G6021: Comparative Programming

Exercises on the λ-calculus

1. Insert all the missing parentheses and λ's into the following abbreviated λ-terms.

$$\begin{array}{llll} (i) & xx(xxx)x & (ii) & vw(\lambda xy.vx) \\ (iii) & (\lambda xy.x)xy & (iv) & w(\lambda xyz.xz(yz))uv \end{array}$$

2. Mark all the occurrences of xy in the following terms:

$$\begin{array}{llll} (i) & (\lambda xy.xy)xy & (ii) & (\lambda xy.xy)(xy) \\ (iii) & \lambda xy.xy(xy) & (iv) & (\lambda xy.x)yxy \end{array}$$

3. Do any of the terms in (1) or (2) contain any of the following terms as subterms? If so, which contains which?

$$\begin{array}{llll} (i) & \lambda y.xy & (ii) & y(xy) \\ (iii) & \lambda xy.x & (iv) & (\lambda zyz.xz)yz \end{array}$$

4. Evaluate the following substitutions:

$$\begin{array}{llll} (i) & (x(\lambda y.yx))\{x \mapsto vw\} & (ii) & (x(\lambda x.yx))\{x \mapsto vw\} \\ (iii) & (x(\lambda y.yx))\{x \mapsto ux\} & (iv) & (x(\lambda y.yx))\{x \mapsto uy\} \end{array}$$

5. Reduce the following terms to normal forms:

$$\begin{array}{llll} (i) & (\lambda xy.xyy)uv & (ii) & (\lambda xy.yx)(uv)zw \\ (iii) & (\lambda xy.x)(\lambda x.x) & (iv) & (\lambda xyz.xz(yz))(\lambda uv.v) \end{array}$$

6. Let $I = \lambda x.x$ and $W = \lambda xy.xyy$. Reduce the following to normal form using any strategy.

$$\begin{array}{llll} (i) & WWW & (ii) & WII \\ (iii) & W(II)I & (iv) & W(WI) \end{array}$$