Experiment: Constant-Volume Gas Thermometer

Goals

- Conduct an experiment using a MEMS (microelectromechanical system) gas pressure sensor, one example of the many kinds of solid state sensors that now populate so many products (e.g., smart-phones, automobiles).
- Determine the value of “absolute zero” using an ideal gas.
- Use Python’s, or a similar scripting language’s, built-in linear regression to analyse data and extract a physically measured quantity, including its measurement uncertainty.

Personal Protective Equipment & Safety

This lab involves a Bunsen burner and non-potable water. The Physics Undergraduate Labs (UGL) will supply personal protective equipment. The following safety rules will be in effect for this lab:

- No food or drink will be allowed in the labs.
- Wear long pants, a dress, or the equivalent.
- Wear close-toed shoes.
- Use the UGL-supplied safety goggles and lab coat, or your own.
- Do not drink or ingest either the water or ice used in this lab.
- Be aware of the burner and potentially hot liquids.
- Do not add or remove water/ice from the canister when the temperature is above 60 C.

Required, Suggested, & Optional Equipment for Students

- Lab Book #2 & Pens (Required)
- Lab Coat (Required); Either UGL-Supplied or Student-Owned
- Safety Goggles (Required); Either UGL-Supplied or Student-Owned
- Ruler (Suggested)
- USB key (Suggested)
- Laptop (Suggested)

1. Background

Ideal gases

There is always a well-defined relationship between the thermodynamic variables of a gas: pressure, temperature, and volume, regardless of its chemical composition. When the density of a gas is relatively low, this relationship can be described by the ideal gas law. The equation of state for an ideal gas is:

$$PV = nRT, \tag{1}$$

where P is the pressure in N/m^2 (also known as Pascals, Pa) exerted by a confined gas, V is the volume in m^3 occupied by the confined gas, n is the number of moles of the gas (1 mole = 6.022×10^{23} molecules), R is the universal gas constant, 8.314 J/mol·K, which is the same for all gases, and T is the absolute temperature in units of Kelvin, K (Recall: 0°C = 273.15 K).

While the ideal gas law is an idealized model that simplifies the behaviour of real gases, it is a good representation of most gases at high temperatures and low pressures. An ideal gas can be considered to be composed of particles that undergo only elastic billiard-ball collisions, and are otherwise non-interacting. This is a reasonable approximation for low-density gases due to the lack of intermolecular attraction between the particles in the gas. However, all gases eventually deviate from the ideal gas law at relatively low temperatures or high pressures, where intermolecular attraction becomes significant.

The ideal gas law emerges from microscopic behaviour (as you may someday see in a statistical mechanics course), when the number density — the number of gas particles per unit volume (N/V) — is constant. The kinetic energy of the gas' atoms and molecules determines the temperature, and the rate of momentum exchange in collisions determines the pressure of the gas (on the walls of a container). At constant density (N/V), both temperature and pressure are proportional to the square of the average gas-particle speed, yielding a direct linear proportionality between pressure and temperature. By measuring this linear relationship over a range of pressures/temperatures, the value of *absolute zero* can be determined through extrapolation.

Measuring pressure

Pressure gauges come in two basic varieties: those that measure absolute pressure and those that measure relative pressure. Absolute pressure is measured against a reference of zero pressure, whereas relative pressure gauges use atmospheric pressure as a reference and require a correction to calculate absolute pressure. Although one may prefer an absolute pressure gauge, relative pressure gauges are cheaper and more compact.

To determine the absolute pressure from a relative pressure gauge (or sensor), the measurement P_{gauge} needs to be adjusted using the atmospheric pressure, P_{atm}. The atmospheric pressure must be recorded at the same time and conditions that relative pressure measurements are being taken — it is ideal to check the atmospheric pressure periodically over the course of an experiment as the atmospheric pressure may drift. Then one can calculate the absolute pressure P from

$$P = P_{\text{rel}} + P_{\text{atm}}, \tag{2}$$

which can be used in Equation 1.

This experiment uses a relative pressure gauge, an "TruStability Board Mount Pressure Sensors Part number: SSCDRRN005PD2A5" from Honeywell, which is rated to measure relative pressures P_{rel} between ± 5 psi

(psi = "pound-force per square inch — how could you convert this to kPa?). Although it is often advertised as having a 0.25% accuracy, the full datasheet indicates the sensor has a full rate measurement error maximum of 2%. This sensor measures the relative pressure between a top port and a bottom port. In this experiment, one port will be connected to your apparatus and the other left at atmosphere (P_{atm}).

This type of microelectromechanical system (MEMS) pressure sensor uses a thin membrane suspended across supports to measure the pressure differential on the two sides of the membrane, as in Fig. 1. The strain caused by the deflection of the membrane is measured using piezoelectric materials deposited on the membrane. Piezoelectrics are crystals that produce an electric field when strained; this electric field is converted to a voltage that can be measured by the computer in our devices. In this device, this information is digitized and passed to the computer as a digital (I2C) signal; the Raspberry Pi computer (see addendum for more information on the Raspberry Pis) uses a Python program to convert the digitized voltage to a relative pressure (in kPa) for you.

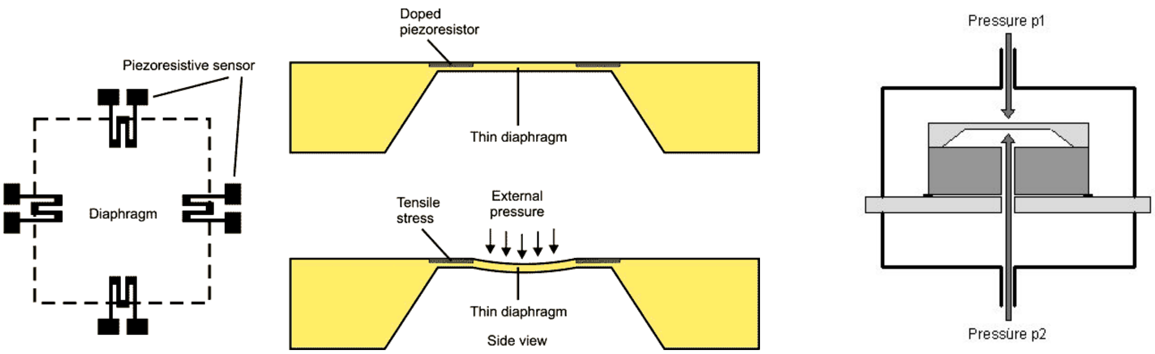

Figure 1: *Left/Center*: The MEMS pressure sensor detects the tensile stress on a thin silicon membrane that changes the piezo-resistance of elements on the membrane's surface. *Right:* The differential sensor senses the deflection of the membrane as a result of different pressures on either side. Images courtesy: https://www.radiolocman.com/review/article.html?di=148185 (accessed 30 August 2023) and https://www.first-sensor.com/en/products/pressure-sensors/pressure-sensors-and-transmitters/pressure-types.html (accessed via the Wayback Machine, 30 August 2023)

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In contrast, pressure gauges were originally built as liquid-filled devices, taking advantage of the property of liquids to resist compression. A manometer, a liquid-based pressure gauge, typically uses a set of U-shaped (glass) tubing to hold the liquid, with one end open to the atmosphere and one end connected to the pressure source needing to be measured. As pressure is applied to the tubing the liquid rises or falls on the side open to the atmosphere, its resulting height directly measuring the amount of pressure applied to the tube. See Figure 2 for an example of a mercury manometer.

Manometers were frequently made using mercury; its very high density (13.5 g/cm^3) limited the height of tubing needed to measure low pressures (as in near-vacuum), while its low vapour pressure meant negligible contribution from the mercury to the pressure in evacuated volumes at low pressures. A closed-tube version of such a manometer – often called a barometer – is provided to measure the atmospheric pressure during the experiment. Using this barometer, the pressure is measured in millimetres of mercury, mmHg, where the standard atmosphere $\equiv 760\,\text{mmHg} = 1.01325 \times 10^5\,\text{Pa}$ at $T = 0\,\text{C} = +273.15\,\text{K}$.

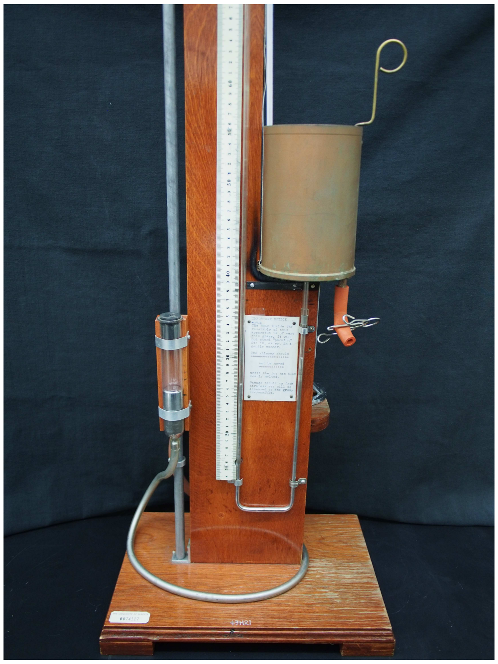

Figure 2: Original version of the experiment apparatus.

Figure 3: Image of the glass bulb within the copper can mounted on a retort stand.

2. Methods

Apparatus

The apparatus being used is the remnant of a mercury manometer equipped with a air-filled glass bulb placed inside a copper canister (see Figures 3 and 4). In the new version of the experiment, the glass bulb is connected via vinyl tubing to a pressure sensor, sealing the air within to create a constant volume system. Heating or cooling of the bulb changes the pressure exerted by the gas on the sensor, registering as a voltage signal recorded by a Raspberry Pi.

The temperature of the bulb is regulated by a water bath in the copper canister, submerging both the bulb and the D-shaped hollow handle (see Figure 5). This approach allows for even heating of both the water and the bulb by placing a Bunsen burner beneath the handle of the canister. The inner wall of the canister also has a stirrer and thermometer attached (Figure 4); the stirrer can be used to ensure even heating in the water bath.

Procedure

1. Measure and record the atmospheric pressure P_{atm} using the barometer in CCIS L2-010. Ensure the barometer is zeroed before recording a measurement. Take several atmospheric pressure measurements over the course of the experiment. Make sure to estimate the uncertainty of your measurements.

   Note: To zero the barometer, adjust the height of the mercury in the bottom well until it just touches the pointer tip visible inside. The height of the mercury can be changed by turning the thumb screw behind the well.

Figure 4: Side view of the copper can showing the stirrer handle.

Figure 5: Correct placement of the Bunsen burner under the can arm.

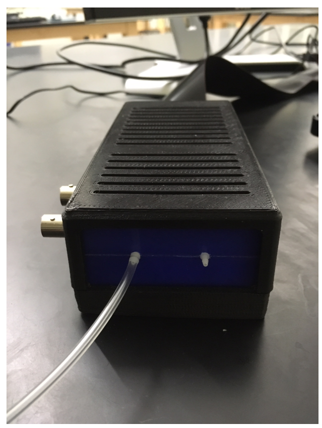

Figure 6: Differential pressure sensor showing the correct connection to the positive pressure sensor port

2. Before adding water to the copper canister, measure and record the gas pressure in the bulb and the temperature of the room temperature using the thermometer in the canister.

   To take a pressure measurement using the Differential Pressure sensor connected to the Raspberry Pi computer:

   (a) Make sure the power strip is turned on, and the Raspberry Pi and monitor power cords are plugged in.

   (b) Ensure that the tubing is connected both to the glass bulb and to the pressure sensor. The positive port of the pressure sensor is the plastic extension closest to the coaxial connectors.

   (c) On the Raspberry Pi computer, open the **Differential Pressure.py** script found in the **CVGT** folder on the Desktop by double-clicking on the icon, then run the script by pressing F5. If preferred, the script can also be run by opening an LXTerminal window using the monitor icon beside the *Raspberry* menu and entering:

   "python /home/pi/Desktop/CVGT/Differential\Pressure.py"

(d) In the graphing window, check the *Save Data* button when thermal equilibrium is established to collect pressure measurements. These pressures are calculated from raw data into units of kPa. The values from this file can be averaged at a later time to find the average pressure over this time. The program will collect a fixed number of data points (25 is the default) each time you click the "Save Data" button.

Note: The file created will contain the number of points specified. The files will be saved sequentially since the opening of the program in a folder called **data** with filenames **pressure_###.csv**. It is strongly suggested that you intelligently rename the files at each temperature measurement point to avoid this. Record the file names and the parameter settings (temperature, any other relevant information) at the time for which each set of data is saved. While any method for saving data you choose is acceptable, make sure to record in your lab notebook the file names and parameters associated with all the data points in your saved files.

3. Pour water into the canister such that the bulb and handle are submerged, leaving room to later add ice. Allow a few minutes for the gas in the bulb to reach the temperature of the water, stirring occasionally to keep the temperature uniform. Once the system has appeared to reach a steady state, record both the water temperature and the gas pressure.

4. Measurements can be performed over a range of temperatures from room temperature to about 80°C, and down to freezing if ice is available. You should make measurements for at least ten separate temperature points. Choose these "target" temperature points for your measurements.

5. If ice is available, add the ice first. Each time you reach one of your desired temperatures, acquire 25 pressure measurements by clicking the "Save Data" button.

6. If either ice is unavailable, or after the iced water has reached a stable temperature, heat the canister by placing a Bunsen burner below the arm. Each time you reach one of your desired temperatures, acquire 25 pressure measurements by clicking the "Save Data" button.

7. Once the maximum temperature has been reached allow the water to cool to room temperature. You should record more measurements at your target temperatures as the system cools.

8. For all your measurements, consider how you will estimate the uncertainty in all measured quantities and record these in your lab book.

9. When the experiment is done, drain as much water from the apparatus as possible into one of the provided copper vessels. Also please ensure that the gas supplying the Bunsen burner is turned off (please close both the needle valve on the bottom of the burner, and the handle on the gas supply tap at back of the bench).

3. Analysis

Lab Book and Lab Report Evaluation

For this lab, you will be completing the full analysis in your lab book and this will be evaluated under the standard “Lab Book” rubric. You will also be completing a “Half Lab Report” that will be evaluated according to the following rubric (out of 40 points total):

- Cover Page [Title, Names, Course Section, **No Abstract**] (/2 points)
- Results and Analysis [incl. Figures and possibly Tables, as in Lab Reporting rubric] (/16 points)
- Discussion [as in standard Lab Report rubric] (/8 points)
- Conclusion [as in standard Lab Report rubric] (/4 points)
- References [as in standard Lab Report rubric] (/2 points)
- Acknowledgements [as in standard Lab Report rubric] (/2 points)
- Appendix [sample calculations, etc.; as in Lab Reporting rubric] (/2 points)
- Formatting, Neatness, Overall Presentation (/4 points)

The heart of the Half Report – Results and Analysis (including Figures and Tables), Discussion, Conclusion, References, Acknowledgements – should be *no more* than 6 pages in length. **Graphs must be prepared using Python or equivalent, and include error bars. Cover Page and Appendix are not included in the 6-page length limit.**

Important Notes: Read carefully the Lab Book Guidelines to include In-Lab and Post-Lab Notes. Make sure you add your observations about the experiment and discussion/conclusion requested for each tasks. In addition, don’t forget to include setup detail including diagram (with appropriate and clear labels/captions), uncertainties including apparatus uncertainty and justifications.

Required Analysis to be included

1. As we saw above, a linear relationship exists between temperature and pressure. Formulate an equation in the form:

$$y = mx + b, \tag{3}$$

   but replace each of the variables in this equation with the appropriate physical variables, including the measured variables (pressure and temperature). Given your choice of x and y, identify the independent and dependent variable, and show how the slope and/or the intercept from a fit to your data will let you find the value of absolute zero (in Celsius). Show all the details of this analysis in your Lab Book (before, during, or after the lab period) and include the final results in your report. Consider carefully the units of the variables in this equation.

2. Analyse the data you saved from your pressure sensor at each temperature point. Determine the best estimate of temperature and pressure at each measurement point, and the uncertainty associated with each of these points. Present a table of this in, at least, your Lab Book.

3. Using a linear fit to your data (you may use the built-in linear regression package in Python, for example), find your experimentally determined value for absolute zero from the fit parameters, including the appropriate uncertainty. Compare this value to the accepted value of -273.15°C. Include all uncertainties as error bars in your plot of the independent versus the dependent variable. Discuss the sources of error and improvements to be made to reduce the errors. Results of this analysis should be in both your Lab Book and Lab Report.

Required additional discussion points to include in the lab report

- Why is it important that the volume of the gas in this apparatus remains constant? How true is that assumption in this experiment?
- Predict what would occur to a gas if both its temperature and volume are kept constant while the atmospheric temperature P_a changes.
- Evaluate whether mercury vapour in the manometer is a significant source of error in the measurement of absolute pressure.

  Note: Mercury liquid is slightly volatile at room temperature, causing it to vapourize in the manometer bulb. The mercury vapour adds $1.85 \times 10^{-4}\,\mathrm{mmHg}$ of pressure at 0°C and $0.27\,\mathrm{mmHg}$ at 100°C.
- Justify whether the height of the meniscus in the manometer is a significant source of error in the measurement of absolute pressure.

  Note: The internal diameter of the glass, the purity of the mercury, and the cleanliness of the glass can all affect the height of the meniscus.