                x[t] = theta + rho*0.0 + v[t]
                r[t] = alpha + beta*0.0 + u[t]
            else
            x[t] = theta + rho*x[t-1] + v[t]
            r[t] = alpha + beta*x[t-1] + u[t]
            end
        end
        X = hcat(ones(T),x)
        estimator = (X'X)\(X'r)
        est_beta[b] = estimator[2]
    end
    beta_percentiles = [percentile(est_beta, 5),mean(est_beta),percentile(est_beta,95)]
    return beta_percentiles
end
```

```
┌ Info: Precompiling Plots [91a5bcdd-55d7-5caf-9e0b-520d859cae80]
└ @ Base loading.jl:1317
```

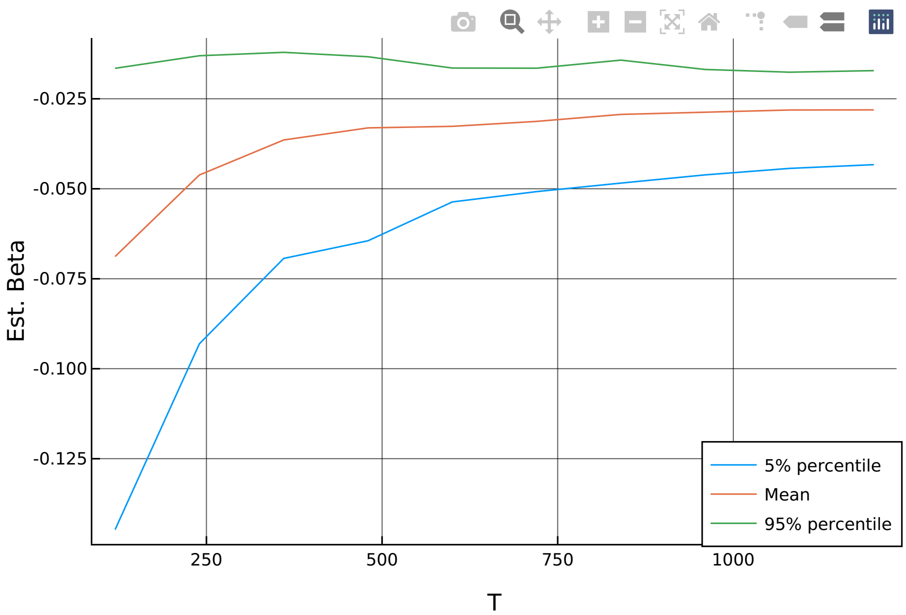
Out[1]: `Stambaugh (generic function with 1 method)`

Part 1: Finite Sample Bias

In [4]:

```
Ts = 120:120:1200
beta_plots = zeros(length(Ts),3)
for i=1:length(Ts)
    beta_plots[i,:]=Stambaugh(Ts[i],-0.8)
end
plotlyjs();
p1 = plot(xlabel="T",ylabel="Est. Beta",legend=:bottomright)
p1 = plot!(Ts,beta_plots[:,1],label = "5% percentile")
p1 = plot!(Ts,beta_plots[:,2],label = "Mean")
p1 = plot!(Ts,beta_plots[:,3],label = "95% percentile")
```

Out[4]:



The larger T becomes, the smaller bias is in absolute sence. The 95 percentile of the biases approaches to zero. However, the gain is not significant if we have more than 1000 observations and the estimator has negative bias.

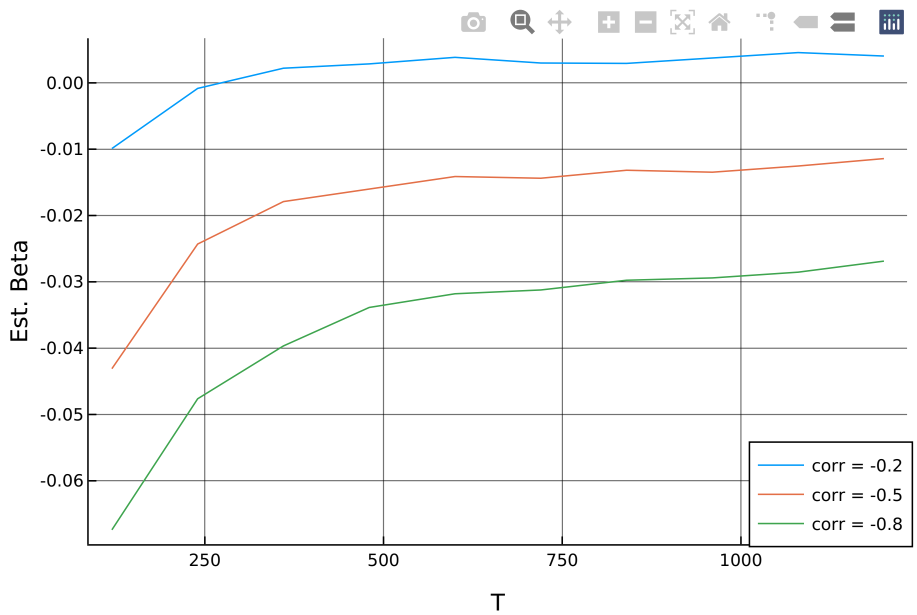
Part 2 - Effect of residual correlation

In [3]:

```
rho_uvs = [-0.2,-0.5,-0.8]
beta_plots_corr = zeros(length(Ts),length(rho_uvs))
for j = 1:length(rho_uvs)
    for i=1:length(Ts)
        beta_plots_corr[i,j]=Stambaugh(Ts[i],rho_uvs[j])[2]
    end
end

p2 = plot(xlabel="T",ylabel="Est. Beta",legend=:bottomright)
p2 = plot!(Ts,beta_plots_corr[:,1],label = "corr = -0.2")
p2 = plot!(Ts,beta_plots_corr[:,2],label = "corr = -0.5")
p2 = plot!(Ts,beta_plots_corr[:,3],label = "corr = -0.8")
```

Out[3]:



As the corrleation between the errors becomes stronger, the estimates have larger negative bias. Larger sample size mitigates the bias.

In [ ]: