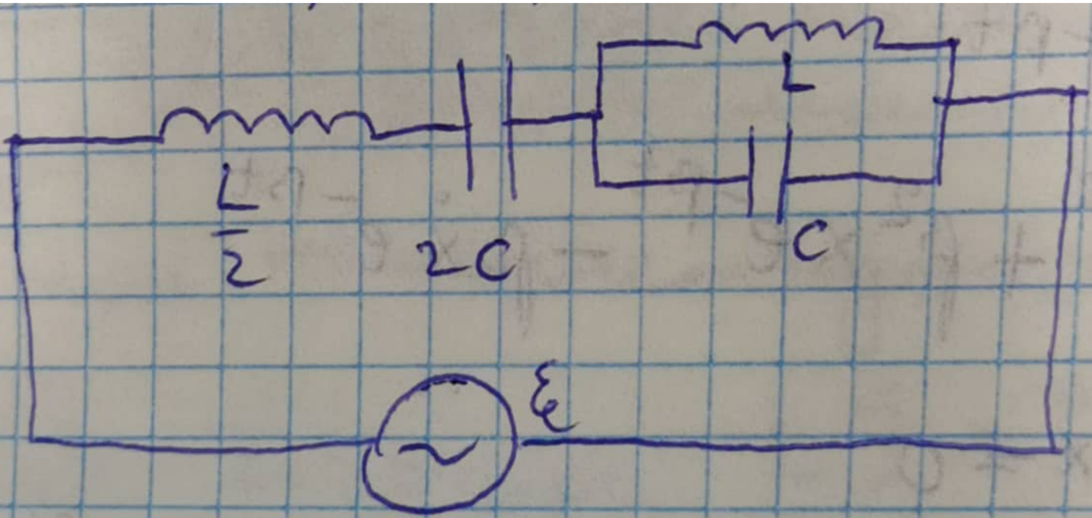
10.26



Импеданс цепи $\hat{Z} = i\omega\frac{L}{2} - i\frac{1}{\omega 2C} - i\frac{i\omega L \cdot \frac{1}{i\omega C}}{\omega L - \frac{1}{\omega C}} =$

$$= i\left(\frac{\omega^2 LC - 1}{2\omega C} - \frac{\omega L}{\omega^2 LC - 1}\right)$$

Резонанс напряжен. наступает при $\hat{Z} = 0$

$(\omega^2 LC - 1)^2 = 2\omega^2 LC$ обозн $\omega^2 = x$

$LC = a$

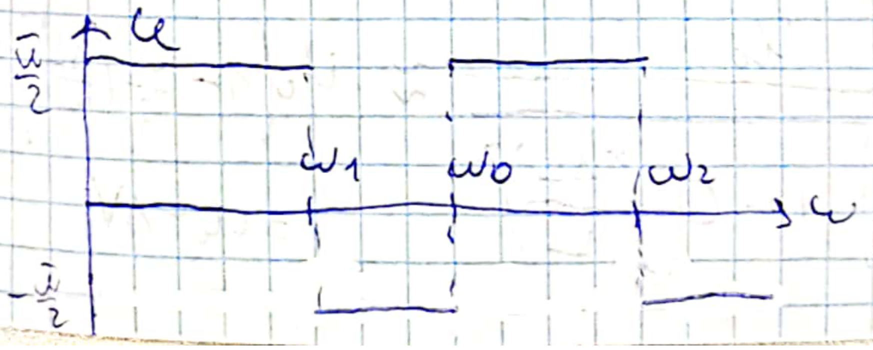
$x^2a^2 - 4ax + 1 = 0 \Rightarrow x = \frac{2}{a} \pm \sqrt{\frac{4}{a} - \frac{1}{a^2}} = \frac{2}{a} \pm \frac{\sqrt{3}}{a}$

$\omega_1 = \sqrt{\frac{2-\sqrt{3}}{LC}}$, $\omega_2 = \sqrt{\frac{2+\sqrt{3}}{LC}}$

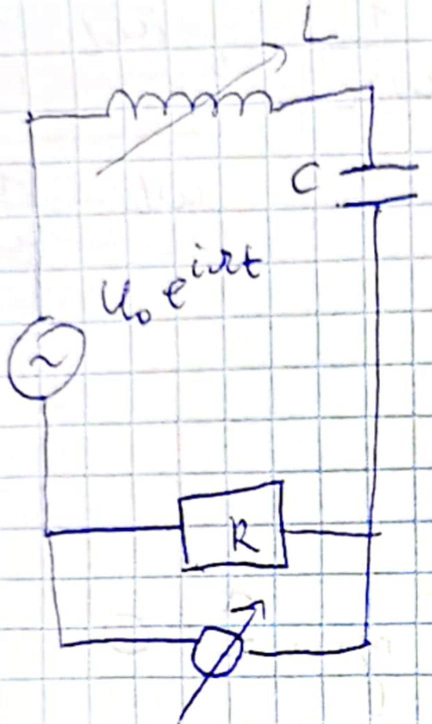
7A

Резонанс токов $Z^{-1} = 0$

$$\frac{2\omega C(\omega^2 LC - 1)}{(\omega^2 LC - 1) - 2\omega^2 LC} = 0 \Rightarrow \omega_0 = \frac{1}{LC}$$



(10.22)

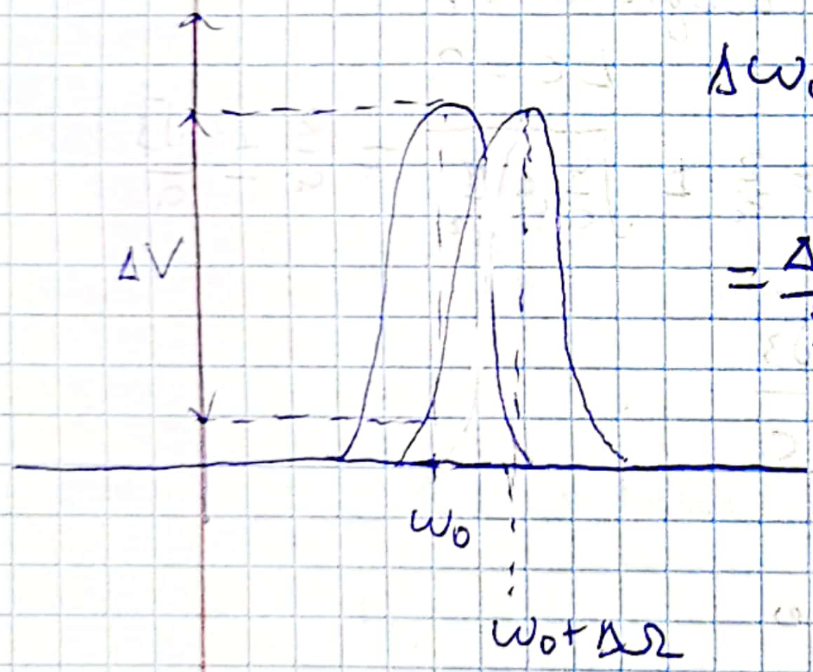


$U_0 = 100\,В$; $\Delta U_R = 10\,мкВ$

$Q = 100$

$$V = JR = \frac{U_0}{\sqrt{1 + Q^2\left(\frac{2\Delta\Omega}{\omega_0}\right)^2}}$$

Если $\Delta L \ll L$, то Q и ω_0 меняются мало



$$\Delta\omega_0 = \Delta\left(\frac{1}{\sqrt{LC}}\right) = \frac{1}{\sqrt{C}}\left(-\frac{1}{2}L^{-3/2}\right) = \frac{\Delta L}{2L}\omega_0$$

$$\frac{2\Delta\Omega}{\omega_0} = \frac{\Delta L}{L}$$

$$\Delta Q = \Delta\left(\frac{\omega_0 L}{R}\right) = \frac{1}{R}\Delta\left(\sqrt{\frac{L}{C}}\right) = \frac{1}{R\sqrt{C}}\frac{1}{2}L^{-\frac{1}{2}}\Delta L \cdot \frac{L}{L} = \frac{Q}{2}\cdot\frac{\Delta L}{L}$$

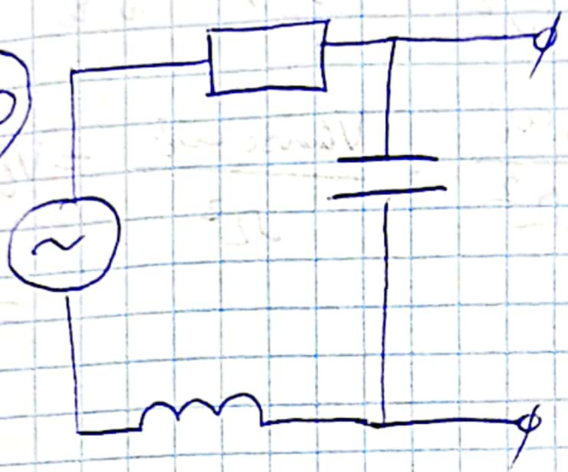
На резисторе $$V = \frac{U_0}{\sqrt{1 + Q^2\left(\frac{\Delta L}{L}\right)^2}} \approx U_0\left(1 - \frac{Q^2}{2}\left(\frac{\Delta L}{L}\right)^2\right) = U_0 - \Delta V$$

$$\Delta V = u_0 \frac{Q^2}{2}\left(\frac{\Delta L}{L}\right)^2 \Rightarrow \frac{\Delta V}{u_0}\cdot\frac{2}{Q^2} = \left(\frac{\Delta L}{L}\right)^2$$

$$\frac{\Delta L}{L} = \frac{1}{Q}\sqrt{\frac{2\Delta u}{u_0}} \sim 10^{-6}$$

Ответ: 10^{-6}

10.20



$f_0 = 1{,}6$ кГц – максимум напряжения

На $f \ll f_0$

$V_{вых_0} = 1$ В

$f_1 = 16$ кГц

$$\delta = \frac{R}{2L}$$

Амплитуда тока вблизи резонанса

$$J_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\Omega L - \frac{1}{\Omega C}\right)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2\frac{4L^2}{4L^2} + \frac{L^2}{\Omega^2}\left(\Omega^2 - \frac{1}{LC}\right)^2}} =$$

$$= \frac{\mathcal{E}_0}{L\sqrt{\underbrace{4\delta^2}_{\text{мало}} + \frac{1}{\Omega^2}(\Omega^2-\omega_0^2)^2}} \approx \frac{\mathcal{E}_0\,\Omega}{L\,|\Omega^2-\omega_0^2|}$$

На конденсаторе $V_0^{вых} = I_0 X_c = \frac{\mathcal{E}_0 \Omega}{\Omega L C|\Omega^2-\omega_0^2|} =$

$$= \frac{\mathcal{E}_0 \cdot \omega_0^2}{|\Omega^2-\omega_0^2|}$$

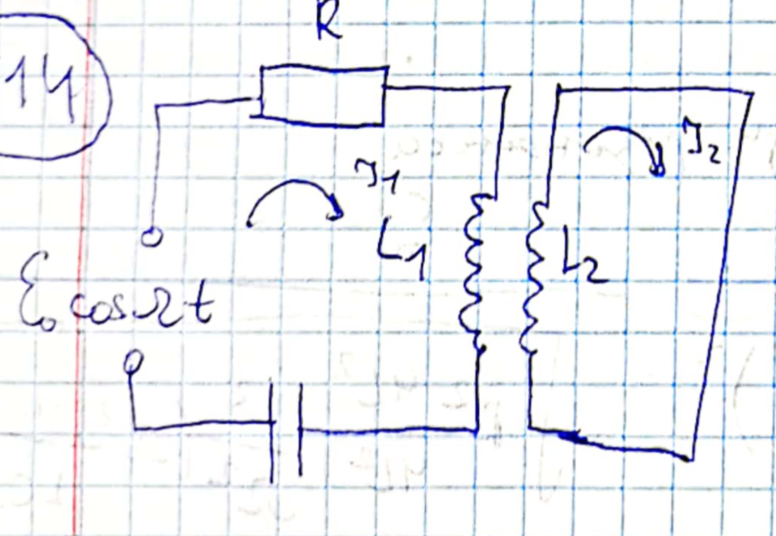
при $f \ll f_0$ $V^{вых} \approx \frac{\mathcal{E}_0 \cdot \omega_0^2}{\omega_0^2} \Rightarrow V^{вых} \approx \mathcal{E}_0$

при $f \gg f_0$ $V^{(1)}_{вых} \approx \frac{\mathcal{E}_0 \omega_0^2}{\Omega^2} = \frac{V_{вых}\, \omega_0^2}{\Omega^2} = V_{вых}\left(\frac{f_0}{f}\right)^2 =$

$= 10^{-2}$ В

Ответ: 10^{-2} В

T14



Правило Кирхгофа для колебательного контура

$$\frac{q}{C} + RI_1 = \mathcal{E}_0 \cos\Omega t - L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

Для второго контура

$$L\frac{dI_2}{dt} + M\frac{dI_1}{dt} = 0 \Rightarrow \frac{dI_2}{dt} = -\frac{M}{L}\frac{dI_1}{dt}$$

$\frac{q}{C} = \frac{1}{C}\int I_1 dt$. Перейдём в комплексную плоскость

$$\left(L_1 - \frac{M^2}{L_2}\right)\frac{d\hat{I}_1}{dt} + R\hat{I}_1 + \frac{1}{C}\int \hat{I}_1 dt = \mathcal{E}_0 e^{i\Omega t}$$

Найдём частное решение вида $\hat{I}_1 = I_{10} e^{i\Omega t}$

$$I_{10} i\Omega\left(L_1 - \frac{M^2}{L_2}\right) R I_{10} \frac{i}{\Omega C} I_{10} = \mathcal{E}_0$$

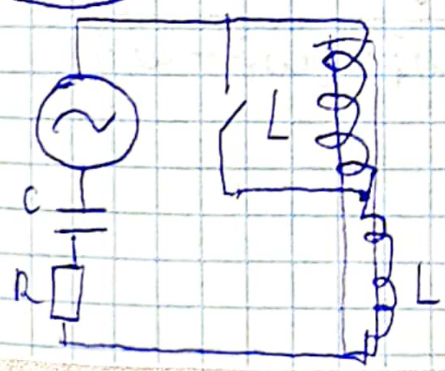
$$I_{10}\underbrace{\left(R + i\left(\Omega\left(L_1 - \frac{M^2}{L_2}\right) - \frac{1}{\Omega C}\right)\right)}_{\hat{Z}} = \mathcal{E}_0$$

$\hat{I}$ отстаёт по фазе на $\frac{\pi}{4}$ $\Rightarrow$ действительная часть импеданса = мнимой

т.е. $R = \Omega\left(L_1 - \frac{M^2}{L_2}\right) - \frac{1}{\Omega C} \Rightarrow \frac{R + \frac{1}{\Omega C}}{\Omega} = L_1 - \frac{M^2}{L_2}$

$$\frac{M^2}{L_2} = L_1 - \frac{R + \frac{1}{\Omega C}}{\Omega} \Rightarrow M = \sqrt{L_2\left(L_1 - \frac{R + \frac{1}{\Omega C}}{\Omega}\right)} \approx 5 \text{ мГн}$$

10.82



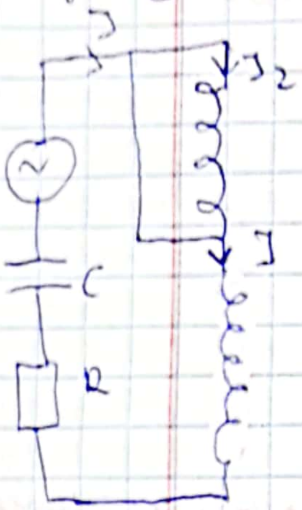
$$Q_1 = \frac{1}{\omega_1 CR} = \frac{1}{2\pi f_1 CR} \approx 89$$

$$Q_2 = \frac{1}{\omega_2 CR} = \frac{1}{2\pi f_2 CR} \approx 79$$

При разомкнутом ключе $\Phi_1 = L_1 J = LJ - MJ + LJ - MJ$
$= 2(L-M)J \Rightarrow L_1 = 2(L-M)$

$$L_1 = \frac{1}{\omega_1^2 C} \Rightarrow M = L - \frac{1}{2\omega_1^2 C}$$

При замкнутом ключе



$$\begin{cases} RJ + \frac{1}{C}\int J\,dt = -L\frac{dJ}{dt} - M\frac{dJ_2}{dt} + \mathcal{E}_0 \cos\Omega t \\ L\frac{dJ_2}{dt} + M\frac{dJ}{dt} = 0 \end{cases}$$

$$\Downarrow$$

$$\left(L - \frac{M^2}{L}\right)\frac{dJ}{dt} + RJ + \frac{1}{C}\int J\,dt = \mathcal{E}_0 \cos\Omega t$$

$$\frac{1}{\omega_2^2 C} = L - \frac{M^2}{L} = L_2$$

$$L^2 - \frac{L}{\omega_2^2 C} = M^2 = L^2 - \frac{1}{4\omega_1^4 C^2} - \frac{L}{\omega_1^2 C}$$

$$L = \frac{\omega_1^2\omega_2^2}{4\omega_1^4 C(\omega_2^2 - \omega_1^2)} = \frac{f_2^2}{16\pi^2 f_1^2 (f_2^2 - f_1^2) C} = 0{,}1 \text{ Гн}$$

$$M = \frac{\omega_2^2}{4\omega_1^2 C(\omega_2^2 - \omega_1^2)} - \frac{1}{2\omega_1^2 C} = \frac{2\omega^2 - \omega_2^2}{4\omega_1^2 C(\omega_2^2 - \omega_1^2)} =$$

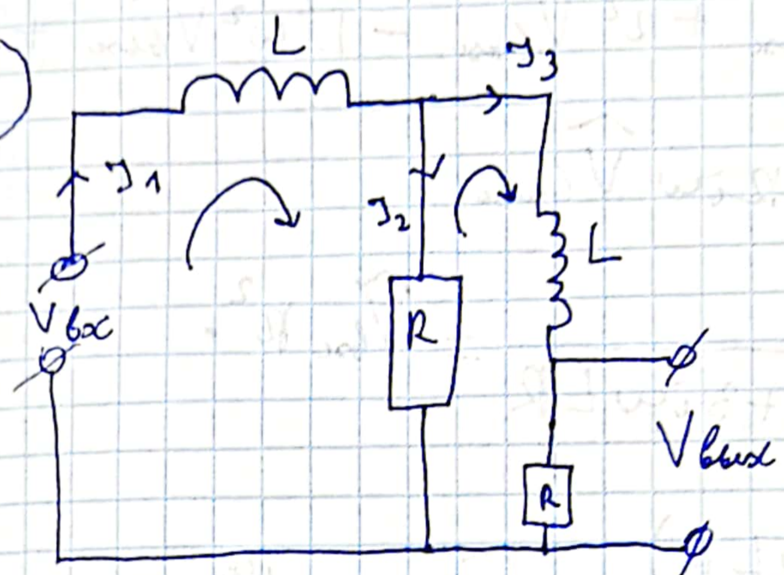
$$= \frac{2f_1^2 - f_2^2}{16\pi^2 f_1^2 C(f_2^2 - f_1^2)} = 0{,}06 \text{ Гн}$$

$L_1 = 0{,}08$ Гн $\quad L_2 \approx 0{,}06$ Гн

Ответ: $L \approx 0{,}1$ Гн; $M \approx 0{,}06$ Гн; $L_1 \approx 0{,}07$ Гн

$L_2 \approx 0{,}06$ Гн

10.6



Правило Кирхгофа

$$V_{вх0} \cos\omega t = R I_2 + L \frac{dI_1}{dt}$$

$$\hat{V}_{вх} = R\hat{I}_2 + L\frac{d\hat{I}_1}{dt} \qquad V_{вх0}\, e^{i\omega t} = R\hat{I}_2 + L\frac{d\hat{I}_1}{dt}$$

$$V_{вых} = R I_3 \qquad I_1 = I_2 + I_3$$

$$-R I_2 + L\frac{dI_3}{dt} + R I_3 = 0$$

$$\frac{d\hat{V}_{вых}}{dt} = i\omega \hat{V}_{вых} \qquad \frac{d\hat{I}_3}{dt} = \frac{1}{R}\frac{d\hat{V}_{вых}}{dt} = \frac{1}{R} i\omega \hat{V}_{вых}$$

$$\hat{I}_2 = \frac{L\frac{d\hat{I}_3}{dt} + \hat{V}_{вых}}{R} = \left(\frac{L}{R^2} i\omega + \frac{1}{R}\right)\hat{V}_{вых}$$

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$$\hat V_{\text{вх}} = R\hat I_2 + L\frac{d\hat I_2}{dt} + L\frac{d\hat I_3}{dt} = \left(\frac{L}{R} i\omega + 1\right)\hat V_{\text{вых}} +$$

$$+ \left(\frac{L^2}{R^2} i\omega + \frac{L}{R}\right) i\omega \hat V_{\text{вых}} + \frac{L}{R} i\omega \hat V_{\text{вых}}$$

$$\hat V_{\text{вх}} R^2 = LRi\omega \hat V_{\text{вых}} + R^2 \hat V_{\text{вых}} - L^2\omega^2 \hat V_{\text{вых}} +$$

$$+ LRi\omega \hat V_{\text{вых}} + LRi\omega \hat V_{\text{вых}}$$

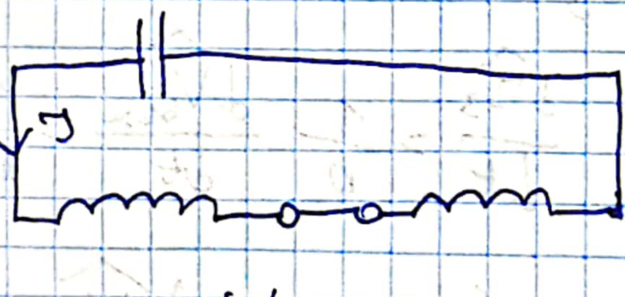
$$\hat V_{\text{вых}} = \frac{\hat V_{\text{вх}} R^2}{R^2-\omega^2L^2+3i\omega LR} = \hat V_{\text{вх}} R^2 \cdot$$

$$\cdot \frac{(R^2-\omega^2L^2-3i\omega LR)}{(R^2-\omega^2L^2-9\omega^2L^2R^2)} = \hat V_{\text{вх}}\,\rho\, e^{i\varphi}$$

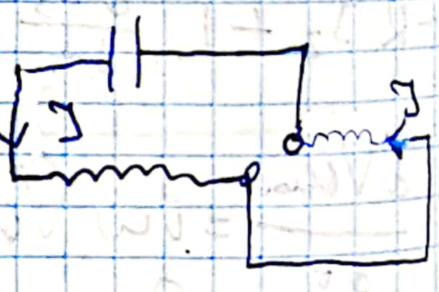
$$\cos\varphi = \frac{\qquad}{R^2-\omega^2L^2} = \cos\frac{\pi}{2} = 0 \Rightarrow R=\omega L$$

Ответ: $R=\omega L$

10.40



ω_1



ω_2

Для первого случая результирующая индуктивность $= L_1$, для второго $= L_2$

Пусть в некоторый момент ток через контур $= J$

1й случай

$$\Phi = L_1 J = LJ + MJ + LJ + MJ \Rightarrow L_1 = 2(L+M)$$

2й случай

$$\Phi = L_2 J = LJ - MJ + LJ - MJ \Rightarrow L_2 = 2(L-M)$$

$$\omega_1 = \frac{1}{\sqrt{L_1 C}} \qquad \omega_2 = \frac{1}{\sqrt{L_2 C}}$$

$$\begin{cases} \dfrac{1}{2\omega_1^2 C} = L+M \\ \dfrac{1}{2\omega_2^2 C} = L-M \end{cases} \Rightarrow \begin{cases} L = \dfrac{1}{4\omega_1^2 C} + \dfrac{1}{4\omega_2^2 C} \\ M = \dfrac{1}{4\omega_1^2 C} - \dfrac{1}{4\omega_2^2 C} \end{cases}$$

Ответ: $L = \dfrac{1}{4C}\left(\dfrac{1}{\omega_1^2} + \dfrac{1}{\omega_2^2}\right)$

$M = \dfrac{1}{4C}\left(\dfrac{1}{\omega_1^2} - \dfrac{1}{\omega_2^2}\right)$

10.92

$\Delta\omega = \dfrac{r}{L} = 20 \qquad V_{max} = 100\,B \quad \left(\begin{matrix}\Delta\omega \text{ изм. на} \\ V_{max}/\sqrt{2} \approx 70\end{matrix}\right)$

$$L = \frac{r}{\Delta\omega} = 0{,}1 \text{ Гн} \qquad \left(= \sqrt{\omega_0^2 - 2\gamma^2}\right)$$

$$\omega_{max} = 1000 = \sqrt{\frac{1}{LC} - \frac{r^2}{2L^2}} = \frac{1}{L}\sqrt{\frac{L}{C} - \frac{r^2}{2}}$$

$$\omega_{max}^2 L^2 = \frac{L}{C} - \frac{r^2}{2} \Rightarrow \omega_{max}^2 L^2 + \frac{r^2}{2} = \frac{L}{C}$$

$$C = \frac{L}{\omega_{max}^2 L^2 + \frac{r^2}{2}} \approx 10 \text{ мкФ}$$

$$Q = \frac{1}{r}\sqrt{\frac{L}{C}} = 50, \quad \delta = \frac{\pi}{Q} \approx 0{,}06$$

$$\mathcal{E}_e = V_{max}/Q = 2 \text{ В}$$

$$R_{кр} = 2\sqrt{\frac{L}{C}} = 200 \text{ Ом} \gg r$$

Ответ: $L = 0{,}1$ Гн, $C = 10$ мкФ, $Q = 50$, $\delta \approx 0{,}06$

$R_{кр} = 200$ Ом, $\mathcal{E}_e = 2$ В