Experiment: Electron Charge-to-Mass Ratio

Goals

- Develop an appreciation for Thomson's method to discover the elementary charge-to-mass ratio.
- Explore the interaction between magnetic fields and electric charges.
- Create and execute a procedure to determine the background magnetic field of Earth.

Personal Protective Equipment & Safety

In addition to the standard safety rules for Second-year Physics Lab Courses, this lab involves electronic equipment with cable connections that you will set up. The Physics Undergraduate Labs (UGL) will supply personal protective equipment upon request. The following safety rules will be in effect for this lab:

- No food or drink will be allowed in the labs.
- Wear long pants, a dress, or the equivalent.
- Wear close-toed shoes.
- If you would like safety goggles or a lab coat, request them from your TA or bring your own.
- Always turn power supplies off before setting up new electronic circuits.

Required, Suggested, & Optional Equipment for Students

- Lab Book #2 & Pens (Required)
- Ruler (Suggested)
- Lab Coat (Optional); Either UGL-Supplied or Student-Owned
- Safety Goggles (Optional); Either UGL-Supplied or Student-Owned
- USB key (Optional)
- Laptop (Optional)

Background

In this experiment J.J. Thomson's original determination of the charge-to-mass ratio of an electron will be replicated using an "e/m tube" and Helmholtz coil assembly manufactured by the Welch Scientific Company. A hot filament will be used to liberate electrons bound to a metal, allowing an electric field to then accelerate the electrons through a potential difference. Subsequent application of a magnetic field by the Helmholtz coils will cause the electrons to be deflected into a circular orbit. By measuring the accelerating potential of the electric field, the intensity of the magnetic field, and the radius of the electrons' resulting orbit, it is possible to compute the velocity of the electrons, as well as their charge-to-mass ratio.

This approach allows for the further determination of two physical values: the ionization potential of mercury gas and the local background magnetic field of Earth. Mercury gas fluoresces when a current of sufficient energy is passed through it: collisions occur between the gas and the current, ionizing the mercury's constituent electrons. Any ionized electron rebonds with a mercury ion, releasing its excess energy as light — this is a physical example of orbital theory, one of the first aspects of quantum theory to be explored. The use of the electromagnetic Helmholtz coils also makes it possible to obtain an approximate value of Earth's magnetic field in the laboratory, the presence of which must be accounted for to properly analyze the experiment.

Theory

When a stream of electrons is accelerated through a potential difference V, the maximum kinetic energy each electron gains is given by:

$$\frac{1}{2}mv^2 = eV, \tag{1}$$

where m is the mass of the electron, v is the final velocity of the electrons, and e is the magnitude of the charge of the electron.

When a particle carrying a charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$, it is acted upon by the Lorentz force $\vec{F}$, given by:

$$\vec{F} = q(\vec{v} \times \vec{B}). \tag{2}$$

The unit of magnetic field in the *SI* system of units is the Weber/m^2 (Wb/ m^2), also known as the Tesla (T). If $\vec{B}$ and $\vec{v}$ are at right angles, the magnitude of the force simplifies to $F = qvB$ and the particle moves in a circular path perpendicular to both the magnetic field and its velocity (see Figure 1). The radius of this circle is such that the required centripetal force is supplied by the Lorentz force, meaning that for the electrons we get:

$$\frac{mv^2}{r} = evB \tag{3}$$

where r is the radius of their circular path.

Substituting the electron velocity v from Equation 1 into Equation 3 obtains an expression for the charge-to-mass ratio of an electron:

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{4}$$

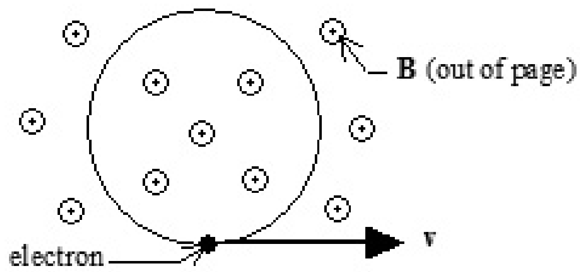


Figure 1: Resulting circular path of electrons exposed to a magnetic field.

In this experiment, the magnetic field is produced by a current in a pair of Helmholtz coils. The magnetic field inside the coils is fairly uniform, the magnitude of which is given by:

$$B_H = \frac{8\mu N I}{\sqrt{125}R}, \tag{5}$$

where N is the number of turns in each coil, R is the radius of the coil, I is the current through the wire and $\mu_0 = 4\pi \times 10^{-7}$ T·m/A is the permeability of free space. The direction of the field is shown in Figure 2.

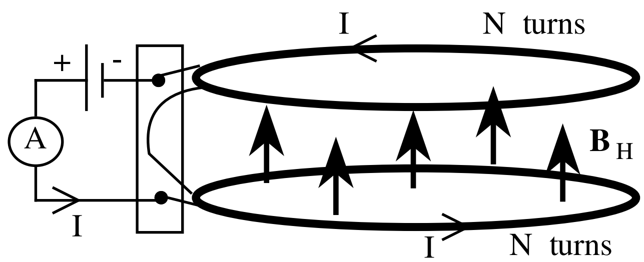


Figure 2: Magnetic field lines inside a Helmholtz coil.

Earth's magnetic field is strong enough to cause an appreciable deflection of the electron beam. It can be observed that the path of the electrons bends away from the crossbar with no Helmholtz field present, as shown in Figure 3. Thus, it is necessary to account for the effect of Earth's magnetic field, B_E, on the path of the electrons.

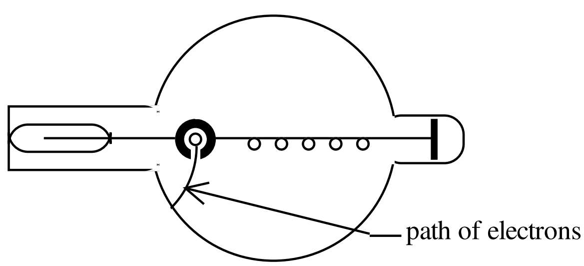


Figure 3: Top view showing the path of the electrons with no Helmholtz field applied.

The method used to account for Earth's magnetic field in this experiment is to align the apparatus so that the variable Helmholtz field is anti-parallel to Earth's magnetic field. Utilizing this approach the total magnetic field present along the vertical axis of the Helmholtz coil is $(B_H - B_E)$, changing equation Equation 4 into

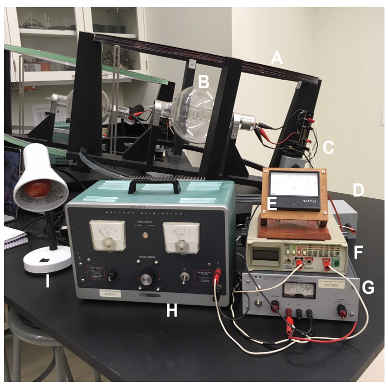
$$\frac{e}{m} = \frac{2V}{(B_H - B_E)^2 r^2}, \tag{6}$$

which can be used to find the charge-to-mass ratio of an electron in this apparatus.

Methods

Apparatus

The entire apparatus for Steps 1–4 of the Procedure for this experiment is illustrated in Figure 4. In Step 5 of the Procedure, you will add components to this setup.



A) Helmholtz: Helmholtz Coils
B) Helmholtz: "e/m" Tube
C) Filament: Brightness Control
D) Filament: 6.3 V AC Transformer
E) Whole System: Analog Voltmeter
F) Whole System: Digital Ammeter
G) Anode: 50 V Power Supply
H) Helmholtz: Power Supply
I) Red Light

Figure 4: An overview picture of the entire set of devices for the Steps 1–4 in the Procedures for this experiment.

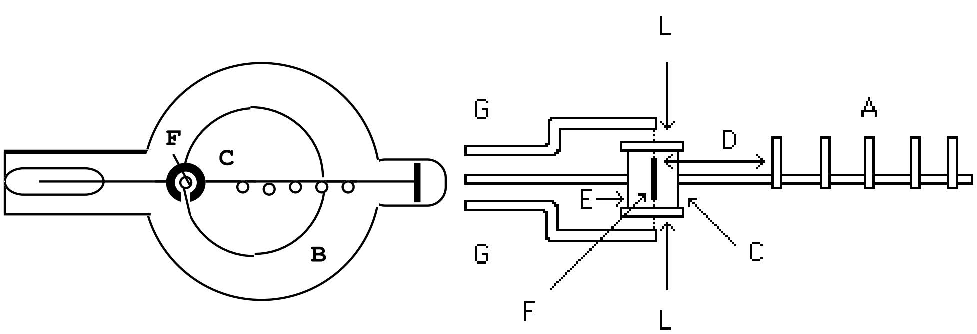


Figure 5: (a) A sectional view of the tube and filament assembly

(b) A detailed section of the filament at right angles to Figure 5a.

Legend: A) Cross bars on a stiff wire B) Typical path of beam of electrons C) Cylindrical anode D) Distance from filament to far side of each of the crossbars E) Lead wire supporting anode F) Filament G) Lead wires and supports for filament L) Insulating plugs

The beam of electrons is produced by an electron gun composed of a straight filament surrounded by a coaxial anode containing a single axial slit as shown in Figure 5. Referencing Figure 5, electrons emitted from the heated filament F are accelerated by the potential difference applied between F and the anode C. Some of the electrons emerge as a narrow beam through the slit in the side of C.

A small amount of mercury vapour is sealed within the tube to allow for visualization of the beam. Whenever an accelerated electron ionizes the mercury it bonds to a free electron in the gas, emitting the characteristic mercury arc-colour, pale blue light. Since the recombination of the mercury ion with an electron occurs quickly it also occurs very near the point where the ionization originally took place. Thus, the blue light serves as an accurate indicator of the path of the electron beam. Furthermore, as the electron beam is barely affected by the collision with the mercury gas, the blue light serves as an even better indicator; the beam is barely affect by the mercury, as its low concentration ensures that only a small fraction of the electron beam loses energy to the ionization process.

The Helmholtz coils are used to provide the magnetic field that bends the electron beam as their shape and positioning produce an approximately uniform magnetic field within their circumference. Each coil consists of 72 turns of copper wire with a radius of 0.33 m. The coils can further be used to affect the radius of the beam. Increasing the current in the coils increases the strength of the magnetic field they apply, which in turn decreases the radius of the beam. Proper adjustment of either the current applied to the Helmholtz coils or the accelerating voltage applied to the beam can make the electron beam bend to coincide with any one of the five bars secured inside the tube, the distances of which are listed in the following table:

Cross Bar Number	**Distance from Filament To Far Side of Crossbar** [m]
1	0.065 ± 0.001
2	0.078 ± 0.001
3	0.090 ± 0.001
4	0.103 ± 0.001
5	0.115 ± 0.001

Procedure

1. In Edmonton, Earth's magnetic field B_E points downward making an angle of approximately $72°$ with the horizontal. To account for this background magnetic field, the Helmholtz apparatus should be orientated so that the compass on its base points to magnetic North with the plane of the apparatus tilted upward at an angle of $18°$ relative to horizontal North, as displayed in Figure 6.

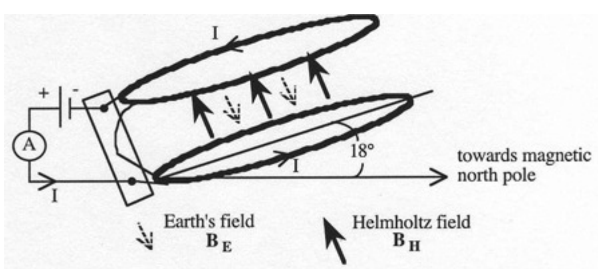


Figure 6: Orientation of the Helmholtz coil relative to magnetic north.

2. The e/m apparatus involves three separate electrical circuits as shown in Figure 6 and Figure 7. Construct the circuits according to the outlines below:

   ◇ Filament Circuit: A 6.3 volt AC transformer is connected to the filament in order to heat it. Check that the filament is on by noting a white glow inside the circular anode. Its brightness can be varied by using the rheostat knob mounted on the frame of the apparatus.

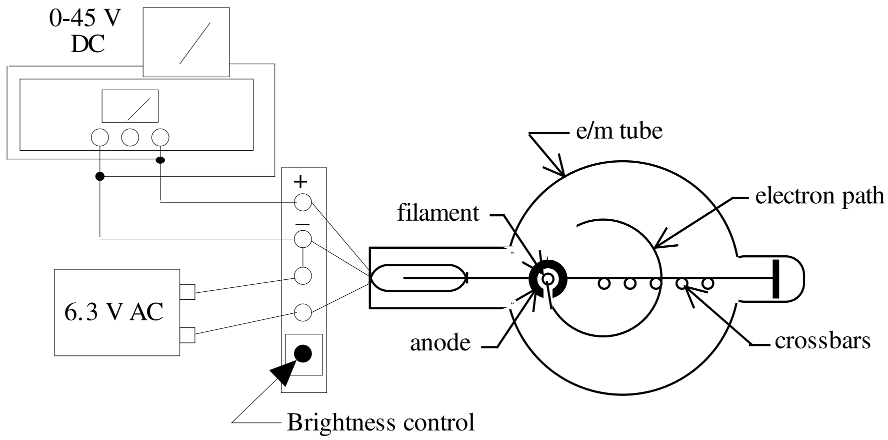


Figure 7: A block wiring diagram for the Helmholtz tube.

   ◇ Anode Circuit: A 50 V DC power supply is connected to the anode terminals to establish the accelerating voltage. This voltage is measured with a voltmeter and should be kept below 45 V. As the power supply is slowly turned on, a bluish-colored beam is observed at ≈12 V emerging from the slit within the anode.

   ◇ Helmholtz Coil Circuit: A DC power supply and ammeter are connected in series with the Helmholtz coils. The current in the coils can be varied to bend the electron beam so it coincides with a given crossbar. The current should be kept less than 5 A at all times.

3. Set the accelerating voltage to 20 V. Measure the Helmholtz current I required to make the circular beam of electrons coincide with each peg on the crossbar (it is easiest to view the beam from overhead and adjust the brightness for optimum sharpness). Consider the uncertainty in your ability to align the beam with the pegs for the analysis.

4. Repeat Steps 1 – 3 while increasing the accelerating voltage, up to a maximum of 45 V.

5. Equation 2 implies that the electron beam will not bend when a magnetic field is absent. Create and conduct a procedure to directly measure the magnitude of Earth's magnetic field by adjusting the magnetic field of the Helmholtz coil. Several rheostats (adjustable resistors) will be available to connect in series to the Helmholtz coil circuit.

Analysis

Lab Book and Lab Report Evaluation

For this lab you will be completing full analysis in your lab book, where this will be evaluated under the standard "Lab Book" rubric. You will also be completing a "Half Lab Report" that will be evaluated according to the following rubric (out of 40 points, total):

- Cover Page [Title, Names, Course Section, Abstract [as in standard Lab Report rubric] (/4 points)
- Introduction [as in Lab Reporting rubric] (/8 points)
- Theory [as in Lab Reporting rubric] (/8 points)
- Experimental Details [as in Lab Reporting rubric] (/12 points)
- Images and Diagrams (evaluated with the appropriate sections)
- References [as in Lab Reporting rubric] (/2 points)
- Acknowledgements [as in Lab Reporting rubric] (/2 points)
- Formatting, Neatness, Overall Presentation (/4 points)

The heart of the Half Report – Introduction, Theory, Experimental Details, References, and Acknowledgements – should be *no more* than 6 pages in length. **The Cover Page is not included in the 6-page length limit.**

Important Notes: Read carefully the Lab Book Guidelines to include In-Lab and Post-Lab Notes. Make sure you add your observations about the experiment and discussion/conclusion requested for each tasks. In addition, don't forget to include setup detail including diagram (with appropriate and clear labels/captions), uncertainties including apparatus uncertainty and justifications. The Post-Lab Notes is mandatory, make sure to include print-outs of your plot, any data analysis, and your final results in the lab book.

Required Analysis in the Post-Lab Section of the Lab Book

1. Record the directions of $\vec{v}$, $\vec{B}$, $\vec{F}$ in a sketch similar to Figure 7, using $\odot$ to indicate **up** and $\otimes$ to indicate **down** vectors. Determine if the direction of the Lorentz force for an electron is correctly given by $\vec{F} = q(\vec{v} \times \vec{B})$. Discuss the effect on the Lorentz force from rotating the electron beam horizontally or vertically relative $\vec{B}$.
2. Clearly explain the procedure used to obtain your measurement of Earth's field using the Helmholtz coils and state the value for B_E it yielded. Record this value in your lab book and clearly label/box this value. Compare the experimentally determined value of B_E to the accepted value of $4.8 \pm 0.3 \times 10^{-5}$ T previously measured on campus.
3. Using graphing techniques with Equation 5 and Equation 6 to obtain values for the charge-to-mass ratio of an electron, e/m, as well as Earth's magnetic field, B_E. Include the final graphs used in your lab book. Record these in your lab book and clearly label/box these values. Compare your value of e/m to the accepted value of 1.76×10^{11}C/kg and B_E to $4.8 \pm 0.3 \times 10^{-5}$ T.

   Hint: It is simplest to generate the relation to graph by isolating the Helmholtz coil current I.

Required additional derivations and discussion points to include in the Post-Lab section of the Lab Book

- Use Equation 1 and Equation 2 to find the base units for Volts and Teslas. Also use equation Equation 4 and only base units to confirm that the units for e/m in *SI* are C/kg.
- Derive Equation 6 using equations Equation 1, and Equation 3, and Equation 4.
- The Helmholtz coil was orientated such that it was anti-parallel to Earth's magnetic field. Briefly discuss why this orientation was an important part of the procedure, considering the result of the coil being misaligned.
- In deriving an expression for e/m, it has been assumed that (a) the final velocity of the electrons is small compared with that of light, i.e., no relativistic corrections are necessary, and (b) the thermal kinetic energy of electrons coming from the hot filament is negligible compared with the kinetic energy given to the electrons by the accelerating field between filament and anode.
  - Briefly discuss whether assumption (a) is justified given an electron's speed as provided by the accelerating potential.
  - Regarding assumption (b), we note that, according to the Maxwell-Boltzmann distribution, the number of electrons emitted from the filament with speed between v and $v + dv$ is proportional to $v^2 dv \cdot e^{\frac{-mv^2}{2kT}}$, where k is Boltzmann's constant and T is the (absolute) temperature of the filament. It can then be shown that the average energy of the electron is $3kT/2$. Assuming a filament temperature of 2000°K, this implies that each electron has an average energy of about 0.27 eV before it begins to be accelerated by the potential applied between filament and anode. Briefly discuss if this thermal energy is negligible compared to the accelerating potential.

Supplementary Comment(s)

The action of the e/m tube and the Helmholtz coils demonstrates the principle underlying the mass spectrometer. Particles of equivalent charge but differing in mass move in orbits of different radii in a given magnetic field. Thus, it is possible to separate ions of different atomic mass in a magnetic field. Indeed, the first evidence that all chlorine atoms were not identical was obtained in this way, showing that chlorine consisted of three-quarters ^{35}Cl and one-quarter ^{37}Cl.