G6021: Comparative Programming

Exercises on the λ-calculus

1. Insert all the missing parentheses and λ's into the following abbreviated λ-terms.

$$\begin{array}{llll} (i) & xx(xxx)x & (ii) & vw(\lambda xy.vx) \\ (iii) & (\lambda xy.x)xy & (iv) & w(\lambda xyz.xz(yz))uv \end{array}$$

Answer:

$$\begin{array}{llll} (i) & (((xx)((xx)x))x) & (ii) & ((vw)(\lambda x.(\lambda y.(vx)))) \\ (iii) & (((\lambda x.(\lambda y.x))x)y) & (iv) & (((w(\lambda x.(\lambda y.(\lambda z.((xz)(yz))))))u)v) \end{array}$$

2. Mark all the occurrences of xy in the following terms:

$$\begin{array}{llll} (i) & (\lambda xy.xy)xy & (ii) & (\lambda xy.xy)(xy) \\ (iii) & \lambda xy.xy(xy) & (iv) & (\lambda xy.x)yxy \end{array}$$

Answer:

$$\begin{array}{llll} (i) & (\lambda xy.\underline{xy})xy & (ii) & (\lambda xy.\underline{xy})(\underline{xy}) \\ (iii) & \lambda xy.\underline{xy}(\underline{xy}) & (iv) & (\lambda xy.x)yxy \end{array}$$

3. Do any of the terms in (1) or (2) contain any of the following terms as subterms? If so, which contains which?

$$\begin{array}{llll} (i) & \lambda y.xy & (ii) & y(xy) \\ (iii) & \lambda xy.x & (iv) & (\lambda zyz.xz)yz \end{array}$$

Answer: $\lambda y.xy$ is a subterm of $(\lambda xy.xy)xy$ (2(i) and 2(ii))
$\lambda xy.x$ is a subterm of $(\lambda xy.x)xy$ (1(iii))

4. Evaluate the following substitutions:

$$\begin{array}{llll} (i) & (x(\lambda y.yx))\{x \mapsto vw\} & (ii) & (x(\lambda x.yx))\{x \mapsto vw\} \\ (iii) & (x(\lambda y.yx))\{x \mapsto ux\} & (iv) & (x(\lambda y.yx))\{x \mapsto uy\} \end{array}$$

Answer:

$$\begin{array}{llll} (i) & ((vw)(\lambda y.y(vw))) & (ii) & ((vw)(\lambda x.yx)) \\ (iii) & ((ux)(\lambda y.y(ux))) & (iv) & ((uy)(\lambda z.z(uy))) \end{array}$$

5. Reduce the following terms to normal forms:

(i)	$(\lambda xy.xyy)uv$	(ii)	$(\lambda xy.yx)(uv)zw$
(iii)	$(\lambda xy.x)(\lambda x.x)$	(iv)	$(\lambda xyz.xz(yz))(\lambda uv.v)$

Answer:

(i)	uvv	(ii)	$z(uv)w$
(iii)	$(\lambda yx.x)$	(iv)	$\lambda yz.yz$

6. Let $I = \lambda x.x$ and $W = \lambda xy.xyy$. Reduce the following to normal form using any strategy.

(i)	WWW	(ii)	WII
(iii)	$W(II)I$	(iv)	$W(WI)$

Answer:

(i)	Does not terminate	(ii)	$WII \to III \to^* I$
(iii)	$W(II)I \to (II)II \to^* I$	(iv)	$W(WI) \to \lambda y.WIyy \to \lambda y.Iyyy \to \lambda y.yyy$